Nonlinear ponderomotive force by low frequency waves and nonresonant current drive

Zhe Gao
Department of Engineering Physics, Tsinghua University, Beijing 100084, China

Nathaniel J. Fisch and Hong Qin
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

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The collisionless nonresonant force by low frequency waves has been thought to be capable of driving the nonresonant current. However, for a single particle, the ponderomotive force is in the direction of the gradient of the wave field energy. For cold plasmas, the Reynolds stress acting on the Lagrangian fluid element fully counteracts the nonresonant force offered by the quasilinear electromagnetic force. For hot plasmas, the collisionless nonresonant force is also cancelled by the nonlinear kinetic stress force. Therefore, in collisionless plasmas, none of the ponderomotive forces by low frequency waves can drive the nonresonant current. *© 2006 American Institute of Physics*. [DOI: 10.1063/1.2397584]

I. INTRODUCTION

Low-frequency waves, Alfvén waves or fast waves, were considered as an attractive mechanism of driving plasma current because of its potential high efficiency, no density limit, and the convenience of high power rf generating and launching.[1,2] However, electron trapping may dramatically reduce the current drive efficiency in the subthermal resonant regime, although possibly the momentum carried by trapped electrons will be retained and returned by collisions or the bootstrap current in the steady state.[1,3] Alternatively, the possibility of increasing the current drive efficiency by helicity injection has been proposed by Ohkawa.[4] This idea was later described[5,6] as helicity balance between the input by the wave and the dissipation by resistivity and viscosity, and developed[7,8] for arbitrary polarization waves. The concept of helicity conservation during plasma relaxation has also been demonstrated theoretically[9] and applied to form and sustain the plasma current in toroidal pinches, spheromaks and low aspect ratio tokomaks.[10–12]

The scheme of helicity injection current driven by low frequency waves has also been referred to as a "dynamo effect" or a "ponderomotive force" of the applied waves. The general form of the rf-induced force has been developed by Klima,[13] Elfimov *et al.*,[14] and Tyspin *et al.*[15] in a two-fluid model; and by Chan and Chiu[16] and Fukuyama *et al.*[17] in the kinetic formalism. It was found that the parallel plasma current can be maintained by means of forces due to the radial gradient of rf field amplitude. Moreover, this mechanism was considered to be typically nonresonant because the ponderomotive force acts on the bulk plasma rather than on resonant particles. It has been argued, even in the collisionless case, that the nonresonant force can exist due to spatial nonuniformity or spatial dispersion. If this is true, this nonresonant current drive in toroidal geometry does not depend on trapped particle effects and the current drive efficiency is expected to be strongly increased.

However, there is a significant disconnection between these nonresonant rf forces[13–17] and the ponderomotive force in the single particle picture. The well-known ponderomotive force is only in the direction of the gradient of second-order field quantities. For pure propagating waves without any dissipation, such as collisions, resonant (Landau) damping or mode conversion, the rf field is fully symmetrical in the propagating direction. It is thus expected that nonresonant plasma cannot feel any force. However, previous results nonetheless find a parallel force, which depends on the cross-field variation of the rf field and acts on the nonresonant plasma. Therefore, it is of great interest both in theory and in practice to clarify the incongruity between the physical single-particle picture and the mathematical treatment of the nonresonant current drive.

In fact, Litwin[18] found that the only single particle force in the parallel direction is the frictional ponderomotive force and that the steady-state collisionless dynamo effect is absent in the double-adiabatic magnetohydrodynamics (MHD) theory. Although Tsypin *et al.*[15] pointed out that the Landau damping due to viscosity is important but is omitted in Litwin's model, it should be noted that the Landau damping implies that the force relies on the resonant effect. However, in Ref. 15, where the contributions from Reynolds stress and nonlinear viscosity are included in the total ponderomotive force and the first order perturbation is calculated from the drift kinetic equation, no result is given to show the absence of the collisionless nonresonant drive.

