2006 Day 1, question 3b
Bootstrap Current

Martin Griswold March 28, 2009

A good reference for this question is this paper .

a) A tokamak needs toroidal current for its equilibrium. Inductive current drive cannot be maintained in steady state and non-inductive methods (RF or neutral beam driven current) have a low efficiency. Therefore a high bootstrap current fraction is needed to make the reactor economically viable.

b) From the Peeters paper:

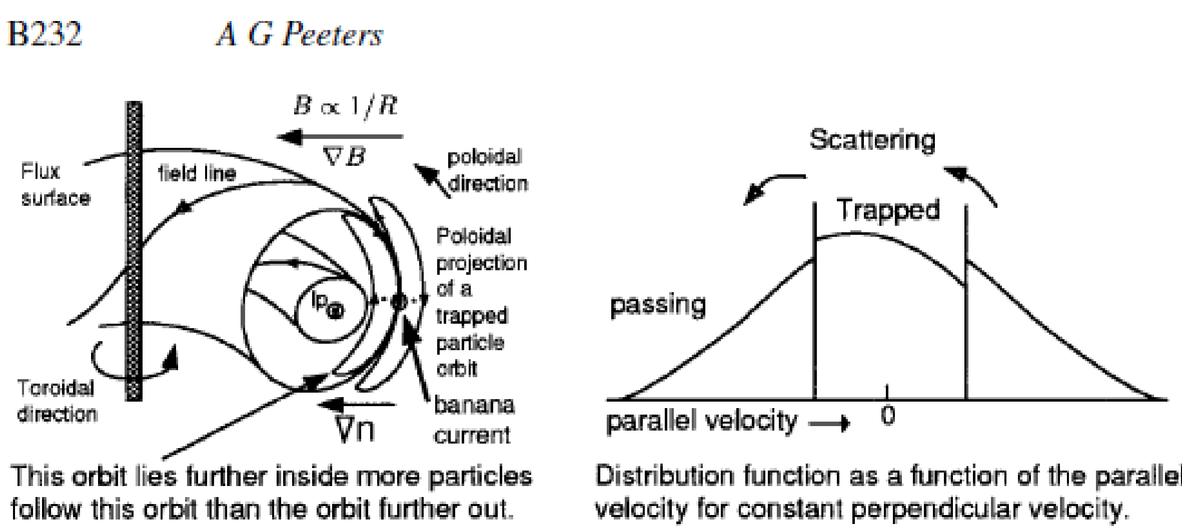

Figure 1. The banana current driven by the density gradient.

Trapped particles execute banana orbits as shown in figure 1. Because of the density gradient adjacent banana orbits are populated by different numbers of particles. Also, as you can see in figure 1, adjacent banana orbits have counter-streaming particles where they overlap. Because of the different number of particles populating each orbit, there is a net trapped particle current. The bootstrap current is carried by passing particles that are collisionally coupled to the trapped particle current

The bootstrap current and its consequences B233

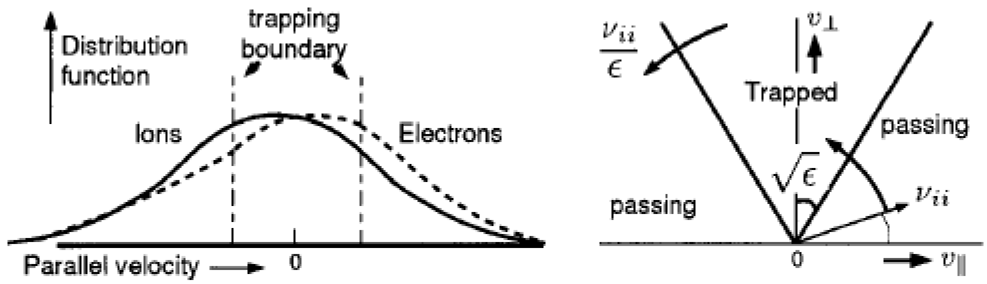

Figure 2. Diagram of the distribution function and scatter processes.

c) The heuristic derivation of the bootstrap current has two important steps. First, you need to calculate the trapped particle current, then you need to calculate the passing particle current that it produces through collisions (which is larger by a factor of 1/epsilon)

--Trapped Particle Current--

The trapped particle current is given by:

$$J_t = e \cdot (\delta n) \cdot (v_\parallel)$$

$$\delta n = \left(\Lambda \cdot \frac{dn}{dr} \cdot f_t \right)$$

where Λ is the banana orbit width. From Tang's notes:

$$\Lambda = \frac{\rho q}{\sqrt{\varepsilon}}$$

The fraction of trapped particles is given by $f_t = \sqrt{\varepsilon}$ and you get the parallel velocity of the trapped particles from the mirror ratio:

$$\left(\frac{v_\parallel}{v_\perp}\right)^2 < 2\varepsilon \quad \Rightarrow \quad v_\parallel \sim \sqrt{\varepsilon} \cdot v_{th}$$

putting this all together gives the trapped particle current:

$$J_t = \frac{c\,\varepsilon^{\frac{3}{2}}}{B_p}\,T\,\frac{dn}{dr}$$

--Bootstrap Current--

I got this part from the "Tokamaks" book by Wesson

In order to calculate how much current is carried by the passing particles because of collisional coupling, you set the momentum exchange between passing electrons and trapped electrons equal to the momentum exchange between passing electrons and trapped ions.

To rephrase, there are two collisional drag forces on the passing electrons. The first is because the passing electrons are moving at a different speed than the trapped electrons. The second is because the pasing electrons are moving at a different speed than the passing ions. These forces ar in opposite directions, and for equilibrium they must be equal.

momentum exchange between passing electrons and passing ions is:

$$\nu_{ei}\,m_e\,\frac{j_b}{e}$$

momentum exchange between passing electrons and trapped electrons is:

$$\nu_{eff} \cdot m_e\,\frac{j_t}{e}$$

$$\nu_{eff} = \frac{\nu_{ee}}{\sqrt{\varepsilon}}$$

Setting the momentum exchanges equal to each other and taking the e-e collision frequency to be equal to the e-i frequency you get that the bootstrap current is larger than the trapped particle current by a factor of 1/epsilon:

$$j_b = \frac{j_t}{\varepsilon} = -\frac{\sqrt{\varepsilon}}{B_p} T \frac{dn}{dr}$$

part d is pretty straightforward, so I'm not going to type it.