Three-Dimensional MHD Simulations of Co- and Counter-Helicity Spheromak Merging in SSX using the HYM Code

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Introduction to Spheromak Merging

Spheromaks and Field-Reversed Configurations (FRCs) belong to the Compact Torus (CT) subset of magnetic fusion configurations. The magnetic field in spheromaks has both toroidal and poloidal components, while the field in FRCs is purely poloidal. Both configurations are "self-organized" and simply connected such that they must rely on internal currents to sustain their respective magnetic topologies.

Spheromak merging is a process by which two spheromaks in close proximity combine via magnetic reconnection in order to reach a lower energy state. Experimentally, this process is studied by launching two spheromaks at each other inside a flux conserving shell.

Helical Fieldlines — Spheromak

Poloidal Fieldlines / Toroidal Current — Field-Reversed Configuration

Merging Configuration

There are several possible spheromak merging geometries due to the availability of spheromaks with both right-handed (RH) and left-handed (LH) helicity. Each merging geometry leads to different behaviors based on the total helicity in the system and the direction of the currents in each spheromak. The two configurations highlighted in red are those that were studied here.

The configurations with "attractive currents" are expected to merge axisymmetrically, while those with "repulsive currents" will not merge without instability.

Motivation for studying spheromak merging:
- **Slow formation of high-flux FRCs (counter-helicity)**
- **Magnetic reconnection physics in all merging experiments**
- **Taylor relaxation physics (co-helicity)**

The HYbrid Magnetohydrodynamic (HYM) Code

- The **HYM code** was developed at PPPL by Elena Belova to study FRC formation and stability.
- HYM is a fully explicit nonlinear 3D code with three primary modes of operation:
    - (1) Resistive MHD (single fluid & Hall MHD (two fluid))
    - (2) Hybrid (fluid electrons with PIC ions)
    - (3) MHD/Particle (single-fluid thermal plasma with an additional kinetic ion particle distribution
- Includes self-consistent equilibria with kinetic ion effects.
- Fully parallelized with 3D domain decomposition/MPI and good processor scaling.
- Code is designed to run with realistic initial and boundary conditions for experimental modeling.
- The resistive MHD mode was used for SSX simulations.
- **Visualization:**
    - Existing HYM visualization routines (IDL) suitable for 2D analysis
    - Visualization has now been expanded to 3D using the **VisIt** visualization software package
    - HYM data was ported to the SILO binary database format for use with **VisIt**
    - **VisIt** facilitates 3D fieldline tracing and 3D rendering of other variables (pressure, velocity)
- Working on further quantitative analysis (local Fourier mode activity, energy & helicity evolution).

The Swarthmore Spheromak Experiment (SSX)

- Merging experiments were first started by Yamada & Ono at the University of Tokyo in 1990.
- Studies of spheromak merging have continued on the Magnetic Reconnection Experiment (MRX) at Princeton (M. Yamada, S. Gerhardt), on the Swarthmore Spheromak Experiment (SSX) at Swarthmore College (M. Brown, et al.), and elsewhere.
- The HYM simulations discussed here were set up to closely match the initial and boundary conditions of merging experiments on SSX with a 3:1 flux conserver.
- SSX uses two coaxial magnetized plasma guns to launch spheromaks at each other (in both the co- and counter-helicity merging geometries).
- SSX measures the magnetic topology of the plasma with 96 magnetic probes that are placed at a variety of axial, radial, and toroidal locations.
- SSX also has an Ion Doppler Spectroscopy (IDS) system to measure line-integrated velocities.

Counter-Helicity Merging Simulations

- The currents in the two merging spheromaks are attractive, so the FRCs will be pulled together and reconnect quasi-axisymmetrically.
- The system has zero total helicity, so the reconnection process is expected to annihilate the toroidal field such that the configuration will settle into an FRC-like state with only poloidal field remaining.
- The configuration that is observed in both the simulations and the SSX experiment is a partially merged compact torus with residual toroidal field (the "Doublet CT").

Run parameters:
- Physics parameters:

$B_0 = 1.0$ kG	$R_c = 20.3$ cm	$\tau_{ci} \simeq 0.1$ μs	$Re \simeq 500$
$n_0 = 10^{15}$ cm^{-3}	$L_c = 61.0$ cm	$t_{A0} \simeq 2.9$ μs	$S \simeq 1000$
$v_{A0} \simeq 70$ km/s	$c/\omega_{pi} \simeq 0.7$ cm	$t_{sim} \simeq 200$ μs	

- Initial & boundary conditions closely model the experiments on SSX.
- 257×129×32 (z×r×φ) grid used to resolve the midplane reconnection region.

Initial Configuration & Early Time Evolution

- The initial magnetic configuration (see left) has a private flux region for each spheromak and a public flux region that has only poloidal field.
- The x-point that is visible in the initial configuration is the seed point for reconnection as the two spheromaks are pulled together.

Cycle 01 / $t = 0.0\ t_{A0}$ / $t = 0.0$ μs

Pressure / Velocity

- The initial spheromak currents ohmically heat the plasma early in the simulation. This causes the plasma pressure to quickly rise (see the first two frames on the left).
- After just a few Alfvén times (~5), the spheromaks begin to reconnect in the midplane. A pronounced reconnection layer is visible in the velocity profile (in the second frame).
- At $t \sim 13\ t_{A0}$ the reconnection outflow peaks, with the maximum velocity reaching ~15% of the Alfvén velocity.
- The high reconnection flow is almost exclusively in the toroidal direction. This phenomena is discussed in the slingshot effect section on the right.

