

Limits for light intensification by reflection from relativistic plasma mirrors

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Ultraintense laser pulses might be capable of producing plasma mirrors through ultrarelativistic oscillating electrons. Two kinds of such mirrors with relativistic factors $\gamma \sim 100\text{--}300$ were contemplated recently as a tool for producing huge frequency upshifts $4\gamma^2$ of reflected laser pulses as well as the compressing and focusing of those pulses. The combination of these effects would result in dramatic light intensification toward the vacuum breakdown intensities (Schwinger limit) [S. V. Bulanov *et al.*, Phys. Rev. Lett. **91**, 085001 (2003) and S. Gordienko *et al.*, Phys. Rev. Lett. **94**, 103903 (2005)]. The analysis performed in these publications was limited, however, to idealized situations of cold uniform plasmas and uniform laser intensities. The analysis here of effects of electron thermal motion and random inhomogeneities in the plasma density or laser intensity indicates that the largest relativistic factors allowed within these schemes are much smaller than those assumed in the idealized models, unless essentially new physical mechanisms are adduced in addition to those already considered. © 2006 American Institute of Physics.
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I. INTRODUCTION

The technology of lasers has advanced to the point that it is possible to envision lasers capable of producing vacuum breakdown intensities (see, for instance, Refs. 1–3). The probability of an electron-positron pair creation from vacuum by a constant field E or electromagnetic wave of amplitude E is known to be proportional to $\exp(-\pi E_c/E)$,⁴ where

$$E_c = \frac{m^2 c^3}{e \hbar} = 1.32 \times 10^{16} \frac{\text{V}}{\text{cm}}, \quad (1)$$

where c is the speed of light in vacuum, m is the electron mass, e is the positron charge, and \hbar is the Planck constant.

The intensity (i.e., power density) of a linearly polarized laser pulse having maximal electric field E_c is

$$I_c = \frac{c E_c^2}{8\pi} = 2.3 \times 10^{29} \frac{\text{W}}{\text{cm}^2}. \quad (2)$$

If the critical electric field E_c is to be obtained by means of compression and focusing laser pulses to a spot of linear sizes about the laser wavelength λ , the energy located within such a spot would be

$$\varepsilon \sim \frac{E_c^2 \lambda^3}{8\pi} \sim 8 \text{ MJ} \frac{\lambda^3}{\mu\text{m}^3}, \quad (3)$$

or about a MJ (megajoule) at half-micron wavelength. Thus, the megajoule laser systems that are currently built in the US [NIF—National Ignition Facility (Ref. 5)] and France [LMJ—Laser Megajoule (Ref. 6)] are, in principle, sufficient for producing vacuum breakdown intensities. However, it would be necessary to compress megajoule pulses to femto-

second durations and to focus the compressed pulses to sub-micron spots.

In principle, achieving extremely high intensities is easier at shorter laser wavelengths. A dramatic reduction of laser wavelengths was recently suggested in Ref. 3, using huge frequency upshifts occurring at light reflection from ultrarelativistic counterpropagating mirrors. The seed laser of laboratory-frame frequency ω_s would have the Lorentz-transformed frequency $\omega'_s \approx 2\gamma\omega_s$ in the rest-frame of the counterpropagating mirror of the relativistic factor γ . The reflected light of the mirror-frame frequency ω'_s would have the laboratory-frame frequency $\omega''_s \approx 2\gamma\omega'_s \approx 4\gamma^2\omega_s$. The laser pulse wavelength and length are then reduced by $4\gamma^2$. If the pulse of diameter D_s is focused to the spot of diameter about the laser wavelength in the mirror-frame $\lambda'_s \approx \lambda_s/2\gamma$, it would give an additional intensity amplification factor $(2\gamma D_s/\lambda_s)^2$. If the mirror intensity reflection coefficient in the laboratory frame is R (which already includes the longitudinal compression factor), the total intensity amplification factor would be

$$I_f/I_s \sim R(2\gamma D_s/\lambda_s)^2. \quad (4)$$

The major idea of Ref. 3 is to use as the ultrarelativistic mirror in the plasma spike in electron concentration produced by the nearly breaking Langmuir wave driven by another laser pulse of frequency ω_d in a highly underdense plasma of electron plasma frequency $\omega_p \ll \omega_d$. The mirror speed is then the group velocity of the drive laser, and the mirror relativistic factor is $\gamma = \omega_d/\omega_p \gg 1$ (which is immediately seen, since the dispersion law of electromagnetic waves in plasmas is the same as for a relativistic particle of the rest energy ω_p). The intensity reflection coefficient of such a mirror would be, according to Ref. 3, $R \sim 2\gamma(\omega_d/\omega_s)^2$, so that the total intensity amplification factor would be

