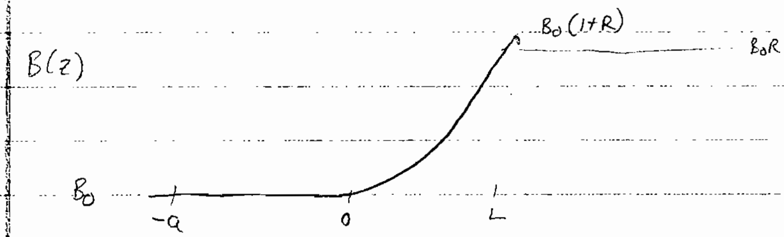
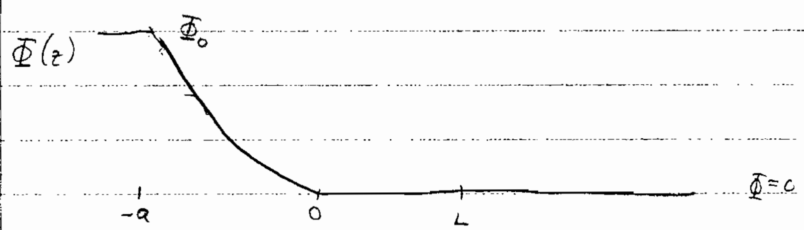
2006 Part II Q3 GPP I

$$B = \begin{cases} RB_0 & z > L \\ B_0\left(1 + R\frac{z^2}{L^2}\right) & L > z > 0 \\ B_0 & z \le 0 \end{cases} \qquad \Phi = \begin{cases} 0 & z > 0 \\ \Phi_0 \left(\frac{z^2}{a^2}\right) & -a < z \le 0 \\ \Phi_0 & z \le -a \end{cases}$$



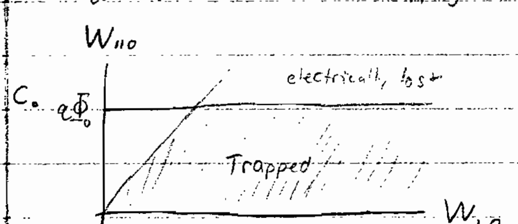


a. For $z>0$, ions are trapped by the magnetic mirror. For $z<0$, ions are confined by the electric potential. Electrons are not confined at all for $z<0$.

b. Requirement for magnetic trapping: $E \le \mu B_{max}$: $W_{\parallel 0} + W_{\perp 0} < W_{\perp 0} \frac{B_{max}}{B_0}$ $\quad \frac{B_{max}}{B_0} = 1+R$

$$\frac{W_{\parallel 0}}{W_{\perp 0}} < R$$

Requirement for electric trapping: $W_{\parallel 0} < q\Phi_0$



(not quite correct)

$E \gtrsim \frac{R}{R-1} q\Phi_0$: scatters into electrically lost region, lost on $z<0$ side d Electric potential changing in time: $\Phi_0 = \Phi_0(t)$ (slow changes)

Second adiabatic invariant is conserved: $J = \oint \vec{v}_{\parallel} \cdot d\vec{s}$

$$J = 2\int_{z_{tp1}}^{z_{tp2}} v_z(z)\,dz = 2\left[\int_{z_{tp1}}^{0} v_z(z)\,dz + \int_{0}^{z_{tp2}} v_z(z)\,dz\right]$$

For $z<0$,
$$W_{\parallel 0} = q\Phi_0 \frac{z_{tp1}^2}{a^2} \qquad z_{tp1}^2 = a^2 \frac{W_{\parallel 0}}{q\Phi_0} \qquad z_{tp1} = -a\sqrt{\frac{W_{\parallel 0}}{q\Phi_0}}$$
$$W_{\parallel}(z) = W_{\parallel 0} - \frac{q\Phi_0 z^2}{a^2} = W_{\parallel 0}\left(1 - \frac{z^2}{z_{tp1}^2}\right)$$
$$v_{\parallel}(z) = \sqrt{\tfrac{2}{m}}\sqrt{W_{\parallel 0}}\left(1-\frac{z^2}{z_{tp1}^2}\right)^{1/2}$$