In this work, we resolve the apparent incongruity between the force on a single particle and that on a Lagrangian fluid element. It is found that the force due to the movement of a single particle along the gradient of the electric field disappears in the fluid picture; however, another force due to the charge accumulation by the divergence of flow appears. This electric force, combined with the Lorentz force, constitutes the so-called nonresonant force. However, the stress tensor, which should be included in the fluid picture, is fully or partly ignored in previous studies. Considering this term, the nonresonant force is cancelled and the fluid picture becomes consistent with the single particle picture. Also, the kinetic formalism of the rf force is derived based on the second-order rf kinetic theory.[19] The collisionless nonresonant force offered by the quasilinear electromagnetic (EM) force is completely counteracted by the nonlinear stress pressure force. Thus, in collisionless plasmas, all the ponderomotive forces that can lead to net current depend on Landau damping.

The paper is organized as follows: Sec. II gives a description of the single particle ponderomotive force. The Lagrangian fluid element analysis based on the momentum equation is given in Sec. III. Section IV is devoted to the kinetic theory of rf force and Sec. V gives the discussion and summary.

II. SINGLE PARTICLE PONDEROMOTIVE FORCE

We consider a constant background magnetic field $B_0\hat{\mathbf{z}}$ and a rf wave with electric and magnetic fields, $\mathbf{E}_1$, $\mathbf{B}_1 \propto \exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$. If the spatial perturbed displacement and the average drift displacement are small compared to the scale of the field, the ponderomotive force can be represented as

$$\mathbf{F} = \frac{q}{2}\,\mathrm{Re}[(\boldsymbol{\delta}_1\cdot\nabla)\mathbf{E}_1^* + \mathbf{v}_1\times\mathbf{B}_1^*], \tag{1}$$

where $\boldsymbol{\delta}_1$ and $\mathbf{v}_1$ is the perturbed displacement and velocity, and $\mathbf{B}_1$ are the wave magnetic field. The perturbed displacement $\boldsymbol{\delta}_1$ can be written straightforwardly as

$$\boldsymbol{\delta}_1 = (\mathbf{T}^{-1}\boldsymbol{\alpha}\mathbf{T})\mathbf{E}_1, \tag{2}$$

where

$$\alpha = \frac{-1}{\omega B}\begin{pmatrix} \dfrac{\Omega}{\omega-\Omega} & 0 & 0 \\ 0 & \dfrac{\Omega}{\omega+\Omega} & 0 \\ 0 & 0 & \dfrac{\Omega}{\omega} \end{pmatrix},$$

and

$$T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & +i & 0 \\ 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}.$$

Using $\mathbf{v}_1=-i\omega\boldsymbol{\delta}_1$ and $\mathbf{B}_1=(1/i\omega)\nabla\times\mathbf{E}_1$, we get the ponderomotive force

$$\mathbf{F} = \frac{q}{2}\,\mathrm{Re}[(\boldsymbol{\delta}_1\cdot\nabla)\mathbf{E}_1^* + \boldsymbol{\delta}_1\times(\nabla\times\mathbf{E}_1^*)] = -q\nabla\Phi, \tag{3}$$

where Φ is the ponderomotive potential and is defined as[20]

$$\Phi \equiv -\frac{1}{4}(\mathbf{E}_1^*\cdot\mathbf{T}^{-1}\boldsymbol{\alpha}\mathbf{T}\cdot\mathbf{E}_1) = \frac{e}{4m\omega}\left[\frac{|E_{x1}+iE_{y1}|^2}{2(\omega-\Omega)} + \frac{|E_{x1}-iE_{y1}|^2}{2(\omega+\Omega)} + \frac{|E_{z1}|^2}{\omega}\right]. \tag{4}$$

Therefore, for a single particle, the $\hat{\mathbf{z}}$ component force exists only when the gradient of the second-order rf field quantities in the $\hat{\mathbf{z}}$ direction does not vanish.