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$$I_f I_s \sim 8\gamma^3 (\omega_d / \omega_s)^2 (D_s / \lambda_s)^2. \quad (5)$$

For $\lambda_s = \lambda_d = 1 \mu\text{m}$, $D_s = 400 \mu\text{m}$, $I_s = 10^{17} \text{ W/cm}^2$ and $\gamma = 100$ (which corresponds to plasma electron concentration $n \sim 10^{17} \text{ cm}^{-3}$), the focal spot intensity would be, according to Ref. 3, $I_f \approx 1.3 \times 10^{29} \text{ W/cm}^2$, which would be enough to produce the vacuum breakdown.

Another kind of relativistic plasma mirror for reaching the Schwinger limit has been proposed in Ref. 7. The proposal considers production of ultrashort highly focused laser pulses by reflection of ultraintense laser pulses from concave plasma surface. The ultrarelativistic electron motion at the overdense plasma surface causes an efficient generation of high harmonics. Coherent focusing of such harmonics in the (very small) focal volume could open a new way to extremely high intensities.

If it could work, these schemes might provide remarkable reduction in size and costs of lasers capable of producing vacuum breakdown intensities. However, the applicability conditions of the schemes in Refs. 3 and 7 were not sufficiently analyzed. To determine these conditions is the goal of this paper.

The calculation of the mirror reflection coefficient in Ref. 3 deserves more attention. It assumes that the width of the electron concentration spike in the plasma wake wave is much smaller than the seed pulse wavelength, in the wake reference frame. This condition might be difficult to satisfy, because the seed pulse wavelength gets much smaller in the wake reference frame (Doppler effect), while the plasma wavelength gets much larger there and the thermal motion tends to make the spike in electron concentration smoother, reducing the overbarrier reflection.

Apart from this, random bends of the mirror spoil the focusability of reflected light, which problem touches both schemes of Refs. 3 and 7. The bends can be associated with random fluctuations of the plasma concentration and the driving pulse intensity (for the plasma wake mirror of Ref. 3) or incident laser intensity (for the concave plasma surface of Ref. 7).

The plan of the paper is as follows: First, the seed pulse reflection from the electron concentration spike in the near-breaking plasma wave is analyzed. Then, the focusability of the frequency upshifted laser pulses reflected from the plasma wave mirror and the concave plasma surface is considered. In conclusion, possible applications and modifications of relativistic plasma mirrors are discussed.

II. SEED PULSE REFLECTION FROM THE DENSITY SPIKE IN THE NEAR-BREAKING PLASMA WAVE

Consider the well-known one-dimensional cold-plasma model equation for electrostatic wakefield potential ϕ measured in units of mc^2/e (see, for instance, Ref. 8):

$$k_p^{-2} \frac{\partial^2 \phi}{\partial \zeta^2} = \frac{n}{n_0} - 1 = \gamma^2 \left\{ \beta \left[1 - \frac{1+a^2}{\gamma^2(1+\phi)^2} \right]^{-1/2} - 1 \right\}. \quad (6)$$

Here a is the amplitude of vector potential of driving laser field in units of mc^2/e , $\zeta = z - v_d t$, v_d is the driving laser group velocity, $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = v_d/c$, n and n_0 are the current

and initial electron plasma concentrations, respectively, $k_p = \omega_p/c$, $\omega_p = (4\pi e^2 n_0/m)^{1/2}$. Note that the axial electric field of the wake is given by $E_z \equiv E = -E_0 k_p^{-1} \partial \phi / \partial \zeta$, where $E_0 = m\omega_p c/e$ is the nonrelativistic wavebreaking field.

