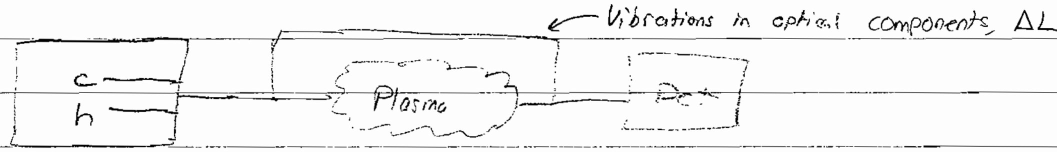
2004 Part 1 Q1 Exp.

a. $\lambda_c = 10.6\ \mu m$ $\qquad \omega_c = 1.78 \cdot 10^{14}\ s^{-1}$

$\lambda_h = 0.633\ \mu m$ $\qquad \omega_h = 2.98 \cdot 10^{15}\ s^{-1}$



$$\Delta\phi_c = \int (k_{0c} - k_c)\,dx + k_{0c}\Delta L$$

$$\Delta\phi_h = \int (k_{0h} - k_h)\,dx + k_{0h}\Delta L$$

$n^2 = 1 - \frac{\omega_{pe}^2}{\omega^2}$ $\qquad k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)$ $\qquad k = \frac{\omega}{c}\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2} \approx \frac{\omega}{c}\left(1 - \frac{\omega_{pe}^2}{2\omega^2}\right)$

$$\Delta\phi_c = \frac{\omega_c}{c}\int \frac{\omega_{pe}^2}{2\omega_c^2}dx + \frac{\omega_c}{c}\Delta L = \frac{1}{2c\omega_c}\int \omega_{pe}^2 dx + \frac{\omega_c}{c}\Delta L$$

$$\Delta\phi_h = \frac{\omega_h}{c}\int \frac{\omega_{pe}^2}{2\omega_h^2}dx + \frac{\omega_h}{c}\Delta L = \frac{1}{2c\omega_h}\int \omega_{pe}^2 dx + \frac{\omega_h}{c}\Delta L$$

$\omega_{pe}^2 = \frac{4\pi e^2}{m_e} \circ n_e$ (proportional to n_e)

eliminate $\frac{\Delta L}{c}$ using $\Delta\phi_h$: $\quad \frac{\Delta L}{c} = \frac{\Delta\phi_h}{\omega_h} - \frac{1}{2c\omega_h^2}\int \omega_{pe}^2 dx$

$$\Rightarrow \Delta\phi_c = \frac{1}{2c\omega_c}\int \omega_{pe}^2 dx + \frac{\omega_c}{\omega_h}\Delta\phi_h - \frac{\omega_c}{2c\omega_h^2}\int \omega_{pe}^2 dx$$

$$= \frac{\omega_c}{\omega_h}\Delta\phi_h + \frac{1}{2c\omega_c}\left(1 - \frac{\omega_c^2}{\omega_h^2}\right)\int \omega_{pe}^2 dx$$

$$\Rightarrow \int \omega_{pe}^2 dx = 2c\omega_c\left(\frac{\Delta\phi_c - \frac{\omega_c}{\omega_h}\Delta\phi_h}{1 - \frac{\omega_c^2}{\omega_h^2}}\right)$$

$$\int n\,dl = \frac{\frac{2c\omega_c m_e}{4\pi e^2}\left(\Delta\phi_c - \frac{\omega_c}{\omega_h}\Delta\phi_h\right)}{1 - \frac{\omega_c^2}{\omega_h^2}}$$

b. ϕ_h uncertainty $= \pm\pi$

$\Rightarrow \int n\,d\ell$ uncertainty $= \dfrac{2cw_c m_c}{4\pi e^2}\dfrac{w_c}{w_h}\pi = 6.3\cdot10^{18}\ m^{-2}$

$= 6.3\cdot10^{14}\ cm^{-2}$

c. If $\ell = 1\ m$, $n = 10^{20}$

$\int n\,d\ell = 10^{20}\ m^{-3}$

Fractional error $= \dfrac{6.3\cdot10^{18}}{10^{20}} - 6.3\cdot10^{-2} = 6.3\%$