The absence of forces in the symmetry directions can be thought of as a result of the cancellation between the electric force and the magnetic force. For clarity, we simplify the electric field as $\mathbf{E}_1=E_{y1}(x)\cos\varphi\hat{\mathbf{y}}+E_{z1}(x)\sin\varphi\hat{\mathbf{z}}$, where $\varphi=\omega t-k_z z$ is the phase of the wave. The perturbed displacement, velocity, and magnetic field are given as follows:

$$\boldsymbol{\delta}_1 = \frac{E_{y1}}{\omega B}\frac{\Omega}{\Omega^2-\omega^2}(\Omega\sin\varphi\hat{\mathbf{x}} + \omega\cos\varphi\hat{\mathbf{y}}) - \frac{E_{z1}}{\omega B}\frac{\Omega}{\omega}\sin\varphi\hat{\mathbf{z}}, \tag{5}$$

$$\mathbf{v}_1 = \frac{E_{y1}}{B}\frac{\Omega}{\Omega^2-\omega^2}(\Omega\cos\varphi\hat{\mathbf{x}} - \omega\sin\varphi\hat{\mathbf{y}}) - \frac{E_{z1}}{B}\frac{\Omega}{\omega}\cos\varphi\hat{\mathbf{z}}, \tag{6}$$

and

$$\mathbf{B}_1 = -\frac{k_z E_{y1}}{\omega}\cos\varphi\hat{\mathbf{x}} - \frac{1}{\omega}\frac{\partial E_{z1}}{\partial x}\cos\varphi\hat{\mathbf{y}} - \frac{1}{\omega}\frac{\partial E_{y1}}{\partial x}\sin\varphi\hat{\mathbf{z}}. \tag{7}$$

Therefore, the $\hat{\mathbf{z}}$ component of the $\mathbf{v}_1\times\mathbf{B}_1$ force is

$$\langle \mathbf{F}_{\mathbf{v}\times\mathbf{B}}\cdot\hat{\mathbf{z}}\rangle = \langle q(v_{x1}B_{y1}-v_{y1}B_{x1})\rangle = -\frac{q\Omega^2}{2\omega B(\Omega^2-\omega^2)}E_{y1}\frac{\partial E_{z1}}{\partial x}, \tag{8}$$

and the parallel force due to the movement of a single particle along the gradient of the electric field is

$$\langle \mathbf{F}_{(\boldsymbol{\delta}\cdot\nabla)\mathbf{E}}\cdot\hat{\mathbf{z}}\rangle = \left\langle \delta_x\frac{\partial}{\partial x}E_{z1} + \delta_z\frac{\partial}{\partial z}E_{z1}\right\rangle = \frac{q\Omega^2}{2\omega B(\Omega^2-\omega^2)}E_{y1}\frac{\partial}{\partial x}E_{z1}. \tag{9}$$

Here, $\langle\,\rangle$ represents the time average. Thus, the two forces counteract each other and the total ponderomotive force on a single particle is zero in the $\hat{\mathbf{z}}$ direction. Understanding the single particle force is this way will be important in understanding the fluid picture.

III. LAGRANGIAN FLUID ELEMENT ANALYSIS

In this section, we will show how previous studies give a parallel force which depends on the cross-field variation of the rf field and, more importantly, what is missing. For a fluid element at a fixed spatial location (Lagrangian fluid element), the force due to the displacement of a single particle, $\langle \mathbf{F}_{(\boldsymbol{\delta}\cdot\nabla)\mathbf{E}}\cdot\hat{\mathbf{z}}\rangle$, disappears. However, a new force, which is the electric field force acting on the charged fluid element, appears. The charge accumulation is caused by the divergence of flow, that is, $\partial(n_1 q)/\partial t=-n_0 q\nabla\cdot\mathbf{v}_1$; therefore we can get the amount of charge using the perturbed velocities obtained in the last section,

$$n_1 q = -\frac{n_0 q \Omega^2}{\omega B(\Omega^2-\omega^2)}\left(\frac{\partial E_{y1}}{\partial x}\sin\varphi - k_z E_{z1}\cos\varphi\right). \quad (10)$$

Then, the parallel electric force on the Lagrangian fluid element is

$$\langle \mathbf{F}_{n\mathbf{E}}\cdot\hat{\mathbf{z}}\rangle = \langle n_1 q E_{z1}\rangle = -\frac{n_0 q\Omega^2}{2\omega B(\Omega^2-\omega^2)}E_{z1}\frac{\partial E_{y1}}{\partial x}. \quad (11)$$