Behind the drive laser pulse, where $a=0$, one can integrate Eq. (6) over ζ and reduce it to the following equation for $x = \phi + 1$:

$$k_p^{-2} \left(\frac{\partial x}{\partial \zeta} \right)^2 = \left(\frac{E}{E_0} \right)^2 = 2\gamma^2 [\beta(x^2 - \gamma^{-2})^{1/2} - x + C], \quad (7)$$

where C is the integration constant.

Wavebreaking occurs when the smallest value of x tends to γ^{-1} , $x_{\min} = \gamma^{-1}$, so that the right-hand side of Eq. (6) tends to infinity. Therefore, at the threshold of Langmuir wave breaking, $C = \gamma^{-1}$ and x oscillates between $x_{\min} = \gamma^{-1}$ and $x_{\max} = 2\gamma - 1/\gamma$. Note, that the maximal electric field (reached at $x=1$, i.e. $\phi=0$) is

$$E_{\max} = \sqrt{2}(\gamma - 1)^{1/2} E_0. \quad (8)$$

Equation (7) can be integrated which gives ζ as a function of x . The result is much more simplified in the two complementary domains $x - \gamma^{-1} \ll \gamma^{-1}$ and $x \gg \gamma^{-1}$. For $x - \gamma^{-1} \ll \gamma^{-1}$, it reads

$$x - \gamma^{-1} = \gamma(k_p \Delta \zeta)^{4/3} 2^{-5/3} 3^{4/3}, \quad (9)$$

where $\Delta \zeta = \zeta - \zeta^{\text{wb}}$ and ζ^{wb} is the wavebreaking point. The conditions $x - \gamma^{-1} \ll \gamma^{-1}$ implies

$$k_p |\Delta \zeta| \ll \gamma^{-3/2}. \quad (10)$$

Substitution of Eq. (9) in the right-hand side of Eqs. (6) and (7), gives expressions for $n/n_0 - 1$ (also obtained in Ref. 9) and E , respectively:

$$n/n_0 - 1 = \gamma(k_p \Delta \zeta)^{-2/3} 2^{1/3} 3^{-2/3}, \quad (11)$$

$$E = -E_0 \gamma (6k_p \Delta \zeta)^{1/3}. \quad (12)$$

In the complimentary domain $x \gg \gamma^{-1}$, the integration of Eq. (7) gives

$$x = k_p \Delta \zeta (\sqrt{2\gamma} - k_p \Delta \zeta / 4). \quad (13)$$

The respective electron plasma concentration is

$$n/n_0 = 1/2 + 1/2x^2. \quad (14)$$

Thus, there is a well-localized spike near the wavebreaking point. This spike contains half of the plasma electrons per the wake wave period. In calculations of the light reflection coefficient in Ref. 3, this spike was assumed to be replaceable by a δ function spike containing half of the electrons on the wake wave period.

However, a noticeable light reflection in an underdense plasma might occur just from inhomogeneities of the spatial scale about the light inverse wave number or shorter. In the spike rest-frame, the incident laser seed wave number $k'_s = 2\gamma k_s$ and the distance from the wavebreaking point is $\Delta \zeta' = \gamma \Delta \zeta$. The light reflection is not exponentially small just in the domain $k'_s \Delta \zeta' \lesssim 1$, i.e.,

$$2\gamma^2 k_s \Delta \zeta \lesssim 1. \quad (15)$$

For $k_s \gg k_p$ and $\gamma \gg 1$, the domain (15) is located deeply inside the domain (10) and, therefore, contains just a small fraction of plasma electrons. This fraction can be evaluated as $(\omega_p/\omega_s)^{1/3} \gamma^{-1/6} = (\omega_d/\omega_s)^{1/3} \gamma^{-1/2}$. The reflection coefficient of Ref. 3 is overestimated by the square of this factor, so that

$$R_{\text{adj}} \sim R(\omega_d/\omega_s)^{2/3} / \gamma \sim (\omega_d/\omega_s)^{8/3}. \quad (16)$$

Note, that the light reflection occurs primarily in the domain $\gamma^2 k_s \Delta \zeta \sim 1$ where the plasma is underdense. Indeed, the plasma concentration there, in the spike rest-frame, is $n' = n/\gamma \sim n_0 (k_p \Delta \zeta)^{-2/3} \sim n_0 (\gamma^2 k_s/k_p)^{2/3}$. The plasma frequency is

$$\omega'_p \sim \omega_p (\gamma^2 k_s/k_p)^{1/3} \sim \omega'_s (\omega_p^2/\gamma \omega_s^2)^{1/3} \ll \omega'_s. \quad (17)$$

The overdense part of the spike is too thin optically to produce a noticeable reflection. More specifically, this part contains a fraction $(\omega_p^2/\gamma \omega_s^2)^{1/3} \ll 1$ of electrons located in the domain (15).

