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Advances in the Numerical Modeling of Field Reversed Configurations *†

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OUTLINE:

I. Introduction

II. Theta-pinch-formed (prolate) FRC

- Behavior of the tilt and rotational instabilities, and the ion toroidal spin-up has been reproduced in the nonlinear simulations.

III. New formation methods

- Numerical study of counter-helicity spheromak merging explains several experimental observations.

IV. Energetic beam ion stabilization

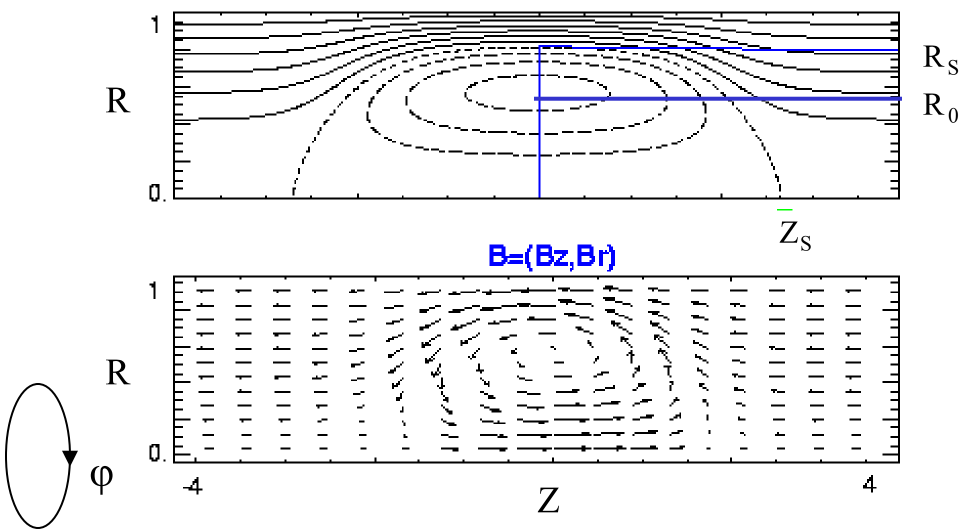
- Numerical study of NBI stabilization of low-n modes in oblate FRCs.

V. New regime of stability: E~1

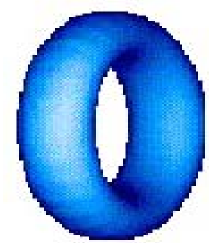
- New stability regime has been found numerically, which requires conducting shell and NBI stabilization.

FRC geometry and parameters

Poloidal flux contours



Prolate FRC
E >> 1



Oblate FRC
E < 1

Parameters:

$E = Z_S / R_S$ - separatrix elongation;

n - toroidal mode number;

$S^* = R_S / \lambda_i$ - kinetic parameter, equals to the ratio of separatrix radius to ion skin depth (Hall effects);

$\bar{s} = (R_S - R_0) / \rho_i$ - number of ion Larmor radii in configuration (FLR effects).

$\bar{s} \sim 1 \rightarrow$ kinetic FRC, $\bar{s} >> 1 \rightarrow$ MHD-like FRC.

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Experimental results

Formation

- Several formation methods have been developed:

  Theta-pinch formation (E>>1)

  Counter-helicity spheromak merging method (E~1)

  Rotating Magnetic Field (RMF) method (E>1)

Parameters

T_i=10-700eV, $T_e \leq T_i$, n~10^{11}-10^{15} cm^{-3}, ion toroidal rotation V_φ~0.1-0.3V_A.

Stability properties

- The n=2 rotational mode is the only global mode observed experimentally in prolate FRCs with E>>1, and which often terminates the FRC.
- The n=2 mode can be stabilized by application of quadrupole fields.
- The strong instability of the n=1 tilt mode is predicted by MHD theory, but not observed in the prolate FRC experiments.
- The n=1 tilt mode has been observed in oblate FRC experiments, stabilized by conducting shells.

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FRC offers experimental and theoretical challenges

Motivation for this study:

- Existing experimental data (mostly prolate FRCs, and kinetic regime).
- Long-standing disagreement between theory and experiment regarding the n=1 tilt-mode stability.
- Attractive FRC features as a fusion reactor (high beta, translation, compact and simple geometry).
- New formation methods have been developed.