Adding with the $\mathbf{v}_1\times\mathbf{B}_1$ force on the fluid element with density n_0, the total parallel force is

$$\langle \mathbf{F}_{n\mathbf{E}}\cdot\hat{\mathbf{z}}\rangle + n_0\langle \mathbf{F}_{\mathbf{v}\times\mathbf{B}}\cdot\hat{\mathbf{z}}\rangle = -\frac{n_0 q\Omega^2}{2\omega B(\Omega^2-\omega^2)}\frac{\partial E_{y1}E_{z1}}{\partial x}. \quad (12)$$

This force is the so-called nonresonant force of applied waves. The force can be described as the divergence of helicity flux, and can be rewritten as $-(n_0 q/B)\mathrm{Re}(\mathbf{E}_1\cdot\mathbf{B}_1)$ for low frequency waves. Therefore, the dissipation of the wave helicity was thought to have the same effect to sustain the plasma current as that of the dc helicity.

But, it is *not exact*. If we consider a Lagrangian fluid element, we must include the stress in the momentum equation. For cold plasma, the stress reduces to the Reynolds stress $-n_0\nabla\cdot\mathbf{v}_1\mathbf{v}_1 = -n_0(\nabla\cdot\mathbf{v}_1)\mathbf{v}_1 - n_0(\mathbf{v}_1\cdot\nabla)\mathbf{v}_1$. It is easy to verify

$$\langle \mathbf{F}_{\mathrm{Re}}\cdot\hat{\mathbf{z}}\rangle = \langle(-n_0\nabla\cdot\mathbf{v}_1\mathbf{v}_1)\cdot\hat{\mathbf{z}}\rangle = \frac{n_0 q\Omega^2}{2\omega B(\Omega^2-\omega^2)}\frac{\partial E_{y1}E_{z1}}{\partial x}. \quad (13)$$

Therefore, the parallel component of the total force acting on a fluid element reduces to zero, i.e.,

$$\langle \mathbf{F}_{\mathrm{total}}\cdot\hat{\mathbf{z}}\rangle = \langle \mathbf{F}_{nE}\cdot\hat{\mathbf{z}}\rangle + n_0\langle \mathbf{F}_{v\times B}\cdot\hat{\mathbf{z}}\rangle + \langle \mathbf{F}_{\mathrm{Re}}\cdot\hat{\mathbf{z}}\rangle = 0. \quad (14)$$

Thus, the fluid picture becomes consistent with the single particle picture. In the view of helicity conservation, the stress can be considered as the fluid helicity of the fluctuations,[21] which fully compensates the dissipation of the EM helicity.

For hot plasmas, the thermal pressure and viscosity should be included in the fluid picture. A model for the equation of state or a closure scheme needs to be given. For example, the CGL model was used by Litwin.[18] However, different closure schemes may give different conclusions. Instead of adopting a particular closure scheme, we will carry out a nonlinear kinetic analysis in the next section for hot plasmas.

IV. NONLINEAR RF FORCE IN THE KINETIC FORMALISM

Expanding the distribution function in powers of the electric field as $f=f_0+f_1+f_2$, where f_0 is the equilibrium distribution function, f_1 is the linear response to the rf field, and f_2 is the second-order response slowly varying in time, the second-order, time-averaged Vlasov equation can be written as

$$\frac{\partial}{\partial t}f_2 + \mathbf{v}\cdot\nabla f_2 + \frac{q}{m}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial}{\partial\mathbf{v}}f_2 = -\left\langle \frac{q}{m}(\mathbf{E}_1+\mathbf{v}\times\mathbf{B}_1)\cdot\frac{\partial}{\partial\mathbf{v}}f_1\right\rangle. \quad (15)$$