The adjustment of the light reflection coefficient (16) still leaves broad applicability range for the scheme.³ There are, however, other effects that have to be taken into account.

Consider, first, the effect of finite plasma temperature. The thermal motion of electrons can smooth the spike and reduce the overbarrier light reflection. In order to avoid this, the effect of electron thermal motion should be small in the reflection domain. The typical longitudinal thermal energy T entails an adjustment $\gamma \beta_T \equiv \gamma(T/mc^2)^{1/2}$ to the relativistic factor of electrons in the mirror rest-frame (moving with the speed v_d). The electrostatic potential in the mirror rest-frame is $\phi' = \gamma \phi = \gamma(x-1)$. The effect of electron thermal motion is small in the reflection domain for

$$\beta_T < x - \gamma^{-1} \sim \gamma(k_p \Delta \zeta)^{4/3} \sim \gamma^{-3} (\omega_d/\omega_s)^{4/3}. \quad (18)$$

This condition can also be rewritten in the form

$$\gamma < (\omega_d/\omega_s)^{4/9} (mc^2/T_e)^{1/6}. \quad (19)$$

It is a severe limitation. For $\omega_d = \omega_s$ and $T_e = 10$ eV, it reads as $\gamma < 6$.

Formal derivation of the condition (19) and thermally smoothed spike profile are presented in the Appendix.

III. FOCUSABILITY OF THE LASER PULSES REFLECTED BY THE RELATIVISTIC PLASMA MIRRORS

So far, just the effects that might reduce too much of the reflectivity of the plasma wave mirror were considered. In addition to these, there are effects of random bends of the mirror. Such bends might spoil the focusability of reflected light. The mirror bends are associated with fluctuations in distance ζ_L from the drive laser pulse to the wakefield near-breaking point. This distance can be evaluated as

$$k_p \zeta_L \approx 2\gamma \sqrt{2C} \approx 2\sqrt{2}\gamma. \quad (20)$$

It depends on both the drive pulse vector potential a and plasma concentration n_0 , and has the same relative fluctuation value as these quantities, so that

$$k_p \delta \zeta_L \sim \sqrt{\gamma} \delta, \quad (21)$$

where $\delta \sim |\delta n_0/n_0| + |\delta a/a|$. Note that δa here is a quantity integrated (with appropriate weight) over the driving pulse duration. Perturbations with a spatial scale about or longer than the wake wavelength in the longitudinal (δn_0) and transverse (δn_0 and δa) directions are considered.

The reflected light focusability condition is

$$1 > k_s'' \delta \zeta_L = 4\gamma^2 k_s \sqrt{\gamma} \delta / k_p \sim \delta \gamma^{7/2} \omega_s / \omega_d. \quad (22)$$

This condition can also be rewritten in the form

$$\gamma < (\omega_d/\omega_s)^{2/7} \delta^{-2/7}. \quad (23)$$

For $\omega_d = \omega_s$ and just 1% (through the entire reflected pulse aperture) variation of the drive pulse amplitude and/or plasma concentration ($\delta = 0.01$), (23) reads as $\gamma < 4$.

Note, that plasma wavelength, $\lambda_p = k_p^{-1} 4\sqrt{2(\gamma-1)} = \lambda_d(2/\pi)\gamma\sqrt{2(\gamma-1)}$, is small enough for such modest γ .