Experimental issues:

- Formation of large-S* FRCs
- Stability of low-n MHD modes
- Sustainment/current drive methods
- Confinement; scaling with R_s/ρ_i

Theoretical issues:

- Large ion Larmor radius
- Two-fluid effects
- Rotation
- Stochasticity of ion orbits
- High beta
- Relaxation, self-organization

→ Need for sophisticated numerical tools

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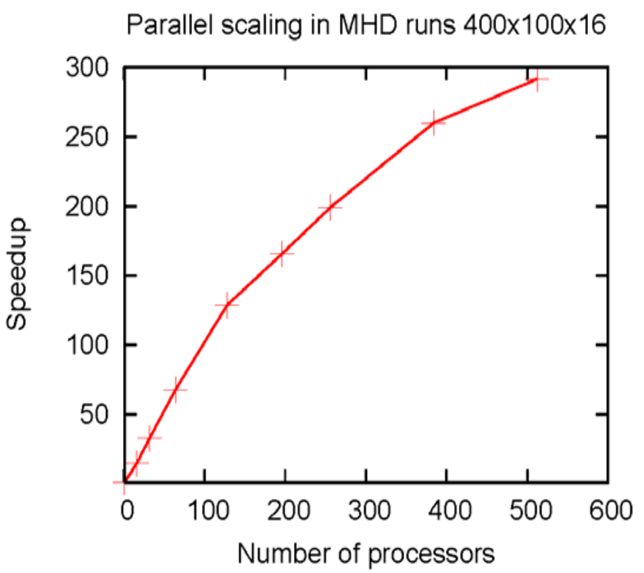
HYM – Parallel Hybrid/MHD Code

HYM code developed at PPPL and used to investigate FRC formation and stability properties

- 3-D nonlinear.
- Three different physical models:
  - Resistive MHD & Hall-MHD.
  - Hybrid (fluid electrons, particle ions).
  - MHD/particle (one fluid thermal plasma, + energetic particle ions)
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)[1].



N_{procs}

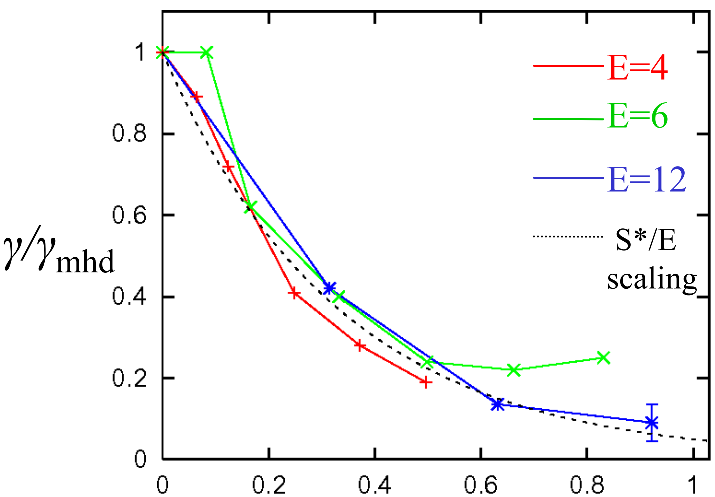
New MPI version of HYM shows good parallel scaling up to 500 processors for production-size jobs, and allows high-resolution nonlinear simulations.

[1]Simulations are performed at NERSC.

HYM Explains S*/E Scaling Observed Experimentally

S/E* parameter determines the experimental stability boundary [*M. Tuszewski,1998*].

Hybrid simulations have shown that ion FLR effects determine linear stability properties of the *n*=1 tilt mode in prolate FRCs.



$$(S^*/E)^{-1} = E\rho_i / R_s$$

Hybrid simulations for equilibria with elliptical separatrix and different elongations: *E*=4, 6, 12. For *E*/*S**>0.5, resonant ion effects are important.

