Figure out how rat race works
Rederive equations in lab report

2007 Part II Q4 — Exp.

a. Interferometry: use interference to measure density by measuring the phase shift of a recombined signal from a reference point.

For example, the phase shift of an electromagnetic wave through a collisionless, unmagnetized plasma relative to traveling through vacuum is

$$\Delta\phi = \int (k - k_0)\, d\ell$$

$$n = \frac{kc}{\omega} \qquad n^2 = 1 - \frac{\omega_{pe}^2}{\omega^2} \Rightarrow k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right) \qquad k_0^2 = \frac{\omega^2}{c^2}$$

$$\Delta\phi = \frac{\omega}{c}\int\left[\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2} - 1\right] d\ell$$

If $\omega \gg \omega_{pe}$, then $\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2} \approx 1 - \frac{\omega_{pe}^2}{2\omega^2}$

$$\Delta\phi \approx \frac{-\omega}{2c\omega^2}\int +\omega_{pe}^2\, d\ell \qquad \omega_{pe}^2 = \frac{4\pi n_e e^2}{m_e}$$

$$\Delta\phi = -\frac{2\pi e^2}{m_e c\omega}\int n_e\, d\ell$$

which is linear in the line integrated density.

The system does not work for $\omega_{pe}^2 > \omega^2$; the wave is cutoff. For simplicity in analysis, $\omega^2 \gg \omega_{pe}^2$ is usually desired.

b. $n^2 = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pCl^-}^2}{\omega^2} - \frac{\omega_{pCl^+}^2}{\omega^2}$

$n^2 \approx 1 - \frac{\omega_{pe}^2}{\omega^2}$ still, so the explanation does not change. One must keep in mind that it is electron density which is measured

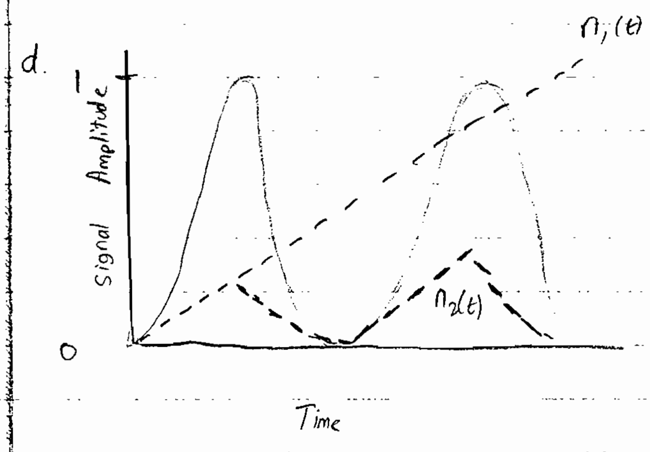
c. A - Gunn Diode: produce microwaves
B - Isolator: allows microwaves to pass through only in one direction
C - Frequency meter
D - Variable attenuator - used to set the relative amplitudes of the beams at the rat race
E - Rat race: Split beam
F - Variable phase shifter - set a reference phase of beams
G - microwave collecting horn

H - detector diode - measures average power at the detector

I - collecting rat race - combine the beams to send to the Cos and Sin detectors. One of the outputs combines the beams in phase and one out of phase

J - reference beam

d.



$n_1(t)$: density rises linearly

$n_2(t)$: density reaches a max, then decreases, and oscillates

The power at either of the detectors is proportional to, say, $\cos^2\left(\frac{\phi}{2}\right)$, where ϕ is the phase difference of the two waves at the rat race. ($\phi=0$ at $n=0$ assumed). If we are in the regime where $\phi \propto n_e$, then if n_e oscillates we will see the above picture. However, if n_e simply increases with time we will also see this picture, as ϕ will increase above 2π. Since it is not known whether ϕ is increasing above 2π or not, the inferred $n_e(t)$ is not unique.