Integrating Eq. (15) along unperturbed orbits and choosing $f_{2|t=0}=0$, we can get[19]

$$f_2 = -\int_0^t dt'\left\langle \frac{q}{m}(\mathbf{E}_1(\mathbf{r}',\mathbf{v}',t')+\mathbf{v}'\times\mathbf{B}_1)\cdot\frac{\partial}{\partial\mathbf{v}'}f_1(\mathbf{r}',\mathbf{v}',t')\right\rangle. \quad (16)$$

The time-averaged parallel momentum equation is

$$m\frac{\partial}{\partial t}\int d\mathbf{v}v_z f_2 = \left\langle \int d\mathbf{v}q[E_{1z}+(\mathbf{v}\times\mathbf{B}_1)_z]f_1\right\rangle - m\nabla\cdot\int d\mathbf{v}\mathbf{v}v_z f_2, \quad (17)$$

where the zero-order momentum balance has been used. The total parallel force is the sum of the EM force

$$F_{\mathrm{EM}z} = \left\langle \int d\mathbf{v}q[E_{1z}+(\mathbf{v}\times\mathbf{B}_1)_z]f_1\right\rangle \quad (18)$$

and the kinetic stress gradient

$$F_{\mathrm{KS}z} = -m\nabla\cdot\int d\mathbf{v}\mathbf{v}v_z f_2. \quad (19)$$

Obviously, the nonlinear kinetic stress term is the same order as the quasilinear EM term.

For a magnetic field $B_0\hat{\mathbf{z}}$ and a rf wave with electric and magnetic fields, $\mathbf{E}_1(\mathbf{r},t)$, $\mathbf{B}_1(\mathbf{r},t)\propto\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$, the first-order perturbed distribution function f_1 is given as[22]

$$f_1(\mathbf{r},\mathbf{v},t) = \frac{qf_M}{T}\sum_{\mathbf{k}}\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\sum_{m,n}J_m(\lambda)\times\exp[i(m-n)(\varphi-\theta)]\frac{1}{i\Delta_n}\mathbf{H}\cdot\mathbf{E}_1, \quad (20)$$

where f_M is Maxwellian for a temperature T, $J_m(\lambda)$ is the Bessel function of order m for $\lambda=k_\perp v_\perp/\Omega$, $\mathbf{H}\cdot\mathbf{E}_1 = (v_\perp E_+/2)J_{n-1}(\lambda)e^{-i\theta} + (v_\perp E_-/2)J_{n+1}(\lambda)e^{i\theta} + v_z E_z J_n(\lambda)$ with $E_\pm = E_{x1}\pm iE_{y1}$, $k_x=k_\perp\cos\theta$ and $k_y=k_\perp\sin\theta$, $\Delta_n = -(\omega - k_z v_z + n\Omega)$, Ω is the gyrofrequency and φ is the gyroangle. The second-order, time-averaged distribution function f_2 can be solved from Eq. (16). When a single wave is traveling in the symmetry directions, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$, the expression of f_2 is derived as Eqs. (34) and (35) in Ref. 19. Substituting f_1 and f_2 into Eqs. (18) and (19), we get

$$F_{\mathrm{EM}z} = \frac{\pi q^2}{T}\mathrm{Re}\sum_{\mathbf{k_R},\mathbf{k_L}}\exp[i(k_{Rx}-k_{Lx})x]\times\int dv_z\int v_\perp dv_\perp\sum_{l,k}\frac{1}{i\Delta_l}\mathbf{H}\cdot\mathbf{E}_{1R}\exp[il(\theta_R-\theta_L)]\times J_{l-k}(\lambda_L)[E_z^* J_k(\delta\lambda_x)+C_+^* J_{k-1}(\delta\lambda_x) + C_-^* J_{k+1}(\delta\lambda_x)]\exp(ik\theta_L), \quad (21)$$

