

2001 Part II Q2

Asymptotics

$$y'' + \frac{y}{1+x} = x \quad x \rightarrow \infty \quad \text{asymptotic behavior} \quad \text{general solution}$$

$$\Rightarrow (1+x)y'' + y = x + x^2$$

$$\text{particular solution: } y \sim x^2$$

$$\text{homogeneous solution: } (1+x)y'' + y = 0$$

$$(1+x)[S'' + (S')^2] + 1 = 0$$

$$S'' + (S')^2 + xS'' + x(S')^2 + 1 = 0$$

$$\underbrace{\hspace{10em}}_{S'^2 = -\frac{1}{x}} \quad S' = \pm \frac{i}{\sqrt{x}}$$

$$S' = \pm \frac{i}{\sqrt{x}} + g'$$

$$S'' = \mp \frac{\frac{1}{2}i}{x^{3/2}} + g''$$

$$(S')^2 = -\frac{1}{x} \pm \frac{2ig'}{\sqrt{x}} + g'^2$$

$$\overset{\sim 0}{\mp \frac{\frac{1}{2}i}{x^{3/2}}} + \overset{\sim 0}{g''} - \frac{1}{x} \pm \frac{2ig'}{\sqrt{x}} + g'^2 \overset{\sim 0}{+ \frac{\frac{1}{2}i}{x^{3/2}} + xg''} - \cancel{1} \pm 2i\sqrt{x}g' + xg'^2 \cancel{+ 1} = 0$$

$$\mp \frac{\frac{1}{2}i}{x^{3/2}} \pm 2ix^{1/2}g' = 0 \quad \frac{1}{2x^{3/2}} = 2x^{1/2}g' \quad g' = \frac{1}{4x}$$

$$S' = \pm \frac{i}{\sqrt{x}} + \frac{1}{4x}$$

$$S = \pm 2ix^{1/2} + \frac{1}{4}\ln x$$

$$y = x^{1/4} e^{\pm i2x^{1/2}}$$

$$\text{real solutions: } y = x^{1/4} (C_1 \cos(2\sqrt{x}) + C_2 \sin(2\sqrt{x}))$$

$$y \sim x^2 + x^{1/4} [C_1 \cos(2\sqrt{x}) + C_2 \sin(2\sqrt{x})]$$