Matt Lucia

$$\vec{J}_\perp \times \vec{B} = \vec{\nabla} P \rightarrow \vec{\nabla} \times (\vec{J}_\perp \times \vec{B}) = \vec{\nabla} \times (\vec{\nabla} P) = 0$$

$$\vec{J}_\perp (\cancelto{0}{\vec{\nabla}\cdot\vec{B}}) - \vec{B}(\vec{\nabla}\cdot\vec{J}_\perp) + (\vec{B}\cdot\vec{\nabla})\vec{J}_\perp - (\vec{J}_\perp\cdot\vec{\nabla})\vec{B} = 0$$

$$-\vec{B}(\vec{\nabla}\cdot\vec{J}_\perp) + \vec{\nabla}(\cancelto{0}{\vec{J}_\perp\cdot\vec{B}}) - \vec{J}_\perp \times (\vec{\nabla}\times\vec{B}) - \vec{B}\times(\vec{\nabla}\times\vec{J}_\perp) - 2(\vec{J}_\perp\cdot\vec{\nabla})\vec{B} = 0$$

$$B^2(\vec{\nabla}\cdot\vec{J}_\perp) = -\vec{B}\cdot\left[\vec{J}_\perp\times(\vec{\nabla}\times\vec{B})\right] - \vec{B}\cdot\left[\vec{B}\times(\vec{\nabla}\times\vec{J}_\perp)\right]$$

$$-2\vec{B}\cdot\left[(\vec{J}_\perp\cdot\vec{\nabla})\vec{B}\right] = 0$$

$$B^2(\vec{\nabla}\cdot\vec{J}_\perp) = \underbrace{-\vec{B}\cdot\left[\vec{J}_\perp\times\vec{J}_\parallel\right]}_{-\vec{J}_\perp\cdot\left[\cancelto{0}{\vec{J}_\parallel\times\vec{B}}\right]} - (\vec{\nabla}\times\vec{J}_\perp)\cdot(\cancelto{0}{\vec{B}\times\vec{B}}) - 2\vec{B}\cdot\left[(J_\perp\cdot\vec{\nabla})\vec{B}\right] = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{J}_\perp = \frac{-2\vec{B}}{B^2}\cdot\left[(\vec{J}_\perp\cdot\vec{\nabla})\vec{B}\right]}$$

---

$$\vec{B} = B_0\vec{\nabla}\phi = \frac{B_0}{R}\hat{e}_\phi$$

$$\vec{\nabla}\vec{B} = \vec{\nabla}\left(\frac{B_0}{R}\hat{e}_\phi\right) = \vec{\nabla}\left(\frac{B_0}{R}\right)\hat{e}_\phi + \frac{B_0}{R}\vec{\nabla}\hat{e}_\phi$$

$$= -\frac{B_0}{R^2}\hat{e}_R\hat{e}_\phi + \frac{B_0}{R}\left[\frac{1}{R}\frac{\partial\hat{e}_\phi}{\partial\phi}\hat{e}_\phi\right] = \frac{-2B_0}{R^2}\hat{e}_R\hat{e}_\phi$$

$$\vec{J}_\perp = J_R\hat{e}_R + J_z\hat{e}_z \longrightarrow \vec{J}_\perp\cdot\left[\frac{-2B_0}{R^2}\hat{e}_R\hat{e}_\phi\right] = \frac{-2B_0J_R}{R^2}\hat{e}_\phi$$

$$\vec{\nabla}\cdot\vec{J}_\perp = \frac{-2}{B^2}\vec{B}\cdot\left[\frac{-2B_0 J_R}{R^2}\hat{e}_\phi\right]$$

$$= \frac{-2}{B^2}\frac{B_0}{R}(\hat{e}_\phi\cdot\hat{e}_\phi)\frac{-2B_0 J_R}{R^2} = \frac{4B_0^2 J_R}{B^2 R^3}$$

$$= \frac{4B_0^2 J_R}{R^3}\frac{R^2}{B_0^2} \rightarrow \boxed{\vec{\nabla}\cdot\vec{J}_\perp = \frac{4J_R}{R}}$$

$$\vec{\nabla}P = \vec{J}_\perp\times\vec{B} = \frac{1}{R}\begin{vmatrix}\hat{e}_R & R\hat{e}_\phi & \hat{e}_z\\ J_R & 0 & J_z\\ 0 & B_0 & 0\end{vmatrix} = \left\langle -\frac{J_z B_0}{R}, 0, \frac{J_R B_0}{R}\right\rangle$$

$$= \left\langle \frac{\partial P}{\partial R}, 0, \frac{\partial P}{\partial z}\right\rangle$$

$$\frac{J_R B_0}{R} = \frac{\partial P}{\partial z} \rightarrow \boxed{\vec{\nabla}\cdot\vec{J}_\perp = \frac{4}{B_0}\frac{\partial P}{\partial z}}$$

$$\frac{\partial \rho}{\partial t} = \frac{-4}{B_0}\frac{\partial P}{\partial z}$$

$z > 0 \rightarrow \frac{\partial P}{\partial z} < 0 \rightarrow \frac{\partial \rho}{\partial t} > 0$

$z < 0 \rightarrow \frac{\partial P}{\partial z} > 0 \rightarrow \frac{\partial \rho}{\partial t} < 0$