Seth Dorfman   Prelims   January 6, 2006

May 2004 SM

1) a. adiabatic $\Rightarrow \delta Q = 0$

$$dU = dW = -P\,dV$$

$$dU = d(V u(T)) = u(T)\,dV + V u'(T)\,dT$$

$$u(T)\,dV + V u'(T)\,dT = -\alpha u(T)\,dV$$

$$V u'(T)\,dT = -(\alpha+1) u(T)\,dV$$

$$\frac{dV}{dT} = \frac{-V}{\alpha+1}\,\frac{u'(T)}{u(T)}$$

$$-dV\,\frac{\alpha+1}{V} = \frac{u'(T)}{u(T)}\,dT$$

$$-(\alpha+1)\ln(V) = \ln(u(T)) + C_1$$

$$\ln\left(u(T)\cdot V^{\alpha+1}\right) = \text{const}$$

$\therefore$ $u(T) V^{\alpha+1}$ is constant for an adiabatic process

isothermal $\Rightarrow dT = 0$

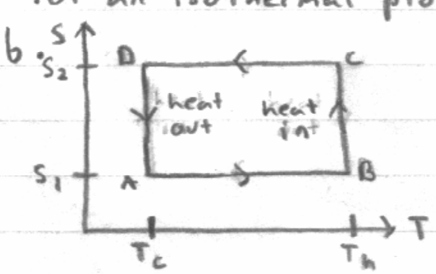
$$dU = \delta Q + dW$$

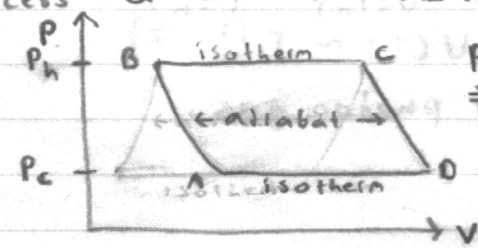
$$u(T)\,dV = \delta Q - \alpha u(T)\,dV$$

$$\delta Q = (\alpha+1) u(T)\,dV$$

For an isothermal process $Q = (\alpha+1) u(T) \Delta V$

b.





$p = \alpha u(T)$
$\Rightarrow$ const T means const P

$$Q_{in} = (\alpha+1) u(T_h)(V_C - V_B)$$

$$Q_{out} = (\alpha+1) u(T_C)(V_D - V_A)$$

$dU_{cycle} = 0 \Rightarrow W_{done} = Q_{in} - Q_{out}$

$$W_{done} = (\alpha+1) u(T_h)(V_C - V_B) - (\alpha+1) u(T_c)(V_D - V_A)$$

$U(T_h) V_B^{\alpha+1} = U(T_c) V_A^{\alpha+1}$

$V_A = V_B \left(\frac{U(T_h)}{U(T_c)}\right)^{\frac{1}{\alpha+1}}$

$V_D = V_C \left(\frac{U(T_h)}{U(T_c)}\right)^{\frac{1}{\alpha+1}}$

$\eta = \frac{W_{done}}{Q_{in}}$

$= \frac{\alpha+1}{\alpha+1}\left[1 - \frac{U(T_c)}{U(T_h)}\left(\frac{V_D - V_A}{V_C - V_B}\right)\right]$ $\quad 1 - \frac{1}{\alpha+1} = \frac{\alpha+1-1}{\alpha+1} = \frac{\alpha}{\alpha+1}$

$= \frac{\alpha+1}{\alpha+1}\left[1 - \frac{U(T_c)}{U(T_h)}\left(\frac{U(T_h)}{U(T_c)}\right)^{\frac{1}{\alpha+1}}\right]$

$= \frac{\alpha+1}{\alpha+1}\left[1 - \left(\frac{U(T_c)}{U(T_h)}\right)^{\alpha/\alpha+1}\right]$

$\eta = 1 - \left(\frac{U(T_c)}{U(T_h)}\right)^{\alpha/\alpha+1}$

c. $\eta = \frac{W_{done}}{Q_{in}}$ $\quad Q_{in} = T_h \Delta S$

$= \frac{Q_{in} - Q_{out}}{Q_{in}}$ $\quad Q_{out} = T_c \Delta S$

$\eta = 1 - \frac{Q_{out}}{Q_{in}}$

Since $\Delta S = S_2 - S_1$ is the same for both isotherms:

$\eta = 1 - \frac{T_c}{T_h} \cdot \frac{\Delta S}{\Delta S}$

$\eta = 1 - \frac{T_c}{T_h}$

d. $\therefore \left(\frac{U(T_c)}{U(T_h)}\right)^{\alpha/\alpha+1} = \frac{T_c}{T_h}$

$\frac{U(T_c)}{U(T_h)} = \left(\frac{T_c}{T_h}\right)^{\frac{\alpha+1}{\alpha}}$

$\Rightarrow U(T) \sim T^{(1+\frac{1}{\alpha})}$

e. A photon gas