How do you express the

- number
- momentum
- kinetic energy

of a species s, in terms of its distribution function, f_s?

What is the pressure tensor, $P_{\alpha\beta}$?

How is temperature generally defined?

number: $n_s = \int d\vec{v}\, \bar{n}_s f_s(\vec{v})$

momentum: (total) $p = \sum_s \int d\vec{v}\, (m\bar{n}\,\vec{v})_s f_s(\vec{v})$

kinetic energy (total) $KE = \sum_s \int d\vec{v} \left(\frac{1}{2} m \bar{n} v^2\right)_s f_s(\vec{v})$

$$P_{\alpha\beta} = p\,\delta_{\alpha\beta} + \Pi_{\alpha\beta}$$

$$T_\alpha(\vec{r},t) = \frac{m_\alpha}{3} \langle (\vec{u} - \vec{v})^2 \rangle$$

What is the pressure, p?

What is the stress tensor, $\Pi_{\alpha\beta}$?

What is the 'chopping procedure' for determining how quantities scale with ϵ_p?

$$p = \frac{nm\langle v^2 \rangle}{3} = nT$$

The stress tensor, $\Pi_{\alpha\beta}$, represents the part of the pressure tensor, $p_{\alpha\beta}$ that arises as a result of the deviation of the distribution from spherical symmetry.

$$\Pi_{\alpha\beta} = nm \langle v_\alpha v_\beta - \left(\frac{v^2}{3}\right) \delta_{\alpha\beta}$$

imagine moving to the continuum limit, $\epsilon_p \to 0$ by 'chopping' particles in half: $(n\lambda_D^3 \to \infty)$

$$m \to \frac{m}{2} \quad e \to \frac{e}{2} \quad \bar{n} \to 2\bar{n} \quad \epsilon_p \to \frac{\epsilon_p}{2}$$

In such a way as to ~~keep~~ preserve $v_{th} \Rightarrow T \to \frac{T}{2}$.

This keeps v_{th} ω_p, λ_D all constant

from avove, see that $m, e, T \sim \mathcal{O}(\epsilon_p)$ $\quad n \sim \mathcal{O}(\epsilon_p^{-1})$

use these to determine scalings of any dimensional quantity

ex: $b \sim \mathcal{O}\left(\frac{e^2}{T}\right) : \mathcal{O}\left(\frac{\epsilon_p^2}{\epsilon_p}\right) \simeq \mathcal{O}(\epsilon_p)$

How are λ_D and b_0 related?

How are collisional effects in a plasma different from those in a Boltzmann gas?

What is the physical interpretation of ϵ_p?

(Related to weak/ strong coupling)

How do $\frac{\nu}{\omega_p}$ and $\frac{\lambda_\nu}{\lambda_D}$ scale with ϵ_p?

$\frac{\lambda_D}{b_0} = \frac{1}{\epsilon_p} = \Lambda$ Boltzmann gas concentrates on collisions on scale of b_0. In plasma, you have additional cumulative effect of interactions w/ particles further away

$\frac{\text{plasma effects}}{\text{Boltzmann effects}} \sim \ln \Lambda$

consider the ratio of potential to kinetic energy for two particles which are separated by a DeBye length:

$$\frac{W}{K} \sim \frac{e^2/\lambda_D}{\frac{1}{2}mv^2 = \frac{1}{2}T} \sim \frac{e^2/T}{\lambda_D} = \frac{ne^2/T}{n\lambda_D} = \frac{1}{n\lambda_D^3} = \epsilon_p$$

ϵ_p is the ratio of potential to kinetic energy for 2 particles separated by a DeBye length. If it is small, 'weakly coupled'

→ tells us how much one particle can 'feel' another.

$\frac{\nu}{\omega_p} \sim \epsilon_p \ll 1$ plasma frequency much greater than collision frequency.

$\frac{\lambda_\nu}{\lambda_D} \sim \frac{1}{\epsilon_p} \gg 1$ mean free path much greater than deBye length.

What is the ONE important dimensionless parameter for a plasma (near thermal equilibrium)?
(no $\vec{B}$ field)

Is the actual friction force, $\vec{R}$, larger or smaller than that computed for a shifted Maxwellian?

Why?

What are transport coefficients?

Why are they important?

Plasma Parameter

$$\epsilon_p \equiv \frac{1}{n\lambda_D^3} \quad [n\lambda_D^3 \cong \# \text{ of particles in DeBye Sphere}]$$

The actual friction force is smaller - because the coulomb cross section goes down w/ higher energy - the dist func. gets a high energy tail higher avg $\vec{v}$ than for a shifted Maxwellian - fewer collisions, lower friction force.

