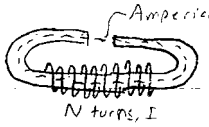
May 2008 #3 (EM)

a. solenoid, $N=1000$, $n = 10$ turns/cm, $I = 100A$

$B = \mu_0 n I = .126\ T$

b. soft iron core — magnetomotive force from Ampere's Law:

$\oint \vec{H} \cdot d\vec{\ell} = I_{enc} = NI$



$NI = H_1 L_1 + H_2 L_2$ $\quad H_1$ in iron, $L_1 = 3m$, $\quad H_1 = \frac{B_1}{\mu_r \mu_0}$

H_2 in air, $L_2 = 0.3m$, $\quad H_2 = \frac{B_2}{\mu_0}$

$$NI = L_1 \cdot \frac{B_1}{\mu_r \mu_0} + L_2 \cdot \frac{B_2}{\mu_0} = \frac{L_1}{\mu_r \mu_0 A_1} \cdot B_1 A_1 + \frac{L_2}{\mu_0 A_2} \cdot B_2 A_2$$

$NI = R_1 \Phi_1 + R_2 \Phi_2$ $\quad R$ = Reluctance, Φ = flux

$\Phi_1 = \Phi_2$: conservation of magnetic flux around a loop (from $\nabla \cdot \vec{B} = 0$)

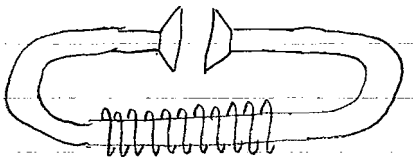
$NI = (R_1 + R_2)\Phi$

$\mu_r = 400$, $A_1 = A_2 = 400\ cm^2 = .04\ m^2$

$\Rightarrow \Phi = .016\ T \cdot m^2$

$B = \frac{\Phi}{A}$ $\quad \boxed{B_{core} = B_{gap} = .41\ T}$

c. $B_{core} = 1.5\ T$



$A_{pole} = 1600\ cm^2 = .16\ m^2$

Conservation of flux: $B_{core} A_{core} = B_{gap} A_{gap}$

$B_{gap} = B_{core} \cdot \frac{A_{core}}{A_{gap}}$ $\quad \frac{A_{core}}{A_{gap}} = \frac{1}{4}$

$B_{gap} = \frac{1}{4} B_{core} = \frac{1}{4} \cdot \frac{3}{2} T = \frac{3}{8} T = .375\ T$

$\boxed{B_{gap} = .375\ T}$