The "Slingshot Effect"

- Due to the magnetic topology of the attractive-current counter-helicity merging geometry, the fieldlines near the reconnection region have sharp kinks (see the first set of figures below).
- The reconnection process releases the tension in these kinks, which allows the fieldlines to straighten as they move away from the reconnection region.
- The straightening fieldlines toroidally "slingshot" the plasma in opposite directions (Yamada, et al., 1990).

Toroidal Flow Channels / Reconnecting Fieldlines

As reconnected fieldlines move away from the X-point, they violently straighten out and slingshot the plasma in the toroidal direction.

The Metastable "Doublet CT" Configuration

Pressure / Velocity

- Reconnection dominates from $t \sim 10$–30 t_{A0} such that at $t \sim 30\ t_{A0}$, the two spheromaks have partially merged
- At this point, reconnection has slowed significantly and the plasma remains in a partially merged metastable state with some remaining toroidal field
- Some of the plasma pressure is now shared within the reconnected fieldlines, but the highest pressure contours remain confined to the original spheromaks
- This metastable configuration that has zero net helicity but it still maintains some toroidal field is called the "Doublet CT" (Cothran, et al., 2003)

- The magnetic configuration of the Doublet CT is shown below. Fieldlines from both private flux surfaces with TF (left) and shared surfaces without TF (right) can be found. The closure of the fieldlines in the shared surface is an open question.
- The 2D vector plots on the right compare the Doublet CT data from the HYM simulations and from SSX experiments. The agreement between the two is remarkable.

Cycle 30 / $t = 30.8\ t_{A0}$ / $t = 90.5$ μs

SSX / HYM

Late Time Instability

Pressure / Velocity

The colors in the two frames on the left have been renormalized to emphasize the tilt instability.

Late in time, before the toroidal fields can be completely annihilated, the configuration becomes unstable and tilts away.

Summary of Counter-Helicity Results

Conclusions:
- This counter-helicity merging geometry reconnects in a quasi-axisymmetric configuration to form a Doublet CT.
- The "slingshot effect" is observed during reconnection.
- The toroidal field of each spheromak is not entirely annihilated (likely due to high ion viscosity) and private flux surfaces persist until the configuration becomes unstable.
- Excellent agreement is observed between HYM simulations and SSX experiments

Future Work:
- Expand analysis to include Fourier mode activity as well as energy & helicity time evolution.
- Scan the viscosity to more closely match SSX timescales and instability growth.

Co-Helicity Merging Simulations

- The currents in the two spheromaks now repel each other, which means that the spheromaks will not merge without first becoming unstable.
- The system has non-zero magnetic helicity, so Taylor relaxation is expected. Though the relaxation process may be very asymmetric, the final state of the magnetic configuration should be the lowest energy helicity-conserving eigenstate.

Run parameters:
- Same physics parameters as the counter-helicity simulation.
- Initial conditions and boundary conditions are designed to closely model the experiments on SSX.
- Run grid of 129×65×32 (z×r×φ)

Cycle 15 / $t = 14.1\ t_{A0}$ / $t = 41.4$ μs	Cycle 30 / $t = 28.2\ t_{A0}$ / $t = 82.9$ μs	Cycle 40 / $t = 37.6\ t_{A0}$ / $t = 110.5$ μs	Cycle 50 / $t = 47.0\ t_{A0}$ / $t = 138.1$ μs	Cycle 67 / $t = 63.0\ t_{A0}$ / $t = 185.1$ μs	Flat Pressure Profile

Time

As the time series above shows, co-helicity merging in this geometry is complex.

The spheromaks tilt before merging asymmetrically and relaxing to a Taylor state.

The spheromaks must first tilt in order to have their currents pull together and drive reconnection.

The tilt begins around halfway through the simulation.

Following the initial tilt, the poloidal field (blue) begins to reconnect asymmetrically.

Topologically, the reconnecting poloidal field becomes a single magnetic axis that threads the original toroidal field axes.

In order for the toroidal field to reconnect, it must twist into a figure-8. This model roughly explains the lowest energy helicity-conserving Taylor state (see below left).

Non-axisymmetric Reconnection Region

Close Examination of Final Taylor State

Final state from HYM simulation

Cycle 67 / $t = 63.0\ t_{A0}$ / $t = 185.1$ μs

Reduce the fieldline count

The final state of the simulation can be compared to ideal MHD eigenstate calculations and experimental data (see left).

The figure-8 form of the calculated ideal eigenstate (PSI-TET) is also seen in the HYM simulation. The other magnetic axis was difficult to locate.

2D vector plots of HYM simulations and SSX experiments have many similarities (see right).

The opposing toroidal twist of the figure-8 is visible in the east and west r-θ projections.

A paper detailing the experimentally observed final co-helicity state has been submitted (Cothran, et al.).

Ideal Taylor Eigenstate (PSI-TET, G. Marklin)

SSX / HYM

Summary of Co-Helicity Results

Conclusions:
- The co-helicity merging geometry undergoes a tilt-then-reconnect process that is very asymmetric.
- A simplified model has been constructed to explain the topological changes in the magnetic configuration during the simulation.
- The final Taylor state matches well with both the PSI-TET ideal eigenstate calculations and SSX experimental data.

Future Work:
- Expand analysis to include Fourier mode activity as well as energy & helicity time evolution.
- Increase the grid resolution to look at changes in the observed behavior.