The concave plasma surface mirror of Ref. 7, though less fragile than the mirror of Ref. 3, is also subject to bends that might spoil focusability of the reflected light. In particular, bends are caused by fluctuations of the plasma skin depth $l_p \sim \sqrt{\gamma}/k_p$, $\gamma \sim a$, associated with fluctuations of the plasma concentration and the incident laser intensity and having the same relative value $\delta l_p/l_p \sim |\delta n_0/n_0| + |\delta a/a| \equiv \delta$. The mirror bends $\delta l_p \sim \delta \sqrt{\gamma}/k_p$ do not prevent the reflected light focusing as long as the respective fluctuations of the light phases are small,

$$1 > k_s'' \delta l_p \sim 4\gamma^2 k_s \sqrt{\gamma} \delta / k_p \sim \delta \gamma^{5/2} \omega_s / \omega_p. \quad (24)$$

For moderately overdense plasmas, favorable for efficient generation of high harmonics, $\omega_s/\omega_p \sim \gamma^{-1/2}$, the necessary focusability condition (24) gives

$$\gamma < 1/2\sqrt{\delta}. \quad (25)$$

For just 1% relative fluctuations of the drive pulse amplitude and/or plasma concentration ($\delta = 0.01$) through the entire aperture of the reflected laser pulse, (25) implies $\gamma < 5$. This is much less than $\gamma \sim 300$ suggested in Ref. 7 to reach the Schwinger limit.

IV. DISCUSSION AND SUMMARY

It was shown above that the coherent laser pulse reflection from the electron concentration spike in the plasma wake wave is too small at large relativistic factors of the mirror for the scheme³ to work. The reflection is reduced, compared to the cold plasma model, because of smoothing the underdense electron plasma spike by the electron thermal motion. In this context, it might be useful to note that incoherent (Thomson) scattering of laser pulses is not so sensitive to the electron concentration spike smoothing. Thomson backscattering of laser pulses by the relativistic electron spike also upshifts the laser frequency by $4\gamma^2$ times. The backscattered radiation propagates in a narrow cone with an opening angle $1/\gamma \ll 1$ (γ is the relativistic factor of electrons in the spike). The x-ray pulse duration $\tau \sim \max\{\tau_s/4\gamma^2, l_d/c\}$, where τ_s is the input seed pulse duration and l_d is the width of the electron concentration spike.

Such scheme of producing x-ray pulses would be similar to the previously suggested Thomson scattering of laser pulses by electrons accelerated in the laser wake fields¹³ (in the standard or self-modulated regime). An advantage of the new scheme might be in much larger fraction of plasma electrons collected into the relativistic mirror. The energy spectrum of electrons in the spike would be broad. It also would maximize and drop abruptly at the highest energy. Ultrashort x-ray laser pulses generated through Thomson scattering could be used in studies of ultrafast processes in solid, molecular, and biological systems (see Ref. 14, and references therein).

In summary, we analyzed the limits for frequency upshift and light intensification within the two recently suggested schemes^{3,7} taking into account finite temperature effects, density inhomogeneities, and focusability issues. In so doing, we found that, without essentially new physical effects being involved, these schemes might work just for very modest relativistic factors of the plasma mirrors, far insufficient for reaching the vacuum breakdown intensities. In particular, the electron thermal motion at the temperature of just 10 eV smooths the electron spike-mirror proposed by Bulanov *et al.* to the extent that a noticeable reflection appears to be possible just for modest relativistic factors $\gamma < 6$. Furthermore, just 1% random variations of plasma density or laser intensities in either of the proposed schemes spoil the focusability of the reflected laser pulses at large relativistic factors, so that the focusing appears to be possible only for $\gamma < 5$. Reducing the inhomogeneity (which is challenging) does not increase the usable γ by much because of the strong γ dependence of the applicability conditions. Thus, to enable the effects predicted by these schemes at much larger relativistic factors would require essentially new physical mechanisms.

Such schemes with modestly relativistic mirrors might be useful in conjunction with Raman backscattering techniques,² as an intermediate step even without achieving focusability. However, using the scheme³ in this manner would require sufficient lifetime of the mirror. The counter-propagating wakes might also be useful for an all-optical x-ray source, where the wakes close to the wavebreaking limit produce narrow pulses of energetic electrons that then generate x rays through Thomson scattering. The scheme⁷ might be useful for generation of ultrashort ultraintense pulses as described in Ref. 15.

