# 2011 - Part II - Question 1A
# Kinetic Effects (Quickie)

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We have a 1-D distribution function of ions $f(x, v, t)$. At $x = 0$ is a wall that can reflect particles.

### Part A

We want the flux of ions into the wall. Any ion moving to the left at x=0 will strike the wall. Thus,

$$\Gamma_{in} = \int_{-\infty}^{0} v f(0, v, t) dv \tag{1}$$

### Part B

Every particle is given a chance of reflecting with probability $\alpha$, independent of energy. When a particle does reflect, however, its energy is reduced by a factor of $\beta$. At $x = 0$, any particles traveling to the right **must** have come from reflections. The probability of reflection reduces the overall right-moving distribution function by a factor of $\alpha$. The energy reduction sends all particles with speed $|v|$ to speed $\sqrt{\beta}|v|$, essentially reducing the width of the distribution. Thus, we get a boundary condition relating the right-moving side ($+|v|$) of the distribution function to the left-moving side ($-|v|$)

$$f(0, |v|, t) = \alpha f(0, -|v|/\sqrt{\beta}, t) \tag{2}$$

This seems to incorporate all the boundary conditions. To find a boundary condition of $\Gamma$, one would integrate the distribution over all positive velocities

$$\begin{aligned}
\Gamma_{out} &= \int_0^\infty v f(0,v,t) dv \\
\Gamma_{out} &= \alpha \int_0^\infty v f(0,-v/\sqrt{\beta},t) dv \\
\Gamma_{out} &= \alpha \int_0^{-\infty} (-\sqrt{\beta}u) f(0,u,t)(-\sqrt{\beta}du) \\
\Gamma_{out} &= -\alpha\beta \int_{-\infty}^0 u f(0,u,t) du \\
\Gamma_{out} &= -\alpha\beta\Gamma_{in}
\end{aligned}$$

In the third line, we used a $u$-sub of $u = -v/\sqrt{\beta}$. In the final line, we used Eq. 1 to relate $\Gamma_{in}$ and $\Gamma_{out}$. Note that since Eq. 1 defines $\Gamma_{in}$ as an integral over negative velocities, it is an inherently negative quantity. Thus, $\Gamma_{out}$ is a positive quantity as one would expect.