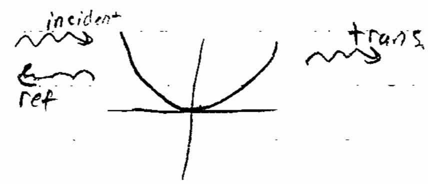
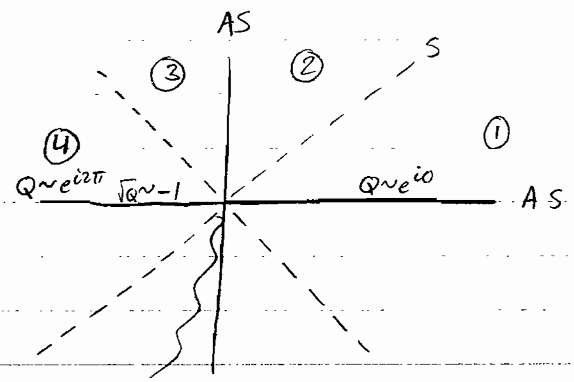
2000 Part I Q8 Asymptotics

$$\frac{d^2\psi}{dz^2} + z^2\psi = 0 \qquad Q = z^2$$



Anti Stokes Line: $\sqrt{Q}dz = \text{real} \rightarrow z\,dz = \text{real}$ $\quad z + dz$ real, or $z + dz$ imaginary



Boundary condition: on the right, there is only a transmitted (rightgoing) wave

Start in region ①

$$(0,z) \sim e^{i\int_0^z \sqrt{Q}dz} \sim e^{\frac{1}{2}iz^2} \qquad e^{\frac{1}{2}iz^2 - i\omega t}$$

As t increases, z^2 increases for stationary phase
$\rightarrow$ z increases (since z is positive)

① $(0,z)$ is the rightgoing wave

dominant or subdominant. Let $z = x + i\epsilon$

$$z^2 = x^2 - \epsilon^2 + i2x\epsilon \qquad e^{iz^2} \sim e^{-2x\epsilon} \rightarrow \text{subdominant}$$

① $(0,z)_s$

② $(0,z)_s$

③ $(0,z)_d$

④ $(0,z)_d + T(z,0)_s \qquad T = \sqrt{2}\,i$

$\quad (0,z)_d + \sqrt{2}\,i\,(z,0)_s$

$$(0,z) \sim \exp\left(i\int_0^z \sqrt{Q}dz\right) \sim \exp\left(-i\int_z^0 \sqrt{Q}dz\right)$$

$\sqrt{Q} \sim -1$ here! $\quad \sqrt{Q} = \sqrt{z^2} = z$, with $z < 0$

$$\sim \exp\left(i\int_0^{\bar{z}} z\,dz\right) \sim \exp\left(\tfrac{1}{2}iz^2\right)$$

$$e^{i(\frac{1}{2}z^2 - i\omega t)}$$

As t increases z^2 increases (and z is negative) so z gets more negative.

$\longrightarrow$ Reflected wave

$(2,0)$ is incoming

$R = \frac{1}{\sqrt{2}i}$ $T = \frac{1}{\sqrt{2}i}$

$|R|^2 = \frac{1}{2}$ $|T|^2 = \frac{1}{2}$