Note: inclusion of effects of collisions w/ other electrons tends to reduce this effect and pull back the tail

The transport eqns. are in terms of $n, \vec{u}, T$ and also $\Pi_{\alpha\beta}$, $\vec{q}$, $\vec{R}$ and Q. In order to solve for n, $\vec{u}$ and T, the other quantities $\Pi_{\alpha\beta}$, $\vec{q}$, $\vec{R}$, and Q must be expressed in terms of $n, \vec{u} + T$. If we assume that the distribution function is a small perturbation from a (local) Maxwellian, then f_1, the deviation, will be proportional to the forces that cause it · gradients, E fields, etc. + thus $\Pi_{\alpha\beta}$ $\vec{q}$, $\vec{R}$ and Q are also proportional to these forces - the constants of proportionality are called Transport coefficients.

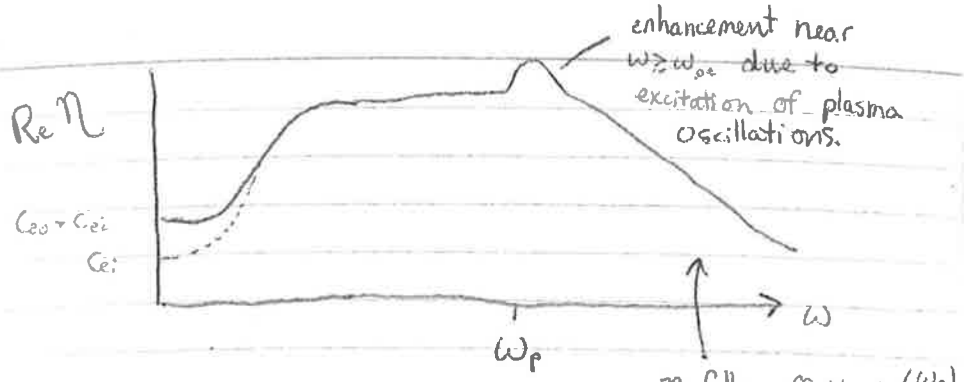
Sketch plasma resistivity vs. ω, indicating reasons for the various features.

What is the density diffusion equation, and how is it related to the continuity equation? (1-D)

What is this?

$$C[f] = -\frac{\partial}{\partial v}\left(\nu v + D_v \frac{\partial}{\partial v}\right) f$$

- What physical situation does it describe?
- What does it conserve?



frequency dependent modifications of $\ln\Lambda$

$$\Lambda = \frac{\lambda_D}{b_0} \rightarrow \frac{v_t/\omega}{b_0}$$

eqn. for density diffusion $\frac{\partial n}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial n}{\partial x}$

relate to continuity eqn:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(n\vec{u}) = 0 \quad \longleftrightarrow \quad \frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

eliminate $\frac{\partial n}{\partial t}$ to get:

$$nu = \Gamma = -D\frac{\partial n}{\partial x}$$

It is the collision operator for a Test Particle undergoing polarization drag and velocity space diffusion
(from HW #5)

- It conserves only number, not momentum or kinetic energy, as those are lost to the bath.

What is $\int d^3v = ?$

What is the thermal correction to the Langmuir oscillation?

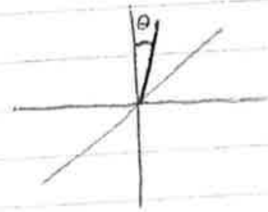
What is this:

$$C_{12}[f] = -\frac{(mn)_2}{(mn)_1} \nu_{21} \frac{\partial}{\partial v_1} \cdot \left[\vec{v}_1 f_1 + \left(\frac{T_2}{m_1}\right) \frac{\partial f_1}{\partial \vec{v}_1} \right] - \frac{1}{(mn)_1} \vec{R} \cdot \frac{\partial f_1}{\partial \vec{v}_1}$$

and where did it come from?

$$\int d^3v = \int v^2 \sin\theta \, dv \, d\theta \, d\phi$$

$$= \int_{\lambda=-1}^{1} d\lambda \int_0^{\infty} v^2 \, dv \int_0^{2\pi} d\phi$$



$\lambda = \cos\Theta$

$$\omega^2 = \omega_p^2 + 3k^2 V_{th}^2$$

It's an approximation to the Landau collision operator for scattering of particles 1 off particles 2, for $m_1 \gg m_2 \rightarrow$ ions scattering off of electrons.

