2002 Part I Q5

Exp.

$R_0 = 1m$, $a = 0.2m$ $B = 5T$ on axis $n_e = n_0\left(1-\frac{r^2}{a^2}\right)$ $T_e = T_0\left(1-\frac{r^2}{a^2}\right)$ $T_0 = 1\,keV$

$n_0 = 10^{20}\,m^{-3}$

a. optical depth for frequency of second harmonic X-mode on axis

$\Omega = \frac{eB}{m} = 8.8\cdot10^{11}\,s^{-1}$

optical depth: $\tau = \frac{L\,\alpha_m}{m\,\Omega}$ $L = \frac{B}{\frac{dB}{dR}} \cong R_0$

$$\alpha_m = \underbrace{\frac{\pi\,\omega_{pe}^2}{2c}}_{1.67\cdot10^{15}\,m^{-1}s^{-1}}\ \frac{m^{2m-1}}{(m-1)!}\left(\frac{T_e}{2m_ec^2}\right)^{m-1}$$

$1.67\cdot10^{15}\,m^{-1}s^{-1}\cdot 8\cdot(9.8\cdot10^{-4})^1$

$\alpha_2 = 1.3\cdot10^{13}\,m^{-1}s^{-1}$

$\tau_2 = 7.4$

b. $1.67\cdot10^{15}\,m^{-1}s^{-1}\cdot\frac{243}{2}\cdot(9.8\cdot10^{-4})^2$ $\alpha_3 = 1.9\cdot10^{11}\,m^{-1}s^{-1}$

$\tau_3 = .074$

c. Take Ω as const $\alpha_2 = 1.67\cdot10^{15}\cdot\left(1-\frac{r^2}{a^2}\right)\cdot8\cdot(9.8\cdot10^{-4})\cdot\left(1-\frac{r^2}{a^2}\right)^{2-1}$

$\alpha_2 = 1.3\cdot10^{13}\left(1-\frac{r^2}{a^2}\right)^2$

$\Rightarrow \tau_2 = 7.4\cdot\left(1-\frac{r^2}{a^2}\right)^2$

where does $\tau_2 = 2$? ($\tau > 2 \Leftrightarrow$ behaves like a blackbody)

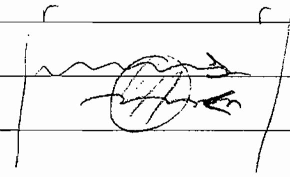
$\sqrt{\frac{2}{7.4}} = 1-\frac{r^2}{a^2}$ $\frac{r^2}{a^2} = 1-\sqrt{\frac{2}{7.4}}$ $\frac{r}{a} = \sqrt{1-\sqrt{\frac{2}{7.4}}}$ $\frac{r}{a} = 0.69 \Rightarrow$

$r = .14\,m$

d. $n_0 = 1\cdot10^{19}$ Then $\tau = .74\left(1-\frac{r^2}{a^2}\right)^2$

$\Rightarrow \tau$ is never > 2.

e. Wall, 90% reflectivity



Initial radiation sourced from plasma: $I_\omega = S_\omega(1-e^{-\tau})$

Each subsequent pass suffers $e^{-\tau}$ loss

Total I_ω passing through plasma:

$$I_\omega = S_\omega(1-e^{-\tau})\left[1 + re^{-\tau} + r^2e^{-2\tau} + \dots\right]$$

$$= \frac{S_\omega(1-e^{-\tau})}{1-re^{-\tau}}$$

For no reflection, "blackbody" was when the factor $1-e^{-\tau} > 0.9$ $(\tau \gtrsim 2)$

Now, require $\dfrac{1-e^{-\tau}}{1-re^{-\tau}} > 0.9 \Rightarrow 1-e^{-\tau} > 0.9(1-0.9e^{-\tau})$

$> 0.9 - 0.81e^{-\tau}$

solve for τ: $0.1 > .19\,e^{-\tau}$ $\quad .53 > e^{-\tau}$

$\rightarrow \tau > 0.64$

Substitute $.64$ for 2 in (c):

$$\frac{r}{a} = \sqrt{1-\sqrt{\frac{.64}{2.4}}} = .84 \qquad r = .168\,m$$

If $n_e = 10^{19}$, $\quad \dfrac{r}{a} = \sqrt{1-\sqrt{\dfrac{.64}{.74}}} = .26 \qquad r = .05\,m$