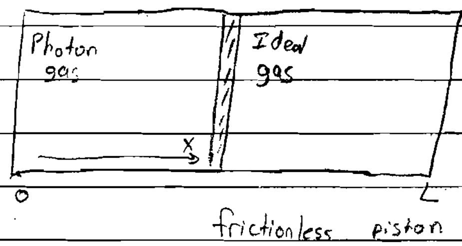
Jan 1997 #3 (SM)



initially, $x = \frac{L}{2}$

a. both sides are in thermal equilibrium with a heat bath of temp T

Photon gas: energy density $u = \frac{4}{c}\sigma T^4$ $\quad p = \frac{1}{3}u = \alpha u$ $\quad (\alpha = \frac{1}{3})$

$(\sigma = 5.67\cdot10^{-8}\ J/s\cdot m^2\cdot K^4)$

Ideal gas: $p = \frac{NkT}{V}$

at constant temperature, $p = \frac{P_0(\frac{L}{2})}{L-x}$ $\quad P_0 =$ initial pressure $= 1$ atm

photon gas: initial pressure $= \frac{4}{3c}\sigma T^4 = 2\cdot10^{-6}\ \frac{N}{m^2} = 2\cdot10^{-11}$ atm

at constant temperature, photon pressure is constant (independent of volume)

• at equilibrium with the heat bath, both gases stay at temp T

Equilibrium would occur when $P_{ph} = P_{gas}$, as x is varied, but this does not happen here. P_{gas} minimum is $\frac{P_0}{2} = .5$ atm while the photon gas pressure is $2\cdot10^{-11}$ atm.

• The piston moves all the way to $x = 0$

b. No heat bath, insulating piston

$\Delta Q = 0$ for both gases (adiabatic)

$dE_{ph} = -P_{ph}\,dV_{ph}$ $\qquad dE_{gas} = -P_{gas}\,dV_{gas}$ $\qquad dE_{ph} + dE_{gas} \neq 0$

(In the quasistatic case, energy of the total system is not conserved, because by moving it quasistatically, the kinetic energy of the piston, which exists because $P_{ph} \neq P_{gas}$ so there is a net force, is continuously removed)

Ideal gas, adiabatic: $P_{gas}V_{gas}^{\gamma} = \text{constant}$ (take $\gamma = \frac{5}{3}$)

$V_{gas} = A\cdot(L-x) \Rightarrow P_{gas}(L-x)^{\gamma} = \text{const.} \Rightarrow P_{gas}\left(1-\frac{x}{L}\right)^{\gamma} = \text{const.}$

Photon gas adiabatic condition:

$dE = -P\,dV$

$E = u(T)V \qquad dE = \left(\frac{\partial E}{\partial T}\right)_V dT + \left(\frac{\partial E}{\partial V}\right)_T dV = \frac{\partial u}{\partial T} V\,dT + u\,dV$

$\frac{du}{dT} V\,dT + u\,dV = -P\,dV \qquad P = \alpha u$

$\cancel{dT}\frac{du}{\cancel{dT}} = -(\alpha+1)\frac{u\,dV}{V} \qquad \frac{du}{u} = -(\alpha+1)\frac{dV}{V}$

$\ln u = -(\alpha+1)\ln V + C$

$u V^{\alpha+1} = \text{const.} \qquad P = \alpha u$

$P_{ph} V_{ph}^{\alpha+1} = \text{const} \qquad V_{ph} = Ax$

$\Rightarrow \quad P_{ph} \cdot \left(\frac{x}{L}\right)^{\alpha+1} = \text{const.}$

ideal gas: $P_{gas}\left(1 - \frac{x}{L}\right)^{\gamma} = P_0 \cdot \frac{1}{2^{\gamma}}$

photon gas: $P_{ph}\left(\frac{x}{L}\right)^{\alpha+1} = P_{ph\text{-}0} \cdot \frac{1}{2^{\alpha+1}}$

Let $u = \frac{x}{L}$

$P_{gas} = \frac{P_0}{2^{\gamma}(1-u)^{\gamma}} \qquad P_{ph} = \frac{P_{ph\text{-}0}}{2^{\alpha+1} \cdot u^{\alpha+1}}$

Equilibrium occurs when $P_{gas} = P_{ph}$

$\frac{P_0}{2^{\gamma}(1-u)^{\gamma}} = \frac{P_{ph\text{-}0}}{2^{\alpha+1} u^{\alpha+1}} \qquad P_0 u^{\alpha+1} = 2^{\gamma-(\alpha+1)} \cdot P_{ph\text{-}0}(1-u)^{\gamma}$

$P_0^{\frac{1}{\gamma}} u^{\frac{\alpha+1}{\gamma}} = 2^{1-\frac{\alpha+1}{\gamma}} P_{ph\text{-}0}^{\frac{1}{\gamma}}(1-u) \qquad \left(\frac{P_0}{P_{ph\text{-}0}}\right)^{\frac{1}{\gamma}} \cdot 2^{\frac{\alpha+1}{\gamma}-1} u^{\frac{\alpha+1}{\gamma}} + u = 1$

$\frac{P_0}{P_{ph\text{-}0}} >> 1$, so the first term is much, much greater than the second term

$u \approx \left(\frac{P_{ph\text{-}0}}{P_0}\right)^{\frac{1}{\alpha+1}} \cdot 2^{\frac{\gamma}{\alpha+1}-1} \qquad \alpha = \frac{1}{3},\ \gamma = 5/3 \qquad \frac{5/3}{4/3} - 1 = \frac{5}{4} - 1 = \frac{1}{4}$

$u \approx \left(\frac{P_{ph\text{-}0}}{P_0}\right)^{\frac{3}{4}} 2^{1/4}$

$u^* = \frac{x}{L} = 1.1 \cdot 10^{-8}$

photon gas temperature: $P_{ph} = \frac{P_{ph\text{-}0}}{2^{\alpha+1} \cdot u^{\alpha+1}} = .315\ \text{atm} = 3.15 \cdot 10^4\ N/m^2$

$P_{ph} = \frac{1}{3}u = \frac{4}{3}\frac{\sigma}{c}T^4 = .315\ \text{atm} \qquad T = \left(\frac{3c}{4\sigma} \cdot .315\ \text{atm}\right)^{1/4}$

$T = 1.05 \cdot 10^5\ K \approx 9\ eV$