2013 I Q1

$$\chi_s = \frac{2\omega_{ps}^2}{k^2 V_{ts}^2}\left\{1 + \frac{\omega - kV_0}{kV_{ts}} Z(\zeta)\right\} \qquad \zeta_s = \frac{\omega}{kV_{ts}}$$

$\frac{\omega_{ps}^2}{\omega^2} \gg 1$ $\quad T_e \gg T_i \Rightarrow \zeta_i \gg 1$ warm ions

$\zeta_e \ll 1$ hot electrons

$$Z(\zeta \gg 1) \simeq -\frac{1}{\zeta} - \frac{1}{2\zeta^3} \qquad Z(\zeta \ll 1) \simeq -2\zeta$$

$V_0 = 0$ $\qquad \varepsilon = 1 + \sum_s \chi_s = 0$

Real part of dielectric:

$$1 + \frac{2\omega_{pi}^2}{k^2 V_{ti}^2}\left\{1 - \zeta_i\left(\frac{1}{\zeta_i} + \frac{1}{2\zeta_i^3}\right)\right\} + \frac{2\omega_{pe}^2}{k^2 V_{te}^2}\left\{1 - 2\zeta_e^2\right\} = 0$$

$$1 - \frac{\omega_{pi}^2}{\omega^2} \cdot \frac{1}{2} + \frac{2\omega_{pe}^2}{k^2 V_{te}^2}\left\{1 - 2\zeta_e^2\right\} = 0$$

Use $\frac{\omega_{ps}^2}{\omega^2} \gg 1$ $\quad \times\omega^2$ on both sides $\Rightarrow$ $\omega_{pi}^2 = \frac{2\omega_{pe}^2}{k^2 V_{te}^2}\left\{1 - 2\zeta_e^2\right\}\omega^2$

To lowest order, $\zeta_e^2 \simeq 0$ $\qquad \omega^2 = \frac{\omega_{pi}^2}{\omega_{pe}^2}\frac{k^2 V_{te}^2}{2} \simeq k^2 \frac{T_e}{m_i} \equiv k^2 C_s^2$

Next order $\qquad \omega^2 = \frac{k^2 C_s^2}{1 - 2k^2 C_s^2}$

$$= \frac{k^2 C_s^2}{1 - m_e/m_i}$$

$\zeta_e^2 = \frac{\omega^2}{k^2 V_{te}^2} = \frac{C_s^2}{V_{te}^2} \sim \frac{m_e}{2m_i}$

Imaginary part: $\gamma'' = \frac{2\omega_{pe}^2}{k^2 v_{te}^2} \pi^{1/2} e^{-\omega^2/k^2 v_{te}^2}$ $\omega^2 \simeq k^2 c_s^2$

$$= \frac{2\omega_{pe}^2}{k^2 v_{te}^2} \pi^{1/2} e^{-m_e/m_i}$$