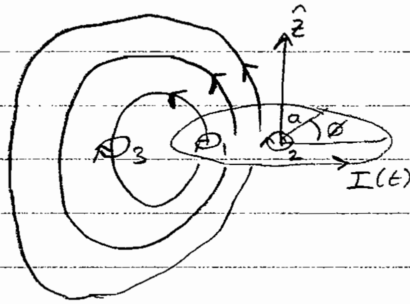
2003 Part II Q6 GPP I



As drawn, particles are moving in left-handed cyclotron orbits (ions)

initially, zero z-directed velocity

Particles are at $z=0$

a. Drifts: ∇B drift $\vec{V}_D = \dfrac{\vec{F}\times\vec{B}}{qB^2}$ $\vec{F} = -\mu\nabla B$ (MKS)

curvature drift: $\vec{F} = \dfrac{mV_{\parallel}^2}{r}\hat{r}$ but $V_{\parallel}=0$

$$\vec{V}_D = \frac{-\mu\nabla B\times\vec{B}}{qB^2} = \frac{-E}{qB^3}\nabla B\times\vec{B} = \frac{E}{qB^3}\vec{B}\times\nabla B$$

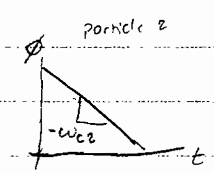
For particle 2, $(\nabla B)_r = 0$ no drift motion, only cyclotron

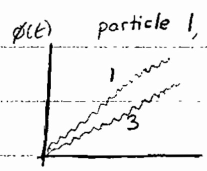
For particle 1, $\nabla B \sim \hat{r}$ (points outward, towards the wire), $\vec{B}\sim\hat{z}$

$V_{D1} \sim \hat{z}\times\hat{r} \sim \hat{\phi}$

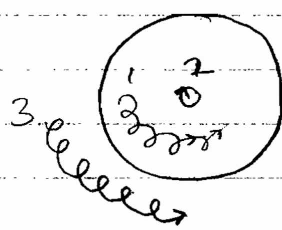
For particle 3, $\nabla B \sim -\hat{r}$ (points inward), $\vec{B}\sim -\hat{z}$

$V_{D3} \sim -\hat{z}\times -\hat{r} \sim \hat{\phi}$





[cannot say which is faster without knowing radius, + calculating $B_z(r)$ from Biot-Savart]



b. particle 2: $\phi_2 = -\omega_{c2}t + \phi_{20}$

particle 1: $\phi_1 = \dfrac{E\,\nabla B_1}{qB_1^2 r_1}t + \phi_{10} = \dfrac{1}{2}\dfrac{V_\perp^2}{\omega_{c2}}\dfrac{(\nabla B)_1}{B_1 r_1}t + \phi_{10}$

particle 3: $\phi_3 = \dfrac{E\,(\nabla B)_3}{qB_3^2 r_3}t + \phi_{30} = \dfrac{1}{2}\dfrac{V_\perp^2}{\omega_{c3}}\dfrac{(\nabla B)_3}{B_3 r_3}t + \phi_{30}$

c. I increases slowly with time → B increases slowly with time

- Gyro motion speeds up for both particles 2,3
- Conservation of μ: Energy of each particle increases
  - Drift of particle 3: $V_D \sim \frac{-\mu \nabla B \times B}{qB^2}$ drift speed does not change
  - Larmor radius of each particle decreases

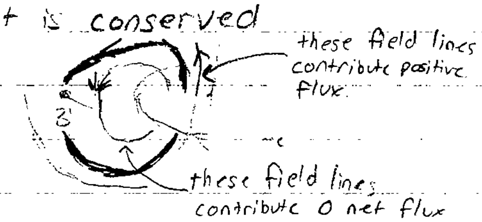
    If B increases by a factor α, then $W_\perp$ increases by a factor α,

    $V_\perp$ increases by $\sqrt{\alpha}$

    $\Rightarrow r_L = \frac{V_\perp}{\omega_c} \sim \frac{1}{\sqrt{\alpha}}$

- Conservation of the third adiabatic invariant ψ
  - Flux within each particle's drift orbit is conserved

    Flux within 3's orbit is net positive



As B increases, if particle 3 stays at the same radius, the net flux within its orbit would increase. Therefore, it must move to a fieldline which, inside the wire, is closer to r=0. This fieldline outside the wire is at a greater radius.

⇒ The particle moves radially outward

d. Particle 2: would mirror back and forth along field line, being marginally trapped

Particle 1: would mirror back and forth along the entire field line, being marginally trapped

Particle 3: not much, stuck in the well of a mirror