Note It is still dependent on the unknown friction force, $\vec{R}$.

What is this:

$$C_{12}[f] \simeq \nu_{12} \frac{\partial}{\partial \vec{v}_1} \cdot \left[(\vec{v}_1 - \vec{u}_2) f_1 + \left(\frac{T_2}{m_1}\right) \frac{\partial f_1}{\partial \vec{v}_1} \right]$$

What is it good for and where did it come from?

Show that the Balescu-Lenard operator conserves number, momentum, and kinetic energy - Quick!

What is $\vec{R}$?

It is an approximation to the Landau operator for $m_1 \gg m_2 \rightarrow$ ions scattering off of electrons, for example. It uses the approximation for the friction force found from taking an electron distribution of a shifted maxwellian, with cold background ions.

Note: It is precisely the Fokker Planck operator for a Brownian ion in an electron fluid moving with velocity $\vec{u}_e$.

- number is straightforward - because C_s is a divergence in velocity space.

- momentum + kinetic energy cons. can be seen by the fact that C_s is symmetric for $\vec{k} \leftrightarrow -\vec{k}$, but antisymmetric for $(s, v) \leftrightarrow (\bar{s}, v) \rightarrow$ sum over species vanishes.

$$\vec{R} = \int m \vec{v} C[f] d\vec{v}$$

Represents the mean change in the momentum of the particles of a given species due to collisions (with all other particles represented by C)

What is this, + where did it come from?

$$(1-\rho_s^2\nabla_\perp^2)\frac{\partial}{\partial t}\delta\varphi + V_*\frac{\partial}{\partial y}\delta\varphi$$
$$+ \vec{u}_E \cdot \vec{\nabla}(-\rho_s^2\nabla_\perp^2\delta\varphi) = 0$$

What is Q?

What are the transport equations, and where did they come from?

It is the Hasegawa-mima equation. It is an approximate equation for the electrostatic potential in the limit $T_i \to 0$.

- assumed electrons Boltzmann response along $\vec{B}$.
- used ion cross field drifts $\vec{u}_E \to \vec{E}\times\vec{B}$ drift.

$V_* = \dfrac{cT_e}{eBL_n}$ = diamagnetic velocity

$\rho_s = \dfrac{c_s}{\nu_{ei}}$

It does not include FLR effects.

$$Q = \int \frac{mv^2}{2} C[f]\, d\vec{v}$$

- It is the heat generated in a gas of particles of a given species as a consequence on collisions w/ particles of other species [as described by C]

Transport equations: Come from velocity moments of the kinetic equation. They describe behavior of the macroscopic parameters n, $\vec{u}$, and T.

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{u}) = 0 \qquad \text{continuity}$$

$$mn\frac{d\vec{u}}{dt} = -\vec{\nabla}p - \vec{\nabla}\cdot\overleftrightarrow{\Pi} + qn\left(\vec{E} + \frac{1}{c}\vec{u}\times\vec{B}\right) + \vec{R}$$

momentum

$$\frac{3}{2}n\frac{dT}{dt} + p\vec{\nabla}\cdot\vec{u} = -\vec{\nabla}\cdot\vec{q} - \Pi : \vec{\nabla}\vec{u} + Q$$

What is $\vec{q}$?

How is the heat gained by ions due to collisions with electrons defined?
What is it equal to?

What is this:

$$C_{s,\bar{s}}[f] = 2\pi \left(\frac{q^2}{m}\right)_s (\bar{n} q^2)_{\bar{s}} \ln\Lambda \cdot \frac{\partial}{\partial \vec{v}} \cdot \int d\vec{v}\, U \cdot \left(\frac{1}{\bar{m}} \frac{\partial}{\partial \vec{\bar{v}}} - \frac{1}{m} \frac{\partial}{\partial \vec{v}}\right) f_s(\vec{v}) f_{\bar{s}}(\vec{\bar{v}})$$

and where did it come from?

$$\vec{q} = nm\left\langle \frac{v^2}{2}\vec{v} \right\rangle$$

is the flux density of heat carried by particles of a given species - represents the transport of energy associated with the random motion in the coordinate system in which the particle gas as a whole is at rest.

$$Q \equiv -\frac{1}{2}\int d\vec{v}_i (m\bar{n})_i \, |\vec{v}_i - \vec{u}_i|^2 \, C_{ie}[f]$$

$$= 3\left(\frac{m}{M}\right) n_e \nu_{ei} \left(T_e - T_i\right)$$

That is the Landau operator - it came from considering the Fokker-Planck operator, with two-body interactions, integrated over distances from b_0 to λ_D. - thus it takes into account only small angle scattering, and only includes the effect of DeBye shielding in the long impact parameter cutoff (no calculation of E field spectrum).