New empirical scaling (hybrid simulations):

$$\gamma = \underbrace{\frac{V_A}{R_s E}}_{\text{MHD}} \underbrace{\exp\left(-3E\frac{\rho_i}{R_s}\right)}_{\text{kinetic (FLR)}}$$

<u>FRC parameters</u>: *E* – elongation, *n* – toroidal mode number, *S** - ratio of separatrix radius to Larmor radius.

Stochasticity of ion orbits is found to have an important effect on FRC stability

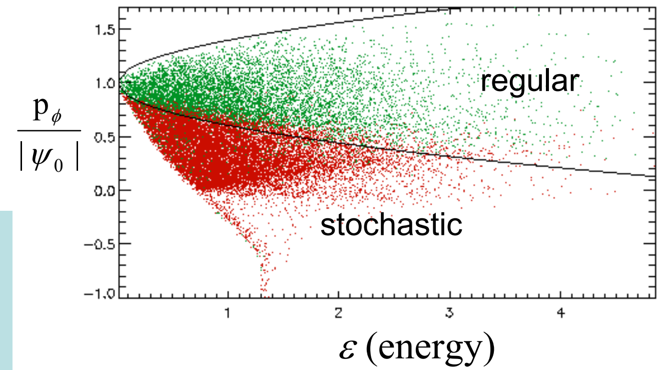
Stochasticity of equilibrium ion orbits -
– is due to large field line curvature near the ends in prolate FRCs .

Main results:

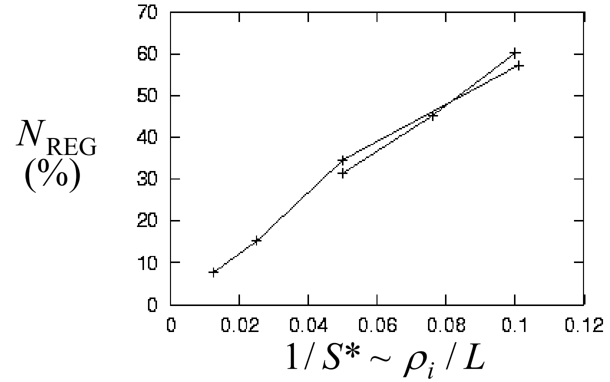
- Regularity condition has been derived.
- Number of regular orbits has been shown to scale linearly with 1/*S**.
- Wave-particle resonances are shown to occur only in the regular region of the phase-space.

Conclusions:

- Stochasticity of ion orbits is not strong enough to prevent resonant instabilities in kinetic regime.



Regular versus stochastic portions of particle phase space for *S*=20*, *E=6*.



Fraction of regular orbits in kinetic FRC equilibria with *E=6* and *E=12*.

Fraction of regular orbits ~ 1/*S**

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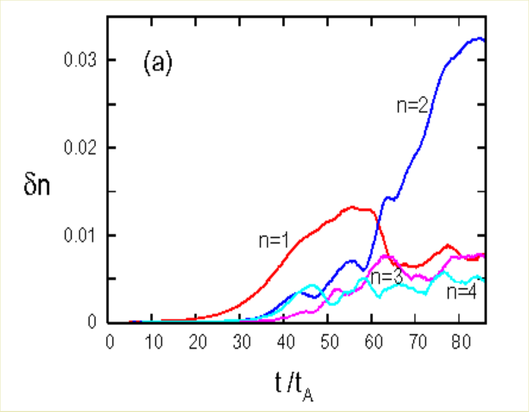
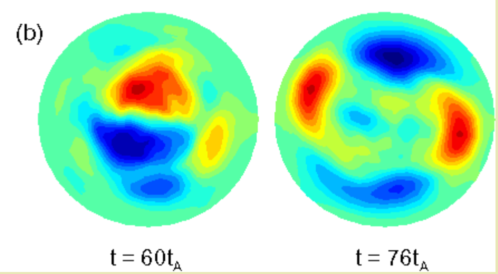
Nonlinear Evolution and Ion Toroidal Spin-up are Now Understood



Time evolution of different Fourier harmonics of density perturbation.

