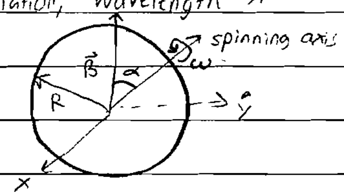
May 1999 #1 (EM)

Radiation, wavelength λ



ρ_c, ρ_m constant

a. The sphere rotating causes a current, which results in a torque $\sim(\vec{J}\times\vec{B}$ force)

The spin axis thus processes around the $\vec{B}$-field at some frequency Ω

$\Rightarrow$ Oscillating magnetic moment at frequency $\Omega \Rightarrow$ radiation

b. Radiation occurs at $\Omega = kc = \frac{2\pi c}{\lambda}$

$\rightarrow$ need to calculate Ω.

$d\vec{F} = \frac{q\vec{v}}{c}\times\vec{B} \qquad q \rightarrow \rho_c dV \qquad \vec{v} = \vec{\omega}\times\vec{r}$

$\Rightarrow dV\frac{\rho_c}{c}(\vec{\omega}\times\vec{r})\times\vec{B} = -dV\frac{\rho_c}{c}\left[\vec{\omega}(\vec{B}\cdot\vec{r}) - \vec{r}(\vec{\omega}\cdot\vec{B})\right]$

$d\vec{\tau} = \vec{r}\times\vec{F} = dV\frac{\rho_c}{c}\vec{B}\cdot\vec{r}\;\vec{\omega}\times\vec{r} = dV\frac{\rho_c}{c}Br\cos\theta\cdot\omega r\;\hat{\omega}\times\hat{r}$

calculate $d\vec{\tau}$ when $\hat{\omega} \in \hat{y}+\hat{z}$: $\quad \hat{\omega} = \sin\alpha\hat{y} + \cos\alpha\hat{z}$

$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$

$$\hat{\omega}\times\hat{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \sin\alpha & \cos\alpha \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{vmatrix} = \begin{matrix} \hat{x}(\sin\alpha\cos\theta - \cos\alpha\sin\theta\sin\phi) \\ +\hat{y}(\cos\alpha\sin\theta\cos\phi) \\ +\hat{z}(-\sin\alpha\sin\theta\cos\phi) \end{matrix}$$

All but 1 term vanishes after integration over ϕ ($\hat{x}$ term)

$$\vec{\tau} = \hat{x}\frac{\rho_c}{c}B\sin\alpha\cdot 2\pi\omega\underbrace{\int_0^R dr\, r^4}_{\frac{1}{5}R^5}\underbrace{\int_{-1}^{1} d(\cos\theta)\cos^2\theta}_{\frac{2}{3}}$$

$\hat{x} \rightarrow -\hat{\phi}$

$$\vec{\tau} = -\frac{4\pi}{15}\frac{\rho_c}{c}B\sin\alpha\,\omega R^5\hat{\phi}$$

$\vec{L} = L_\rho\hat{\rho} + L_z\hat{z} = L\sin\alpha\hat{\rho} + L\cos\alpha\hat{z}$

$\frac{d\vec{L}}{dt} = L\sin\alpha\frac{d\hat{\rho}}{dt} = L\sin\alpha\,\Omega\,\hat{\phi}$, precession at frequency Ω

$\vec{\tau} = \frac{d\vec{L}}{dt}$: $\quad -\frac{4\pi}{15}\frac{\rho_c}{c}B\sin\alpha\,\omega R^5 = L\sin\alpha\,\Omega = I\omega\sin\alpha\,\Omega$

constant density sphere: $I = \frac{2}{5}MR^2 = \frac{8\pi}{15}R^5\rho_m$

$\Rightarrow -\frac{4\pi}{15}\frac{\rho_c}{c}B\sin\alpha\,\omega R^5 = \frac{8\pi}{15}\rho_m\sin\alpha\,\omega R^5\,\Omega$

$$\Rightarrow \left|\frac{\rho_c}{\rho_m}\right| = \frac{2c|\Omega|}{B} \Rightarrow \frac{Q}{M} = \frac{2c\cdot 2\pi c}{B\lambda} \qquad \boxed{\frac{Q}{M} = \frac{4\pi c^2}{B\lambda}}$$

c. polarization $\rightarrow$ electric field direction

for a magnetic dipole, $\vec{B} \sim (\hat{n} \times \vec{m}) \times \hat{n}$

$\vec{E} \sim -\hat{n} \times \vec{B} \sim \vec{B} \times \hat{n} \sim [(\hat{n} \times \vec{m}) \times \hat{n}] \times \hat{n} \sim -[\hat{n}(\vec{m} \cdot \vec{n}) - \vec{m}] \times \vec{n} = \hat{m} \times \hat{n}$

$\underline{\vec{E} \sim \hat{m} \times \hat{n}}$

$\hat{m} \sim \cos \Omega t \, \hat{y} + \sin \Omega t \, \hat{x} \longrightarrow Re[(\hat{y} + i\hat{x}) e^{-i\Omega t}]$

$\hat{m} \sim \hat{y} + i\hat{x}$

$\vec{E} \sim \hat{m} \times \hat{n}$

$\vec{E} \sim Re[(\hat{y} + i\hat{x}) e^{-i\Omega t} \times \hat{n}]$

$= [\cos \Omega t \, \hat{y} + \sin \Omega t \, \hat{x}] \times \hat{n}$