Jan 2008 #1 (QM)

$m_n = 939.5$ MeV $\quad m_p = 938.2$ MeV

$a \approx 1.5$ fm, binding energy $E_b = 2.226$ MeV

$$V(r) = \begin{cases} -V_0 & r<a \\ 0 & r>a \end{cases}$$

Use the reduced mass, $m = \frac{m_n m_p}{m_n + m_p}$

a. Solve Sch-Eq for $l=0$

radial eqn: $\psi = \frac{U(r)}{r}$ $\quad \frac{-\hbar^2}{2m}\frac{d^2U}{dr^2} + V(r)U = EU \qquad U(0)=0$

$r<a$: $\frac{-\hbar^2}{2m}\frac{d^2U}{dr^2} = (E+V_0)U \qquad \frac{d^2U}{dr^2} = \frac{-2m(E+V_0)}{\hbar^2}U = -k^2U$

$k^2 = \frac{2m(E+V_0)}{\hbar^2}$

$U = \sin kr \qquad r<a$

$r>a$: $\frac{-\hbar^2}{2m}\frac{d^2U}{dr^2} = EU \qquad \frac{d^2U}{dr^2} = q^2U \qquad q^2 = \frac{-2mE}{\hbar^2}$

$U = Ae^{-q(r-a)}$

U continuous at a: $\sin ka = A \qquad \tan ka = -\frac{k}{q} \Rightarrow \boxed{\tan ka = \frac{-ka}{qa}}$ quantization condition

U' " " : $k\cos ka = -qA$

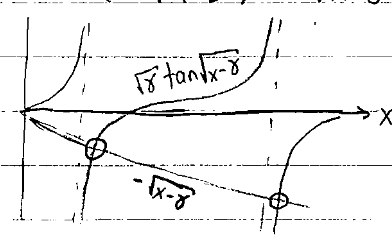
$(ka)^2 = \frac{2mEa^2}{\hbar^2} + \frac{2mV_0a^2}{\hbar^2} = -\gamma + x \qquad \gamma = \frac{-2mEa^2}{\hbar^2} \qquad x = \frac{2mV_0a^2}{\hbar^2}$

$(qa)^2 = \frac{-2mEa^2}{\hbar^2} = \gamma$

We are given the energy of the ground state, $E = -E_b$

$\Rightarrow$ In this state, $\gamma = .12$

$\sqrt{\gamma}\left(\tan\sqrt{x-\gamma}\right) = -\sqrt{x-\gamma}$ solve graphically for x



$x = 3.24,\ 23.01, \ldots \infty$

Correct solution corresponding to a ground state is $x=3.24$. For $x=23.01$, the solution with $E = -E_b = -2.226$ MeV is the second solution; there is a solution with lower energy

$$x = \frac{2mV_0 a^2}{\hbar^2} = 3.2366$$

$$\frac{x}{\gamma} = \frac{V_0}{E} = \frac{3.2366}{.12} = 26.8$$

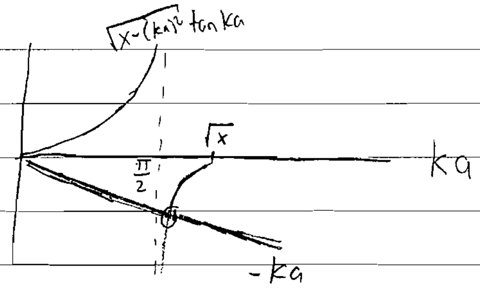
$$\boxed{V_0 = 59.66 \text{ MeV}}$$

b. Check for multiple solutions with this value of V_0

$\tan ka = \dfrac{-ka}{qa}$ $\qquad (ka)^2 + (qa)^2 = x$ $\qquad x = \dfrac{2mV_0 a^2}{\hbar^2}$

$\Rightarrow \quad (qa)^2 = x - (ka)^2, \qquad (qa) = \sqrt{x-(ka)^2}$

$$\sqrt{x-(ka)^2}\,\tan ka = -ka$$



There is only one possible solution, the ground state for $l=0$
$\Rightarrow$ No excited states with $l=0$

c. $$\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2} + \underbrace{\left[V + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]}_{V_{eff}} U = EU$$

$l=1$: $\dfrac{\hbar^2}{ma^2}$