$$F_{\mathrm{KS}z} = \frac{\pi q^2}{T} \mathrm{Re} \sum_{\mathbf{k_R},\mathbf{k_L}} \exp[i(k_{Rx} - k_{Lx})x] \times \int dv_z \int v_\perp dv_\perp \sum_l \frac{1}{i\Delta_l} \mathbf{H} \cdot \mathbf{E}_{1R} \exp[il(\theta_R - \theta_L)] \times \Bigg\langle - \sum_{n\neq 0} J_n(\delta\lambda_x)\exp(in\theta_L)[E_z^* J_{l-n}(\lambda_L) + C_+^* J_{l-n-1}(\lambda_L)\exp(i\theta_L) + C_-^* J_{l-n+1}(\lambda_L)\exp(-i\theta_L)] + v_z \frac{\delta\lambda_x}{v_\perp} \times \Bigg\{ \frac{A_+^*}{2}[J_0(\delta\lambda_x)J_l(\lambda_L) - J_1(\delta\lambda_x)J_{l-1}(\lambda_L)\exp(i\theta_L)] - \frac{A_-^*}{2}[J_0(\delta\lambda_x)J_l(\lambda_L) + J_1(\delta\lambda_x)J_{l+1}(\lambda_L)\exp(-i\theta_L)] + \frac{iB^*}{2} J_0(\delta\lambda_x) \times [J_{l-1}(\lambda_L)\exp(i\theta_L) + J_{l+1}(\lambda_L)\exp(-i\theta_L)] \Bigg\} \Bigg\rangle, \quad (22)$$

where $A_\pm = E_\pm(1 - k_z v_z/\omega) + E_{z1}k_\pm v_z/\omega$, $B = E_{y1}k_x v_\perp/\omega - E_{x1}k_y v_\perp/\omega$, $C_\pm = E_\pm k_z v_\perp/2\omega - E_{z1}k_\pm v_\perp/2\omega$ with $k_\pm = k_x \pm ik_y$, $\delta\lambda_x = \lambda_{Rx} - \lambda_{Lx}$ and the R and L are used to distinguish the two electric fields in the quadratic form. It can be seen that, except for the $J_0(\delta\lambda_x)$ terms, all the components in the quasilinear EM force are cancelled by the kinetic stress gradient force. Also, the kinetic stress gradient force provides an additional contribution due to the divergence in real space (proportional to $\delta\lambda_x$).

For clarity, we give the explicit form under the assumption of low frequency $\omega \ll \Omega$. Retaining the first order of $\rho(=v_t/\Omega)$, we get

$$F_{\mathrm{EM}z} \Big/ \left(\frac{q^2 n_0}{2T}\right) = \mathrm{Re}\Bigg\{ \frac{1+\zeta_0 Z_0}{ik_z}|E_{z1}|^2 + i\rho\frac{k_y v_t}{\omega} \times [1+\zeta_0 Z_0]\mathrm{Im}(E_{x1}E_{z1}^*) + i\rho\frac{k_z v_t}{\omega}\,\mathrm{Im}(E_{y1}E_{x1}^*) + \frac{\rho}{2}\frac{[1+\zeta_0 Z_0]}{ik_z\zeta_0} \times (E_{z1}' E_{y1}^* - E_{y1}' E_{z1}^*) + \frac{\rho}{2}\frac{v_t}{i\omega}\frac{\partial E_{y1}E_{z1}^*}{\partial x} - \frac{\rho}{2}\frac{k_y}{k_z}\frac{v_t}{i\omega}[1+\zeta_0 Z_0]\frac{\partial |E_{z1}|^2}{\partial x} \Bigg\}, \quad (23)$$

where Z_0 is the plasma dispersion function and its argument is $\zeta_0 = \omega/k_z v_t$. When k_y and k_z are real, Eq. (23) reduces to the same expression derived by Chan and Chiu,[16]

$$F_{\mathrm{EM}z} \Big/ \left(\frac{q^2 n_0}{2T}\right) = \mathrm{Re}\Bigg\{ \frac{\zeta_0 Z_0}{ik_z}|E_{z1}|^2 + i\frac{k_y\rho}{k_z} Z_0\,\mathrm{Im}(E_{x1}E_{z1}^*) + \frac{\rho}{2}\frac{Z_0}{ik_z}\mathrm{Re}\left(E_{y1}\frac{\partial E_{z1}^*}{\partial x} - E_z\frac{\partial E_{y1}^*}{\partial x}\right) + \frac{\rho}{2}\frac{Z_0}{k_z}\mathrm{Im}\,\frac{\partial (E_{z1}E_{y1}^*)}{\partial x} - \frac{\rho}{2}\frac{k_y}{k_z}\frac{Z_0}{ik_z}\frac{\partial |E_{z1}|^2}{\partial x} \Bigg\}. \quad (24)$$

