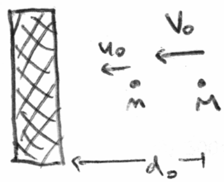
5/04 CM #1 Erik Granstedt



Motion constrained in 1-D
All collisions are elastic
$m \ll M$, $u_0 \gg V_0$

Physically, this is like "Fermi acceleration" - the small mass will be accelerated as its "walls" close in on it.

- Since $M \gg m$, with each collision, the small mass will be almost perfectly reflected, while the large mass will slow a little bit.
- The hard way to do this problem would be to solve for the change in velocity of each mass after each collision, and try to add them up in some sort of series. However, since the masses are very different in size, we can take advantage of the fact that the cannonical momentum of the small mass ~~times~~ the distance between the wall and large mass is an adiabatic invariant. This is justifiable since $V_0 \ll u_0$, the large mass moves only a very small distance between subsequent collisions.

$$S = \oint p\,dq = mud = \text{const}$$

$$\Rightarrow ud = u_0 d_0 \Rightarrow u(t) = \frac{u_0 d_0}{d(t)}$$

conservation of energy: $\frac{1}{2}mu_0^2 + \frac{1}{2}MV_0^2 = \frac{1}{2}mu^2 + \frac{1}{2}MV^2 \rightarrow V^2 = V_0^2 + \mu(u_0^2 - u^2)$

$$\rightarrow V^2 = V_0^2 + \mu u_0^2\left(1 - \left(\frac{d_0}{d}\right)^2\right) \qquad \mu = \frac{m}{M}$$

The large mass turns around when $V=0$. $\Rightarrow \frac{V_0^2}{\mu u_0^2} = \frac{d_0^2}{d_{min}^2} - 1$

$$\Rightarrow d_{min} = \frac{d_0}{\sqrt{\frac{V_0^2}{\mu u_0^2} + 1}}$$

Distance traveled $= d_0 - d_{min} = d_0\left(1 - \frac{1}{\sqrt{\frac{V_0^2}{\mu u_0^2} + 1}}\right)$