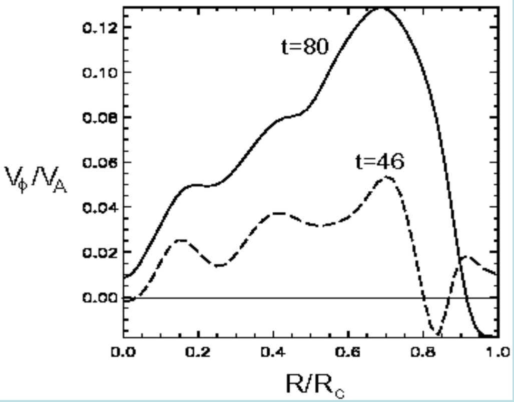
Nonlinear studies of the evolution of MHD modes in FRC configurations demonstrate the importance of ion toroidal spin-up on the nonlinear saturation of the *n*=1 tilt mode, and growth of the *n*=2 rotational instability.



Contour plots of density perturbation from 3D hybrid simulations of a kinetic prolate FRC.

Ion spin-up is related to the resistive decay of magnetic flux and the resulting loss of weakly confined particles from the closed-field-line region.

- Spin-up up to 0.1- 0.3V_A in the direction of the current observed at t>40 t_A.
- Similar ion rotation is seen in the experiments.



Radial profile of ion flow velocity.

New FRC formation methods

- Traditional theta-pinch formation methods are limited to low flux, highly kinetic configurations with s~1-2. ‾

  FRC behavior at low-(S*/E) is best understood and recent theoretical studies provide explanation for experimentally observed FRC properties.

- **Large-S* FRCs:** New (slow) FRC formation schemes and FRC stability properties in the MHD-like reactor-relevant regimes are yet to be investigated both experimentally and theoretically.

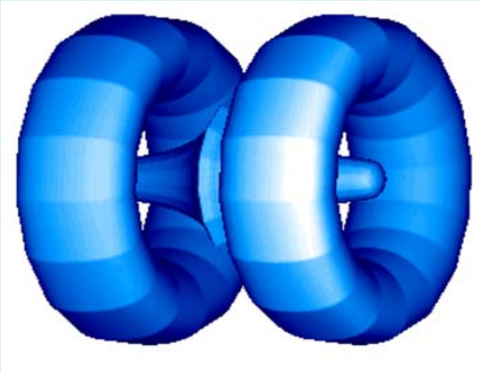
- **New formation methods:**

  - Counter-helicity spheromak merging

    U. Tokyo, SSX-FRC, MRX [S. Gerhardt et al., poster GP1. 104].

  - Rotating magnetic field RMF-FRC

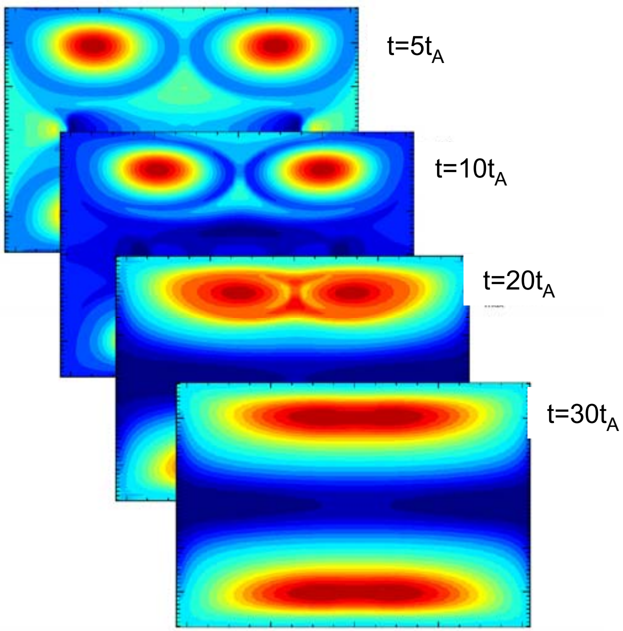
    U. Washington, PPPL.



