2002 Part II Q6

Beam of ions, constant density, velocity v

radial electric field: $\nabla \cdot \vec{E} = 4\pi\rho = 4\pi Zen$

$$E \cdot 2\pi r L = 4\pi Zen \cdot \pi r^2 L \Rightarrow E = 2\pi r Zen \qquad r < r_b \text{ beam radius}$$

azimuthal magnetic field: $\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} = \frac{4\pi}{c} nZe v \hat{z}$

$$B_\phi \cdot 2\pi r = \frac{4\pi}{c} nZeV \cdot \pi r^2 \Rightarrow B_\phi = 2\pi r Zen \frac{v}{c}$$

Force Balance on a beam particle at radius r

$$m\frac{d^2r}{dt^2} = Ze\left(2\pi r Zen - 2\pi r Zen \frac{v^2}{c^2}\right)$$

radial force vanishes when $v \to c$