

# Compression of powerful x-ray pulses to attosecond durations by stimulated Raman backscattering in plasmas

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Backward Raman amplification (BRA) in plasmas holds the potential for longitudinal compression and focusing of powerful x-ray pulses. In principle, this method is capable of producing pulse intensities close to the vacuum breakdown threshold by manipulating the output of planned x-ray sources. The minimum wavelength limit of BRA applicability to compression of laser pulses in plasmas is found.

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The goal of this paper is to examine the possibility of using powerful x-ray sources for producing ultrahigh laser intensities. It might clarify how much the emerging mJ x-ray laser technologies [1] could compete with the emerging MJ optical laser technologies [2,3] in producing ultrahigh electromagnetic fields. The most challenging laser intensity benchmark targeted in recent proposals [4–6] is the vacuum breakdown intensity

$$I_c = \frac{cE_c^2}{8\pi} = 2.3 \times 10^{29} \frac{\text{W}}{\text{cm}^2}. \quad (1)$$

Here

$$E_c = \frac{m^2 c^3}{e\hbar} = 1.32 \times 10^{16} \frac{\text{V}}{\text{cm}}, \quad (2)$$

$c$  is the speed of light in vacuum,  $m$  is the electron mass,  $e$  is the positron charge, and  $\hbar$  is the Planck constant. [Formula (1) gives the intensity (i.e., power density) of a linearly polarized laser pulse having maximal electric field  $E_c$ . The probability of an electron-positron pair creation from vacuum by a constant field  $E$  or plane electromagnetic wave of amplitude  $E$  is known to be proportional to  $\exp(-\pi E_c/E)$  [7].]

If the critical electric field  $E_c$  is to be obtained by means of compression and focusing laser pulses to spots of linear sizes about the laser wavelength  $\lambda$ , the energy located within such a spot would be

$$\varepsilon_c \sim \frac{E_c^2 \lambda^3}{8\pi} \sim 8 \text{ MJ} \frac{\lambda^3}{\mu\text{m}^3}. \quad (3)$$

Table I numerically illustrates the wavelength dependence of energy  $\varepsilon_c$  necessary for the vacuum breakdown.

Table II compares the biggest of currently built lasers: Linac Coherent Light Source (LCLS) [1] in x-ray range and megajoule laser systems in visible range—National Ignition Facility (NIF) in US [2], or Laser Megajoule (LMJ) in France [3].

The table shows that the energy in each of these systems would be, in principle, sufficient for producing vacuum breakdown intensities. The LCLS might even have some advantages. However, it would be necessary to compress the

output laser pulses of any system to several wavelengths and to focus the compressed pulses similarly to spot sizes of no more than several wavelengths.

The currently expected duration of LCLS output pulses is about 200 fs, which is  $2 \times 10^5$  times larger than the attosecond ( $10^{-18}$  sec) duration of a 3 Å long laser pulse. There are several schemes proposed to reduce the duration of powerful x-ray pulses, but only to few times shorter than a femtosecond [8].

The expected diameter of LCLS output pulses is about 100  $\mu\text{m}$ . There are proposals on focusing x-ray pulses to few nm spot sizes [9], while available optics are now capable of x-ray focusing to sub 100 and even sub 50 nm spot sizes [10]. However, these focusing techniques handle relatively low intensity (power density) x rays, and might not be directly applicable to intense LCLS output pulses, for which focusing appears to be more challenging (see, for instance, [11]).

We put forth here the possibility to improve both the longitudinal compression and focusing of intense x ray pulses by means of resonant backward Raman amplification (BRA) of laser pulses in plasmas. The resonant backward amplification mechanism is similar to that previously considered in [5], but now having to operate under physical conditions corresponding to very different plasma and laser parameters.

In the BRA scheme, a less energetic well-focused short laser pulse (seed) consumes a longer more energetic laser pulse (pump), while remaining well-focused. The problem of focusing intense x-ray outputs could thus be reduced to a simpler problem of focusing less intense x-ray seed pulses.