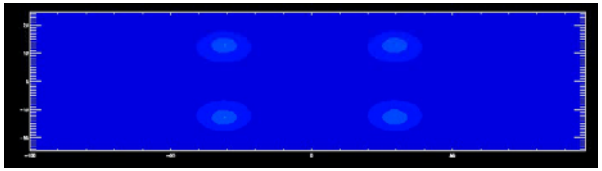
MHD simulation of counter-helicity spheromak merging; pressure isosurfaces are shown.

3D simulations performed with goal of improving the basic understanding of FRC formation techniques by spheromak merging, and large-*S** FRC stability properties

Simulation Study of Counter-Helicity Spheromak Merging and FRC Formation



2D counter-helicity spheromak merging simulations in support of SSX-FRC experiment. Pressure evolution.



New FRC formation method by counter-helicity spheromak merging has been developed in U. Tokyo, MRX and SSX-FRC experiments.

- Three-dimensional MHD simulations of counter-helicity spheromak merging using HYM code provides an explanation for the observed plasma behavior in the SSX-FRC experiment (Swarthmore College):

    Persistence of the toroidal fields and slower-than-MHD growth of the tilt instability are shown to be related to field line-tying effects and large viscosity in SSX plasmas.

- New signatures of Hall-reconnection have been observed in two-fluid simulations of MRX (PPPL) experiment with different magnetic field polarities.

    - MRX posters on Wednesday afternoon [M. Inomoto LP1. 97].

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Oblate FRCs have different stability properties than prolate FRCs

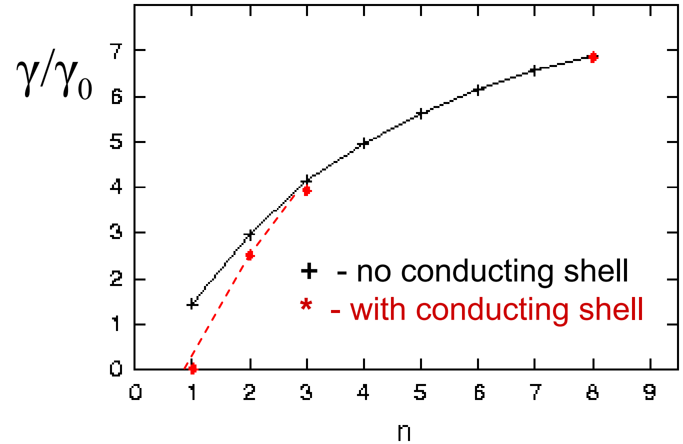
Prolate FRCs (E >> 1)

- All unstable modes (n≥1) are internal.
- Conducting shell has little effect on stability.
- FLR effects are stabilizing for low *S*/E*.

Oblate FRCs (E<1)

- n=1 *tilt and radial shift* modes can be stabilized by conducting shells.
- *Interchange modes* (n≥1) can be stabilized by profile effects.
- n>1 *co-interchange (kink-like)* modes are internal modes, and they remain unstable in the presence of a close-fitting conducting shell.
- S*/ E >>1 → FLR stabilization is weak.

MHD results E=0.7

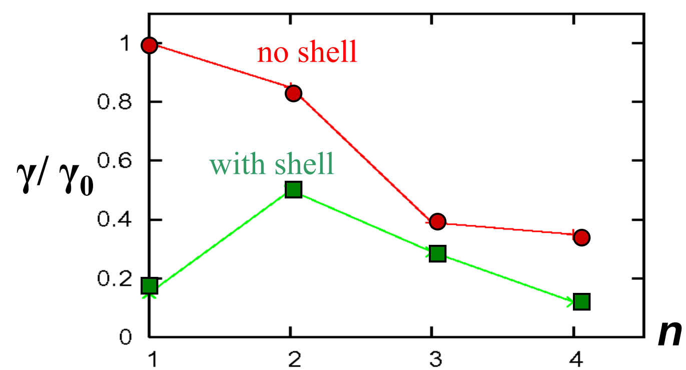


Linear growth rates of n≥1 co-interchange modes from MHD simulations with E=0.7 and peaked current profile. The n=1 tilt mode is an external mode, and it is stabilized by the close-fitting conducting shell.

