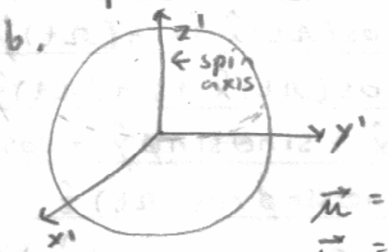
Seth Dorfman Prelims December 31, 2005

May 1999 EM

1) a. The dipole moment of the sphere precesses around the magnetic field at an angle α. The resulting nonzero $\ddot{\vec{m}}$ gives rise to magnetic dipole radiation

b.



$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

$$V = ws = wr\sin\theta$$

$$\vec{J}(r) = \rho\vec{V} = \frac{3Q}{4\pi R^3} wr\sin\theta\,\hat{\phi}$$

$$\vec{m} = \frac{1}{2c}\int \vec{r}\times J(\vec{r})\, d^3r$$

$$\vec{m} = \frac{1}{2c}\int \frac{3Q}{4\pi R^3} wr^2\sin\theta\,(\hat{r}\times\hat{\phi})\, d^3r$$

$$\vec{m} = \frac{w}{2c}\frac{3Q}{4\pi R^3}\int r^2\sin\theta(-\hat{\theta})\, d^3r$$

$$\vec{m} = \frac{w}{2c}\frac{3Q}{4\pi R^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} -r^2\sin\theta(\cos\theta\cos\phi\,\hat{x}' + \cos\theta\sin\phi\,\hat{y}' - \sin\theta\,\hat{z}')\, r^2\sin\theta\, d\theta\, d\phi$$

$\int_0^{\pi}\sin^2\theta\cos\theta\, d\theta = 0 \Rightarrow$ no $\hat{x}'$ and $\hat{y}'$ components

$$\vec{m} = \frac{w}{2c}\frac{3Q}{4\pi R^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^4\sin^3\theta\, d\theta\, d\phi\,\hat{z}'$$

$$= \frac{w}{2c}\frac{3Q}{4\pi R^3}\, 2\pi\frac{R^5}{5}\int_0^{\pi}(\cos^2\theta - 1)\, d(\cos\theta)\,\hat{z}'$$

$$= \frac{w}{c}\frac{3Q}{20}R^2\left(\frac{\cos^3\theta}{3} - \cos\theta\Big|_0^{\pi}\right)\hat{z}'$$

$$= \frac{w}{c}\frac{3Q}{20}R^2\cdot\frac{4}{3}\hat{z}'$$

$$\vec{m} = \frac{1}{5}\frac{w}{c}QR^2\hat{z}' \qquad \hat{z}' = \cos\alpha\,\hat{z} + \sin\alpha[\cos(\Omega t)\hat{x} + \sin(\Omega t)\hat{y}]$$

$$\vec{m} = \frac{1}{5}\frac{w}{c}QR^2[\cos\alpha\,\hat{z} + \sin\alpha(\cos(\Omega t)\hat{x} + \sin(\Omega t)\hat{y})]$$

Ω = frequency of precession

$$\vec{L} = \int\vec{r}\times\vec{p}\, d^3r$$

$$= \int\vec{r}\times(\rho_m s w\hat{\phi})\, d^3r$$

$$= \rho_M w\int\vec{r}\times(r\sin\theta)\hat{\phi}\, d^3r$$

$$= \rho_M w\int r^2\sin\theta\,(\hat{r}\times\hat{\phi})\, d^3r$$

$$= \rho_M w\, 2\pi\frac{R^5}{5}\frac{4}{3}\hat{z}'$$

$$= \frac{M}{\frac{4}{3}\pi R^3} w\, 2\pi\frac{R^5}{5}\frac{4}{3}\hat{z}'$$

$$\vec{L} = \frac{2}{5}wMR^2\hat{z}'$$

$$\frac{d\vec{L}}{dt} = \vec{m}\times\vec{B}$$

$$\frac{2}{5}wMR^2\frac{d\hat{z}'}{dt} = \frac{1}{5}\frac{w}{c}QR^2\hat{z}'\times B\hat{z}$$

$$2M\frac{d\hat{z}'}{dt} = \frac{Q}{c}B(\hat{z}'\times\hat{z})$$

$$\frac{\partial \hat{z}'}{\partial t} = \Omega \sin\alpha \left(-\sin(\Omega t)\hat{x} + \cos(\Omega t)\hat{y}\right) = \frac{QB}{2Mc}\left(\hat{z}' \times \hat{z}\right)$$

$$\hat{z}' \times \hat{z} = \sin\alpha\left(\sin(\Omega t)\hat{x} - \cos(\Omega t)\hat{y}\right)$$

$$\Rightarrow \Omega = \frac{QB}{2Mc}$$

$$c\,\frac{2\pi}{\lambda} = \frac{Q}{M}\left(\frac{B}{2c}\right)$$

$$\frac{Q}{M} = \frac{2c}{B}\cdot c \cdot \frac{2\pi}{\lambda}$$

$$\frac{Q}{M} = \frac{4\pi c^2}{\lambda B}$$

c. $$\ddot{\vec{m}} = \frac{1}{5}\frac{W}{c}QR^2\left[-\Omega^2 \sin\alpha\left(\cos(\Omega t)\hat{x} + \sin(\Omega t)\hat{y}\right)\right]$$

$$\ddot{\vec{m}} \times \hat{r} = -\frac{1}{5}\frac{W}{c}QR^2\Omega^2 \sin\alpha\left[\cos(\Omega t)\hat{x} + \sin(\Omega t)\hat{y}\right] \times \left[\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}\right]$$

$$\ddot{\vec{m}} \times \hat{r} = -\frac{1}{5}\frac{W}{c}QR^2\Omega^2\sin\alpha\left[\sin\theta\sin\phi\cos(\Omega t)\,\hat{z} - \cos\theta\cos(\Omega t)\hat{y} - \sin\theta\cos\phi\sin(\Omega t)\hat{z} + \cos\theta\sin(\Omega t)\hat{x}\right]$$

$$\ddot{\vec{m}} \times \hat{r} = \frac{1}{5}\frac{W}{c}QR^2\Omega^2\sin\alpha\left[\sin\theta\sin(\Omega t - \phi)\,\hat{z} + \cos\theta\left(-\sin(\Omega t)\hat{x} + \cos(\Omega t)\hat{y}\right)\right]$$

$$\vec{B}_{rad} = \frac{1}{cr}\ddot{\vec{m}}\left(t - \frac{r}{c}\right) \times \hat{r}$$

If viewed from $\theta = 0$ or $\theta = \pi$ the polarization is circular. From other angles, the polarization is elliptical.