# May 2004 #3

mass $m$, energy $E$, $\vec{k}_i = k\hat{z}$, $E = \frac{\hbar^2 k^2}{2m}$

$$V = \begin{cases} v & \text{if } |x| \le L,\ |y| \le L,\ |z| \le L \\ 0 & \text{else} \end{cases}$$

$v$ is small, constant energy

a. Use Born Approximation to find $\frac{d\sigma}{d\Omega}(\theta,\phi)$

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2, \quad \text{where}$$

$$f(\theta,\phi) = -\frac{2m}{4\pi\hbar^2}\int e^{-ik\hat{r}\cdot\vec{r}\,'} V(\vec{r}\,')\, e^{ik\hat{z}\cdot\vec{r}\,'}\, d^3\vec{r}\,'$$

The integral should be performed in Cartesian Coords

$$\hat{z}\cdot\vec{r}\,' = z'$$

$$\hat{r}\cdot\vec{r}\,' = \hat{r}\cdot(x'\hat{x} + y'\hat{y} + z'\hat{z})$$

$$= x'\sin\theta\cos\phi + y'\sin\theta\sin\phi + z'\cos\theta$$

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\, v\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} dx'dy'dz'\; e^{ikz'(1-\cos\theta)}\, e^{-ikx'\sin\theta\cos\phi}\, e^{-iky'\sin\theta\sin\phi}$$

z integral: $\frac{1}{ik(1-\cos\theta)}\left(e^{ikL(1-\cos\theta)} - e^{-ikL(1-\cos\theta)}\right) = \frac{2\sin[kL(1-\cos\theta)]}{k(1-\cos\theta)}$

x integral: $1-\cos\theta \to -\sin\theta\cos\phi$ : $\frac{2\sin[kL\sin\theta\cos\phi]}{k\sin\theta\cos\phi}$

y integral: $\frac{2\sin[kL\sin\theta\sin\phi]}{k\sin\theta\sin\phi}$

$$f(\theta,\phi) = -\frac{4mv}{\pi\hbar^2}\,\frac{\sin[kL(1-\cos\theta)]\sin[kL\sin\theta\cos\phi]\sin[kL\sin\theta\sin\phi]}{k^3\sin^2\theta(1-\cos\theta)\sin\phi\cos\phi}$$

$k^6 = \frac{8m^3E^3}{\hbar^6}$

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 = \frac{2\hbar^2 v^2}{m\pi^2E^3}\,\frac{\sin^2[kL(1-\cos\theta)]\sin^2[kL\sin\theta\cos\phi]\sin^2[kL\sin\theta\sin\phi]}{(1-\cos\theta)^2\sin^4\theta\sin^2\phi\cos^2\phi}$$

b. Valid when $v \ll \frac{\hbar^2}{mL^2}$

[at low energies, $k \sim \frac{1}{L}$ $\quad E \gg v \to \frac{\hbar^2k^2}{2m} \gg v \to \frac{\hbar^2}{mL^2} \gg v$]