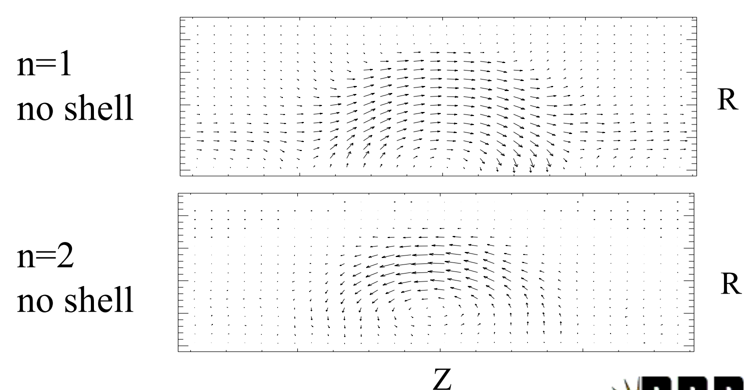
Conducting shell stabilization of low-*n* MHD modes

- Hybrid simulations for S*=18 and E=1.1 show that the *n*=1 tilt mode is the most unstable mode with $\gamma=0.83\gamma_{mhd}$, and the growth rates of the *n*>1 modes are reduced by FLR effects – in contrast to the ideal MHD.
- The unstable modes are axially (or radially) polarized co-interchange modes.
- The modes with *n*=1-4 have an external mode structure, and their growth rates are reduced significantly by the conducting shell.
- With conducting shell stabilization, the *n*=2 co-interchange mode becomes the most unstable mode for E~1.
- Due to mode localization, energetic neutral beam ions may be effective in stabilizing these residual instabilities.

Kinetic simulation results E=1.1



Normalized growth rates of the *n*=1-4 modes from 3D hybrid simulations including kinetic effects (red) and the effects of conducting shell (green) for E=1.1 and flat current profile.



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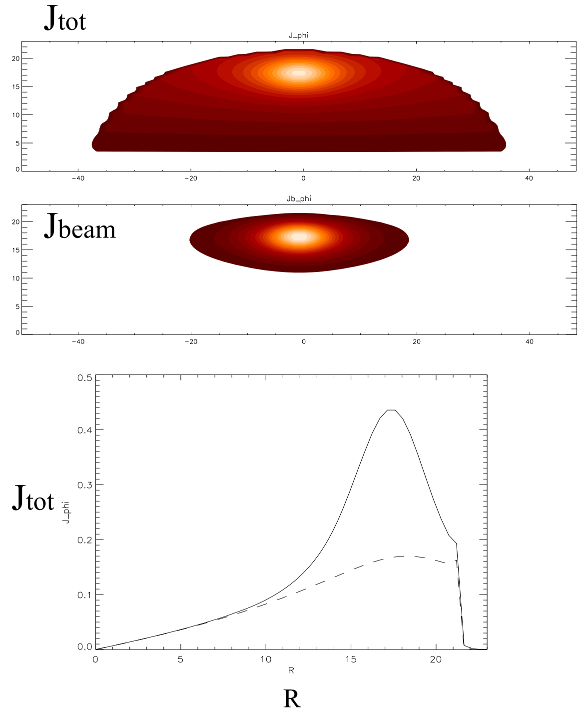
Injection of energetic ion beams may provide additional stabilizing mechanism, as well as plasma heating and current drive

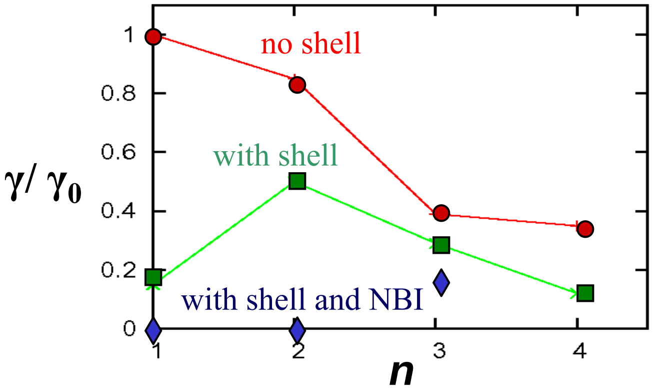
- Large beam ion density n_b or small T_b results in strongly localized beam profiles.
- Due to localization, the peak NBI current density J_b can be comparable to the local thermal plasma current density, even when the fraction of the total current carried by the NBI ions is small.
- Strong beams are highly localized and can be destabilizing - beam parameters have to be chosen carefully.
- Calculations have been performed in support of MRX-FRC experimental proposal, for E~1-2, n_b/n_i=0.01-0.05, and V_0=4-6V_A.