It is often used, and can be written in terms of the Rosenbluth potentials for simplification of evaluation.

What important properties does the Balescu-Lenard operator have?

Briefly describe the Landau operator, and name 4 important properties which it has.

What is this:

$$C_{s,\bar{s}}[f] = \frac{\partial}{\partial \underline{v}} \cdot \left[\pi \left(\frac{q^2}{m}\right)_s \sum_{\bar{s}} (\bar{n} q^2)_{\bar{s}} \cdot \int d\underline{\bar{v}} \int \frac{d\underline{k}}{(2\pi)^3} \frac{\underline{\epsilon}_k \underline{\epsilon}_k^*}{|\mathcal{D}(\underline{k}, \underline{k}\cdot v)|^2} \delta(\underline{k}\cdot(\underline{v} - \underline{\bar{v}})) \cdot \left(\frac{1}{\bar{m}} \frac{\partial \bar{f}}{\partial \underline{\bar{v}}} f - \frac{1}{m} \bar{f} \frac{\partial f}{\partial \underline{v}} \right) \right.$$

and what 6 steps lead to it?

The Balescu-Lenard operator:

- conserves number, momentum, and kinetic energy
- annihilates a local maxwellian
- has a local H theorem - entropy increases monotonically due to collisions, except for a local Maxwellian, where it remains constant.

Landau operator - comes from Fokker-Planck operator for plasmas; under assumption of two-body-type interactions summed from b_0 to λ_D - accounts for small angle scattering, and only heuristically includes shielding.

OFTEN USED

It:
1. conserves number, momentum, and kinetic energy.
2. annihalates (uniqely) a local Maxwellian.
3. has an H theorem - entropy increases montonically due to collisions, except for a local maxwellian where it is constant.
4. $\vec{P}$ and KE conservation arise from simultaneous presence of velocity sp diffusion + polarization drag which enter symmetrically

That is the Balescu-Lenard collision operator. To get it:

1. Make the Markov assumption and derive the Master equation from Chapman Kolmogorov eqn.
2. See that predominant deflections are small, derive the Fokker-Planck eqn by Taylor expanding the Master Eqn.
3. Find general forms for 1st + 2nd Fokker-Planck coefficients (for 1st coeff. - must work to one higher order than straight line trajectories) Note cancellation between part of 1st coefficient + velocity divergence of diffusion tensor.
4. Determine the velocity space diffusion tensor in terms of spectral information.
5. determine the spectrum itself by using the Test Particle Superposition Principle
6. Compute the polarization drag using test particle techniques

$\Rightarrow$ put it all together to get the Balescu Lenard operator!

How do you find a dielectric function by inserting a test Charge into a plasma?

What is to the calculation of the plasma collision operator as the two-body probem is to the Boltzmann operator?

The dielectric function relates the total potential to the test potential: $\delta\varphi^{tot} = \frac{\delta\varphi^{test}}{\mathcal{D}}$. So to find $\mathcal{D}$, starting from ρ^{test}...

① Relate ρ^{test} to φ^{test} via poisson eqn. $k^2\delta\varphi^{test} = 4\pi\rho^{test}$

② Use eqn. of motion for plasma Vlasov eqn, Gyrokinetic Eqn as appropriate, putting in the TOTAL potential for $\vec{E}$: $-ik E^{tot} = \delta\varphi^{tot}$ and linearize around Maxwellian $F = f_M + \delta f$

③ Fourier transform the eqn + solve for δf, in terms of $\delta\varphi^{tot}$.

④ Calculate induced charge density from δf:
$\delta\rho^{ind} = \sum_s (\bar{n}q)_s \int d\vec{v}\,\delta f$

⑤ Use poisson eqn, now for induced potential + induced charge density - use fact that $\delta\varphi^{tot} = \delta\varphi^{test} + \delta\varphi^{ind}$

⑥ Now have everything w/ $\delta\varphi^{tot}$ and $\delta\varphi^{test}$ → put in form $\delta\varphi^{tot} = \frac{\delta\varphi^{test}}{\mathcal{D}}$ to find $\mathcal{D}$.

The theory of the Vlasov eqn.

including single particle motion and the linearized self-consistent response to that motion.