Generals 2011 Part II Section 2

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a) The general dispersion relation for X-waves is $n_\perp^2 = \frac{RL}{S}$. Thus resonances will occur for $S = 0$, provided R and L remain finite:

$$1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2} = 0 \tag{1}$$

In the range $\omega^2 \approx \omega_{UH}^2 = \Omega_e^2 + \omega pe^2$, the ion term is smaller by a factor $\frac{\omega_{pi}^2}{\omega_{pe}^2} = \frac{m_e}{m_i} \ll 1$. Thus you can neglect all ion terms in S. Rearranging a bit, we get

$$\omega^2 = \omega_{pe}^2 + \Omega_e^2 \tag{2}$$

as desired. We can also check that RL remains finite:

$$RL = S^2 - D^2 = -D^2 = -\left[\sum_s \frac{\Omega_s}{\omega} \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2}\right]^2 \approx -\left[\frac{\Omega_e}{\omega} \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2}\right]^2 \tag{3}$$

Clearly this is finite at $\omega^2 = \omega_{UH}^2$, and thus there is a resonance at $S = 0$.

b) We are only interested in the perpendicular x-y block for the X-wave polarization. Using $n_\parallel^2 = 0$:

$$\begin{bmatrix} S & -iD \\ iD & S - n_\perp^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = 0 \tag{4}$$

We use the second line to determine the polarization:

$$\frac{iE_x}{E_y} = \frac{n_\perp^2 - S}{D} \gg 1 \tag{5}$$

since $n_\perp^2 \gg 1$ very near the resonance. Thus $E_x \gg E_y$, yielding polarization in the $\hat{x}$ direction, which is parallel to $\vec{k}_\perp = k_x \hat{x}$, the electrostatic condition.