Contour plots of the toroidal total and beam ion current, and radial profiles of the total and bulk plasma current for E=1.7, V_0=6.2V_A, T_b=10, and beam density n_b=0.04.

FRC stability including close-fitting conducting shell and energetic beam ion effects: I. Linear results

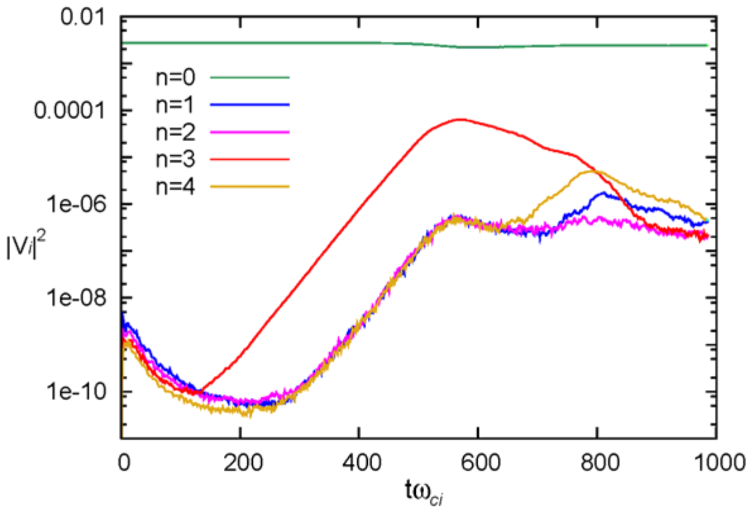
- Close-fitting conducting shell stabilizes all low-*n* radially-polarized (even) modes.
- Due to localization, the ion beams are effective in stabilizing the residual low-*n* instabilities, except for relatively cold beams which have a destabilizing effect on $n \geq 3$ modes.
- The NBI effects are stronger for lower-*n* modes (n=1 and n=2), and smaller V_0.
- The n=1 tilt mode and the n=2 mode are stabilized, and the growth rate of the n=3 mode is reduced for E~1, n_b/n_i=0.03, and $V_0 = 6V_A$.



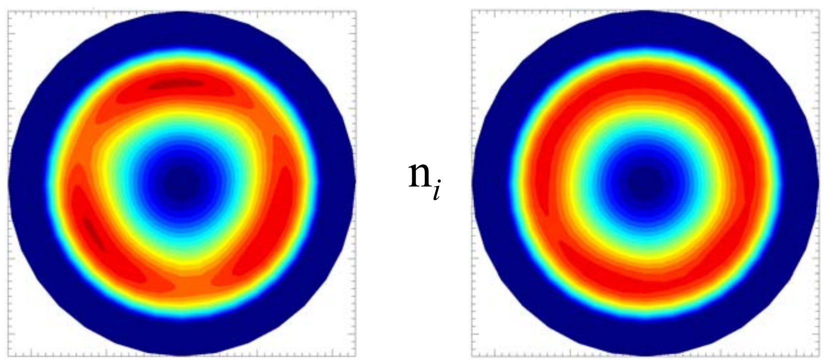
Normalized growth rates of the *n*=1-4 modes from 3D hybrid simulations including the effects of conducting shell and NBI stabilization.

FRC stability including close-fitting conducting shell and energetic beam ion effects: II. Nonlinear simulations

- Nonlinear 3D simulations show that the residual instabilities (n=3 mode) saturate at small amplitudes.
- FRC remains stable with respect to **all MHD modes**, as long as it is sustained.



