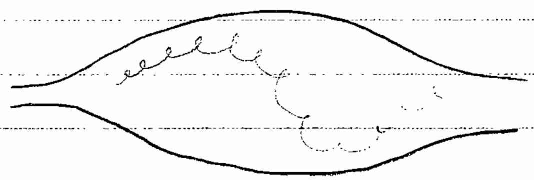
2005 Part II Q7 GPP I

a. Mirror confinement - regions of stronger and weaker magnetic field.

$\mu = \frac{W_\perp}{B}$ is conserved, so $E = W_\parallel + \mu B$, and E is conserved. At higher B,

$W_\parallel$ must decrease $\Rightarrow$ the particle may turn around and be reflected if B_{max} is high enough

Drifts: Curvature and ∇B



b. Suppose the particle has $V_{\parallel 0}$ and $V_{\perp 0}$ on the midplane.

Then $\mu = \frac{\frac{1}{2} m V_{\perp 0}^2}{B_0}$.

$$E = \tfrac{1}{2} m v_{\parallel 0}^2 + \tfrac{1}{2} m v_{\perp 0}^2 = \tfrac{1}{2} m v_\parallel^2 + \tfrac{1}{2} m v_\perp^2$$
$$= \tfrac{1}{2} m v_\parallel^2 + \mu B$$
$$= \tfrac{1}{2} m v_\parallel^2 + \tfrac{1}{2} m v_{\perp 0}^2 \frac{B}{B_0}$$

$$\Rightarrow V_\parallel^2 = V_{\parallel 0}^2 - V_{\perp 0}^2 \left(\frac{B}{B_0} - 1 \right)$$

The particle is trapped if $V_\parallel = 0$ somewhere. Marginal trapping occurs if $V_\parallel = 0$ when $B = B_{max}$

$$0 = V_{\parallel 0}^2 - V_{\perp 0}^2 (R - 1) \qquad R = \frac{B_{max}}{B_0}$$

For trapping, $V_{\perp 0}^2 (R-1) > V_{\parallel 0}^2$

c. $\phi(z) = \phi_0 \frac{z^2}{L^2}$ $\qquad B(z) = B_0 \left(1 + \frac{z^2}{L^2}\right)$

conservation of energy: $\frac{1}{2} m V_\parallel^2 + \underbrace{\frac{1}{2} m V_\perp^2}_{\mu B = \frac{1}{2} m V_{\perp 0}^2 \cdot \frac{B}{B_0}} + q\phi(z) = \frac{1}{2} m V_{\parallel 0}^2 + \frac{1}{2} m V_{\perp 0}^2 + \cancel{q\phi(0)}$

$$V_\parallel^2 = V_{\parallel 0}^2 - V_{\perp 0}^2 \left(\frac{B}{B_0} - 1 \right) - \frac{2q}{m} \phi(z)$$

$$V_{\parallel}^2 = V_{\parallel 0}^2 - V_{\perp 0}^2 \frac{z^2}{L^2} - \frac{2q\phi_0}{m}\frac{z^2}{L^2}$$

Turning point z_{TP} when $V_{\parallel}=0$

$$\Rightarrow \quad V_{\parallel 0}^2 = \left(V_{\perp 0}^2 + \frac{2q\phi_0}{m}\right)\frac{z_{TP}^2}{L^2}$$

$$z_{TP}^2 = \frac{L^2 V_{\parallel 0}^2}{V_{\perp 0}^2 + \frac{2q\phi_0}{m}}$$

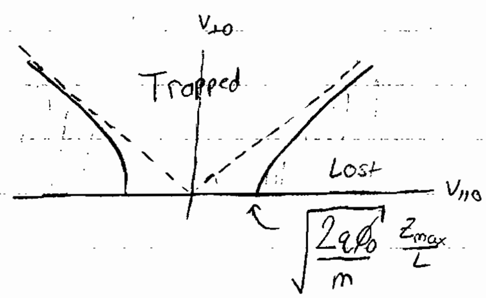
d. Trapped if it has turning point before $z = z_{max}$. Marginally trapped particles have $V_{\parallel}=0$ at z_{max}.

$$V_{\parallel 0}^2 - V_{\perp 0}^2 \frac{z_{max}^2}{L^2} = \frac{2}{m} q\phi_0 \frac{z_{max}^2}{L^2} \qquad \text{Equation of hyperbola}$$

Hyperbola orientation depends on sign of $q\phi_0$

For $q\phi_0 > 0$ (ions + positive potential, or electrons + negative potential)

$$V_{\parallel 0}^2 - V_{\perp 0}^2 \frac{z_{max}^2}{L^2} = \frac{2}{m} q\phi_0 \frac{z_{max}^2}{L^2}$$



For $q\phi_0 < 0$, $\quad V_{\perp 0}^2 \frac{z_{max}^2}{L^2} - V_{\parallel 0}^2 = \frac{2}{m}|q\phi_0| \frac{z_{max}^2}{L^2}$