For $z>0$,
$$W_{\parallel}(z) = W_{\parallel 0} - W_{\perp 0}\left(\frac{B}{B_0} - 1\right) \qquad B(z) = B_0\left(1 + R\frac{z^2}{L^2}\right)$$
$$W_{\parallel}(z) = W_{\parallel 0} - W_{\perp 0}\left(R\frac{z^2}{L^2}\right)$$
$$W_{\parallel 0} = W_{\perp 0} R \frac{z_{tp2}^2}{L^2} \qquad z_{tp2}^2 = L^2 \frac{1}{R}\frac{W_{\parallel 0}}{W_{\perp 0}} \qquad z_{tp} = L\sqrt{\frac{1}{R}\frac{W_{\parallel 0}}{W_{\perp 0}}}$$
$$W_{\parallel}(z) = W_{\parallel 0}\left(1 - \frac{z^2}{z_{tp2}^2}\right)$$
$$v_{\parallel}(z) = \sqrt{\tfrac{2}{m}}\sqrt{W_{\parallel 0}}\left(1 - \frac{z^2}{z_{tp2}^2}\right)^{1/2}$$

$$J = \frac{2\sqrt{2}}{m}\sqrt{W_{\parallel 0}}\left[\int_{z_{tp1}}^{0}\left(1-\frac{z^2}{z_{tp1}^2}\right)^{1/2}dz + \int_{0}^{z_{tp2}^2}\left(1-\frac{z^2}{z_{tp2}^2}\right)^{1/2}dz\right]$$

$$x = -\frac{z}{z_{tp1}} \qquad dx = -\frac{dz}{z_{tp1}} \qquad\qquad x = \frac{z}{z_{tp2}} \qquad dx = \frac{dz}{z_{tp2}}$$

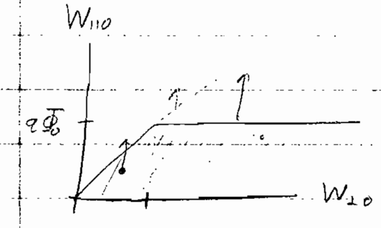
$$J = \frac{2\sqrt{2}}{m}\sqrt{W_{\parallel 0}}\left[-z_{tp1}\int_{-1}^{0}(1-x^2)^{1/2}dx + z_{tp2}\int_0^1 (1-x^2)^{1/2}dx\right]$$

$$J = \frac{2\sqrt{2}}{m}\sqrt{W_{\parallel 0}}\cdot\frac{\pi}{4}\cdot\left[-z_{tp1} + z_{tp2}\right] = \frac{\pi\sqrt{2}}{2m}\sqrt{W_{\parallel 0}}\left[\sqrt{W_{\parallel 0}}\cdot\frac{a}{\sqrt{q\Phi_0}} + \sqrt{W_{\parallel 0}}\,\frac{L}{\sqrt{RW_{\perp 0}}}\right]$$

$$J = \frac{\pi\sqrt{2}}{2m}W_{\parallel 0}\left[\frac{a}{\sqrt{q\Phi_0}} + \frac{L}{\sqrt{RW_{\perp 0}}}\right] = \text{const}$$

$$\Rightarrow W_{\parallel 0}\cdot f(\Phi_0, W_{\perp 0}) = \text{const}$$

e. Φ_0 increases in time. $W_{\perp 0}$ is constant because μ is conserved, B doesn't change. Thus by J conservation, $W_{\parallel 0}$ increases. As $\Phi_0(t)$ increases indefinitely, $W_{\parallel 0}$ saturates at a level
$$\frac{\frac{a}{\sqrt{q\Phi_0(t=0)}} + \frac{L}{\sqrt{RW_{\perp 0}}}}{\frac{L}{\sqrt{RW_{\perp 0}}}} = \left[1 + \frac{a}{L}\sqrt{\frac{RW_{\perp 0}}{q\Phi_0(t=0)}}\right] \times \text{initial } W_{\parallel 0}.$$



Initially trapped particles may leave at $z > L$, because $W_{\parallel 0}$ increases while $W_{\perp 0}$ is constant, and the magnetic trapping boundary is unchanged in velocity space. No particles initially trapped leave at $z < -a$. If we look at the conservation equation $W_{\parallel 0}\left[\frac{a}{\sqrt{q\Phi_0}} + \frac{L}{\sqrt{RW_{\perp 0}}}\right] = \text{const}$, then if Φ_0 increases by a factor α, the most $W_{\parallel 0}$ can increase is a factor $\sqrt{\alpha}$ (and that is only if the electric term is dominant over the magnetic term). The electric trapping boundary in velocity spaces moves with Φ_0, so it moves farther than $W_{\parallel 0}$.

Yes, some particles remain trapped as $\Phi_0 \to \infty$ because $W_{\parallel 0}$ saturates at a finite value. Thus, if this saturated $W_{\parallel 0}$, and $W_{\perp 0}$, is within the magnetic trapping region, it is confined.

$$\Rightarrow \left(W_{\parallel 0}\left[1 + \frac{a}{L}\sqrt{\frac{RW_{\perp 0}}{q\Phi_0(t=0)}}\right], W_{\perp 0}\right) \quad \text{confined?}$$

For $W_{\parallel 0}\left(1 + \frac{a}{L}\sqrt{\frac{RW_{\perp 0}}{q\Phi_0(t=0)}}\right) < RW_{\perp 0}$, then the particle is confined as $\Phi_0 \to \infty$