With current drive



$t=30\ t_A$ $t=50\ t_A$

Nonlinear hybrid simulations of an FRC with E=1.1, including the effects of the beam ions and the close-fitting conducting shell. (a) Time evolution of n=0-4 modes kinetic energy; and (b) contour plots of plasma density in the toroidal cross sections.

Simulations have been performed in support of MRX-FRC experimental proposal.

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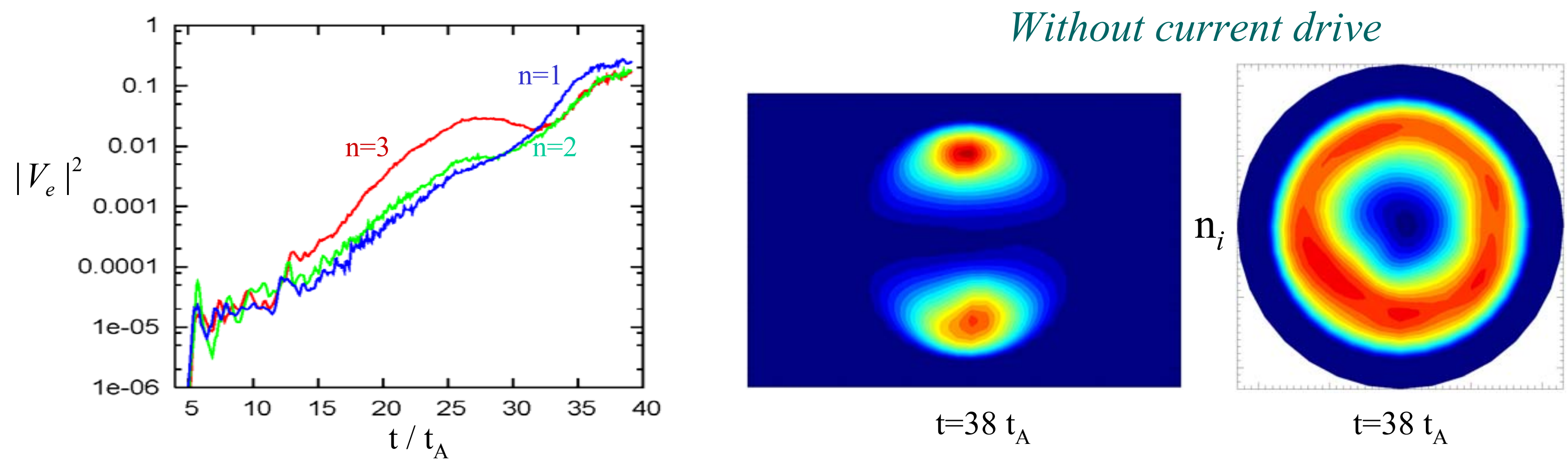
FRC stability including close-fitting conducting shell and energetic beam ion effects: II. Nonlinear simulations

Effects of the current decay:

Nonlinear simulations show that the residual instabilities (n=3 mode) saturate in the nonlinear regime, but the n=1 tilt mode become unstable at $t \sim 30 t_A$ due to reduction of the stabilizing effects of the conducting shell.

→ Current drive is needed for stability!



Nonlinear hybrid simulations of an FRC with E=1.1, including the effects of the beam ions and the close-fitting conducting shell. (a) Time evolution of n=1-3 modes energy; (b) and (c) contour plots of plasma density in the poloidal and toroidal cross sections at $t=38t_A$.

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Conclusions

- Linear and nonlinear stability properties of prolate kinetic (theta-pinch-formed) FRCs have been explained, including behavior of the tilt and rotational instabilities, and the ion toroidal spin-up.

- A new stability regime has been discovered for oblate FRCs with a close-fitting conducting shell and energetic beam ion stabilization.
  - *Linearly stable with respect to the n=1 tilt mode and the n=2 modes,*
  - *Residual instabilities saturate nonlinearly at small amplitudes,*
  - *Configuration remains MHD stable, if current is sustained.*

- New FRC formation method by counter-helicity spheromak merging has been investigated using MHD and Hall-MHD simulations, which contributed to understanding of experimental results from SSX-FRC (Swarthmore) and MRX (PPPL) spheromak-merging experiments.