Here, the first three terms, which depend on $\mathrm{Im}(Z_0)$, drive the current by Landau damping. The fourth term, which depends on $\mathrm{Re}(Z_0)$ and the spatial variation of the quadratic field, was believed to be the collisionless nonresonant ponderomotive force.[16] Combined with the fifth term, it can be rewritten as the form with the divergence of the wave helicity flux.[16,17]

However, the kinetic stress force is the same order as the quasilinear force, that is,

$$F_{\mathrm{KS}z} \Big/ \left(\frac{nq^2}{2T}\right) = \mathrm{Re}\Bigg[\left(-\frac{\rho}{2}\frac{Z_0}{ik_z}\right)\frac{\partial}{\partial x}E_{z1}E_{y1}^* + \frac{\rho}{2}\frac{k_y v_t}{\omega}\frac{1+\zeta_0 Z_0}{ik_z}\frac{\partial}{\partial x}|E_{z1}|^2 - \rho\frac{k_y v_t}{\omega}\left(\frac{1}{i2k_z} + \frac{\zeta_0^2(1+\zeta_0 Z)}{ik_z}\right)\frac{\partial}{\partial x}|E_{z1}|^2\Bigg]. \quad (25)$$

It is clear that the terms related to the divergence of the wave helicity in the EM force are fully cancelled. All the other additional terms depend on the $\mathrm{Im}(Z_0)$ as well as the spatial variation of second-order field quantities. Finally, we can write the total parallel ponderomotive force as

$$F_z \Big/ \left(\frac{nq^2}{2T}\right) = \mathrm{Re}\Bigg[\frac{1+\zeta_0 Z_0}{ik_z}|E_{z1}|^2 + i\rho\frac{k_y v_t}{\omega} \times (1+\zeta_0 Z_0)\mathrm{Im}(E_{x1}E_{z1}^*) + i\rho\frac{k_z v_t}{\omega}\,\mathrm{Im}(E_{y1}E_{x1}^*) - \rho\frac{k_y}{ik_z}\frac{v_t}{\omega}\left[\frac{1}{2} + \zeta_0^2(1+\zeta_0 Z_0)\right]\frac{\partial}{\partial x}|E_{z1}|^2 - \rho\frac{Z_0}{ik_z}\mathrm{Re}\left(E_{z1}\frac{\partial E_{y1}^*}{\partial x}\right) + \frac{\rho v_t}{i\omega}\mathrm{Re}\left(E_{y1}^*\frac{\partial E_{z1}}{\partial x}\right)\Bigg]. \quad (26)$$

This is the kinetic formalism of the ponderomotive force by low frequency rf waves. Although there are terms depending on the gradient of the quadratic field, all the driving terms are multiplied by $\mathrm{Im}(Z_0)$, unless the wave number k_z or k_y is complex. However, for steady state using rf driven current in toroidal systems, the frequency and the toroidal and poloidal wave numbers are real. Therefore, the collisionless nonresonant force is overestimated in the previous studies, where the stress contribution was fully or partial neglected. In fact, this force vanishes.

V. DISCUSSION AND SUMMARY

We have found cancelling terms that negate the apparent ponderomotive current drive effect in cases in which there is no gradient in the ponderomotive potential in the current-drive direction. In fact, the cancellation between the $q\mathbf{v}_1 \times \mathbf{B}_1$ force and $-n_0(\mathbf{v}_1\cdot\nabla)\mathbf{v}_1$ in cold plasma had been verified earlier.[8] There, the author tried to use the anisotropic pressure to reduce v_{1z}, and then to make the $q\mathbf{v}_1 \times \mathbf{B}_1$ drive effective again. However, the second-order modification from the pressure tensor has not been described in detail. Litwin[18] verified that the anisotropic CGL model, which corresponds to the collisionless, low frequency, small Larmor radius limit, gives a zero flux surface average nonresonant force.