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APPENDIX: LIGHT REFLECTION BY THERMALLY SMOOTHED ELECTRON SPIKE

Relativistic wavebreaking of nonlinear plasma waves in thermal plasmas has been analyzed in Refs. 10–12. Nonlin-

ear equations for the wakefield potential have been obtained in Refs. 10 and 12 assuming a small thermal electron momentum spread. Reference 10 uses a waterbag approximation for the electron distribution function, while Ref. 12 does not. Both approaches result, however, in very similar equations. In the following, we follow Ref. 12.

The scalar potential ϕ (in units of mc^2/e) can be presented in the form¹²

$$\phi = \frac{1-v\beta}{\sqrt{1-v^2}} \left[1 + \frac{3}{2}\beta_T^2 \frac{1-v^2}{(1-v\beta)^2} \right] - 1 - \frac{3}{2}\beta_T^2 \quad (\text{A1})$$

and it satisfies to the Poisson equation:

$$k_p^{-2} \frac{\partial^2 \phi}{\partial \zeta^2} = -k_p^{-1} \frac{\partial(E/E_0)}{\partial \zeta} = \frac{n}{n_0} - 1 = \frac{v}{\beta - v}. \quad (\text{A2})$$

Here v is the hydrodynamic electron velocity normalized to the speed of light, $\beta_T = (T/mc)^{1/2}$, and $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic factor corresponding to the plasma wave phase velocity. Both the effects of the wake field electrostatic force and thermal electron pressure are included. Thermal pressure does not allow the electron density to rise to infinity. The maximum electron velocity v_0 at wavebreaking is found from $\partial\phi/\partial v = 0$. The electron velocity and density profiles at the maximum can be found approximately using a substitution $\phi \approx \phi(v=v_0) + \phi''\Delta v^2/2$. It results in

$$\Delta v \approx \Delta v_0 (|\Delta \zeta|/\Delta \zeta_p) \quad (\text{A3})$$

and

$$n/n_0 \approx \Delta v_0^{-1} (1 - |\Delta \zeta|/\Delta \zeta_p), \quad (\text{A4})$$

where $|\Delta \zeta| < \Delta \zeta_p$,

$$\Delta v_0 = \beta - v_0 = 3^{1/4} \frac{\beta_T^{1/2}}{\gamma^{3/2}}, \quad (\text{A5})$$

$$\Delta \zeta_p = 3^{3/8} 2 \left(\frac{\beta_T}{\gamma} \right)^{3/4} k_p^{-1} \quad (\text{A6})$$

for $\beta_T \gamma < 1$ (which is the case only of interest for using the plasma mirror, as we will see below); for $\beta_T \gamma > 1$ Eqs. (A1) and (A2) predict $\Delta v_0 = 3\beta_T^2$, $\Delta \zeta_p = (3/2)^{3/4} \beta_T^{3/2} k_p^{-1}$.

The reflection from the density spike is produced primarily within the domain $|\Delta \zeta| < \Delta \zeta_m = (2\gamma^2 k_s)^{-1}$, see Eq. (15). The reflection coefficient is given approximately by Eq. (16), if $\Delta \zeta_p < \Delta \zeta_m$, which is equivalent to Eq. (19) (requiring also $\beta_T \gamma < 1$). In the opposite case, $\Delta \zeta_p > \Delta \zeta_m$, the reflection coefficient is reduced by the extra factor $(\gamma/\gamma_c)^{15/2} \max\{1, (\beta_T \gamma)^{9/2}\}$, where $\gamma_c = (\omega_d/\omega_s)^{4/9} \beta_T^{-1/3}$ at the limit of the applicability of Eq. (19). If Eq. (19) is not satisfied, the intensity amplification factor Eq. (4) in the scheme of Ref. 3 appears to be a rapidly decreasing function of γ , instead of increasing as γ^3 as in Eq. (5).

Equations (A1) and (A2) do not take into account partial trapping of tail electrons by the plasma wave. These trapped electrons could modify the density profile. Trapping could be important at a distance of a few $\Delta \zeta_p$ from the density maximum. The mean electron velocity differs from the phase velocity by a few thermal spread values (the thermal spread is of order Δv_0 for $\beta_T \gamma < 1$). Should these trapping effects lead

to further smoothing of the density maximum, there would result an even smaller reflection coefficient for $\Delta\zeta_p < \Delta\zeta_m$.

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