2005 Part II Q2

a. Fluid force balance: $m_j n_j \frac{d\vec{v}}{dt} = n_j q_j \frac{\vec{v}\times\vec{B}}{c} - \nabla P_j$ $(\vec{E}=0)$

Take $\frac{d\vec{v}}{dt}=0$. Then $\vec{v}\times\vec{B} = \frac{c}{n_j q_j}\nabla P_j$

$\vec{B}\times(\vec{v}\times\vec{B}) = \frac{c}{n_j q_j}\vec{B}\times\nabla P_j$

$= \vec{V}_\perp B^2$

$$\Rightarrow \vec{V}_\perp = \frac{cT_j}{n_j q_j B_0}\hat{z}\times\nabla n_j$$ diamagnetic drift velocity

This is called "diamagnetic" because the currents, irrespective of the sign of the charge, produce a magnetic field to counter the applied field. (For reasonable density profiles)

e.g. $\nabla n(r) \sim -\hat{r}$ (decreasing density profile)

Then $n_j q_j \vec{V}_\perp = \vec{J} \sim \hat{z}\times(-\hat{r}) \sim -\hat{\theta}$

Current in the $-\hat{\theta}$ direction produces a magnetic field which points in the $-\hat{z}$ direction

b. single particle motion: $\frac{d\vec{v}}{dt} = \frac{q}{m}\vec{E}(t) + \frac{q\vec{v}\times\vec{B}}{mc}$ $\vec{B}$ is uniform, constant

First let $\vec{V}_c$ satisfy $\frac{d\vec{V}_c}{dt} = \frac{q\vec{V}_c\times\vec{B}}{mc}$, contains the 0 order cyclotron motion

Write $\vec{v}=\vec{V}_c+\vec{V}_E$: $\frac{d\vec{V}_E}{dt} = \frac{q}{m}\vec{E}(t) + \frac{q}{m}\frac{\vec{V}_E\times\vec{B}}{c}$

• Assume $\frac{d\vec{V}_E}{dt}$ is small to order $\frac{\omega}{\Omega}$ (this will be verified)

$$\rightarrow \frac{q}{m}\vec{E} + \frac{q}{m}\frac{\vec{V}_E\times\vec{B}}{c} = 0 \Longrightarrow \boxed{\vec{V}_E = c\frac{\vec{E}\times\vec{B}}{B^2}}$$

$\frac{d\vec{V}_E}{dt} \sim \omega\frac{cE}{B} \sim \frac{\omega}{\Omega}\frac{q}{m}E$ is indeed small

To next order, write $\vec{V} = \vec{V}_c + \vec{V}_E + \vec{V}_P$

$$\frac{d\vec{V}_E}{dt} + \cancel{\frac{d\vec{V}_P}{dt}} = \frac{q}{mc}\vec{V}_P \times \vec{B}$$

neglect to this order

$$\Rightarrow \quad \vec{V}_P \times \vec{B} = \frac{mc}{q}\frac{d\vec{V}_E}{dt} \qquad\qquad -\vec{V}_P B^2 = \frac{mc}{q}\frac{d\vec{V}_E}{dt} \times \vec{B}$$

$$\vec{V}_P = -\frac{mc}{qB^2}\frac{d\vec{V}_E}{dt} \times \vec{B}$$

$$\frac{d\vec{V}_E}{dt} = c\frac{d}{dt}\left(\frac{\vec{E}\times\vec{B}}{B^2}\right) = \frac{c}{B^2}\frac{d\vec{E}}{dt}\times\vec{B}$$ for $\vec{B}$ constant, uniform

$$\vec{V}_P = -\frac{1}{\Omega B}\cdot\frac{c}{B^2}\left(\frac{d\vec{E}}{dt}\times\vec{B}\right)\times\vec{B}$$

$$\boxed{\vec{V}_P = \frac{c}{\Omega B}\frac{d\vec{E}_\perp}{dt}}$$

$\vec{E}_\perp$ is the component of $\vec{E}$ perpendicular to $\vec{B}$