However, it is shown in Litwin's work, when the fluid theory is applied, that many nonlinear modifications make things complicated. The fact that the kinetic definitions of the average fluid variables are different form the conventional "fluid" definition was clarified in regulating the neoclassical and turbulent transport theory by Sugama and Horton.[23] All the fluid variables are defined from the total distribution function and divided into the averaged and fluctuating parts. Although the entire stress tensor has the same definition in the kinetic and fluid model, the thermal pressure part, $\int f(\mathbf{v}-\mathbf{u})(\mathbf{v}-\mathbf{u})d\mathbf{v}$, where the fluid velocity $\mathbf{u}$ contains both the averaged (zero order and second order) and fluctuating parts (first order), is different for cases with and without waves. This is why even though the MHD theory[13–15] has included the Reynolds stress term, the related terms in the EM force are not fully cancelled. We can evaluate the ponderomotive force in the MHD theory, where the expression of the Reynolds stress term $F_{\mathrm{Re}z}=(m/2nq^2)\mathrm{Re}[\nabla\cdot(\mathbf{j}_1 j_{1z})]$ can be given as

$$F_{\mathrm{Re}\,z}\Big/\left(\frac{nq^2}{2T}\right)=\mathrm{Re}\left[\rho\zeta_0\frac{1+\zeta_0 Z_0^*}{ik_z}\frac{\partial}{\partial x}E_{y1}E_{z1}^* - \frac{k_y}{k_z}\rho\zeta_0\frac{|1+\zeta_0 Z_0|^2}{ik_z}\frac{\partial}{\partial x}|E_{z1}|^2\right]. \tag{27}$$

It is clear that, although the Reynolds stress in fluid theory gives a similar trend as that of the kinetic stress force shown in Eq. (25), it cannot cancel all the terms related to the EM force. Even when the nonlinear viscosity is considered, no clear cancellation is reported.[15] Moreover, the Reynolds stress contribution was even considered as another mechanism of driving or rearranging plasma current. However, in the kinetic theory, the modifications from the thermal pressure stress and the Reynolds stress are all included in the generalized nonlinear stress term. As a result, the EM force is cancelled by the nonlinear stress tensor forces, and the collisionless nonresonant current drive does not exist.

The above cancellations notwithstanding, ponderomotive forces can drive the current in the toroidal plasma through asymmetry effects[24–28] not discussed here. As opposed to the effects here, these current drive effects entail resonant damping or other similar mechanisms, and they require an asymmetry such as a gradient in the potential in the direction of the current. The current drive then arises from rearranging under energy absorption particle phase space through Hamiltonian forces. These ratchet-type effects produced through ponderomotive potentials can be quite efficient compared to the traditional current-drive resonant mechanisms.[29] However, the physics of these effects is also consistent with a single-particle model in which the rearrangement of particle phase space by the waves is immediately apparent.

In summary, in collisionless plasmas, the rf force by low frequency waves, in the picture of a single particle, is shown to be consistent with that in the fluid and kinetic theory. For a single particle, the parallel component of the Lorentz force is fully compensated by the force due to the movement of a single particle along the gradient direction of the electric field. The ponderomotive force has only the directions where the second rf field quantities are asymmetric. For a Lagrangian fluid element, the force due to the displacement of a single particle is replaced by a force due to the charge accumulation by the divergence of the flow. This electric force, combined with the Lorentz force, constitutes the so-called nonresonant force. However, the stress tensor fully counteracts the nonresonant force. Furthermore, using the second-order rf kinetic theory, we have derived the parallel force by low frequency waves and found that the collisionless nonresonant force offered by the quasilinear EM force is completely cancelled by the nonlinear kinetic stress force. Therefore, in collisionless plasmas, only the Landau resonant forces survive, and the nonresonant ponderomotive forces by low frequency waves cannot drive the toroidal current.

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