This method is also similar to the “beam-cleaning techniques” proposed for Raman amplifiers, in which the output can be made more coherent than the pump at relatively low intensities [12]. However, the important element here is that the output can be made more focusable than the pump, at

TABLE I. Examples of the laser energy  $\varepsilon_c$  necessary for the vacuum breakdown at different laser wavelengths.

$\lambda$	1 $\mu\text{m}$	100 nm	10 nm	1 nm	1 Å
$\varepsilon_c$	8 MJ	8 kJ	8 J	8 mJ	8 $\mu\text{J}$

TABLE II. Laser energy  $\varepsilon$  versus  $\varepsilon_c$  for the biggest of currently built lasers.

Device	NIF or LMJ	LCLS
$\lambda$	0.35 $\mu\text{m}$	0.15 nm
$\varepsilon$	2 MJ	2 mJ
$\varepsilon_c$	1/3 MJ	1/40 mJ
$\varepsilon/\varepsilon_c$	6	80

intensities that would be too high to handle efficiently otherwise.

For longitudinal compression of intense x-ray outputs by BRA, the high intensity is a favorable factor and even a necessary condition. This is because the Raman growth length (which is shorter for higher laser intensities) should be much shorter than the pump length, in order to accomplish significant longitudinal compression of the pump.

By means of a BRA in plasmas, laser pulses could be compressed to durations about the electron plasma wave period

$$t_o \sim \frac{2\pi}{\omega_e}, \quad \omega_e \equiv \left( \frac{4\pi n_e e^2}{m} \right)^{1/2}, \quad (4)$$

where  $n_e$  is the electron plasma concentration.

To reach shorter output pulse durations, more concentrated plasmas are needed. So far, no upper theoretical limits for plasma or laser frequencies in BRA schemes were known. However, it is not clear in advance, if BRA can be practiced in the x-ray regime. A simple extrapolation of the mechanism from the visible light regime, examined earlier [5], is not sufficient, because competing effects might narrow and even close the window in parameter space for efficient BRA operation. What follows clarifies this basic issue.

Apart from the general theoretical consideration that follows, it might be useful to mention here some of today's practical limits. For the largest laboratory plasma concentration feasible now,  $n_e = 10^{26} \text{ cm}^{-3}$ , the BRA output pulse duration could be  $t_o \sim 10$  asec. Such plasma concentrations are planned for compressed inertial fusion targets, but so far there are no plans to join LCLS and NIF in one facility. For a less challenging plasma concentration  $n_e = 2.5 \times 10^{25} \text{ cm}^{-3}$ , the BRA output pulse duration could be  $t_o \sim 20$  asec. Assuming an efficient longitudinal compression of laser energy, the BRA output pulse could carry the energy close to the input pump energy, say,  $\varepsilon = 2$  mJ (roughly the x-ray pulse energy to be produced by LCLS), which would imply the power

$$P \sim \varepsilon/t_o \sim 100 \text{ TW}.$$

The intensity of such a BRA output pulse focused to a spot of diameter, say 0.5 nm, would be

$$I_f \sim 4 \times 10^{28} \text{ W/cm}^2.$$

The existing proposals on increasing the x-ray pulse energies by more than order in value [8] could enable approaching the vacuum breakdown intensities even within less focused and compressed pulses.

To begin the general consideration of BRA applicability

in the x-ray regime, note that a necessary condition for efficient BRA is the smallness of the laser energy absorption due to electron-ion collisions in plasma, the so-called ‘‘inverse bremsstrahlung.’’ The inverse bremsstrahlung rate can be evaluated [13] as

$$\nu_{ib} = \nu_{ei} \omega_e^2 / \omega^2, \quad (5)$$

where  $\nu_{ei}$  is the rate of electron-ion collisions in plasma and  $\omega = 2\pi c/\lambda$  is the laser frequency. Upon traversing a plasma of length  $L$ , a laser pulse loses to inverse bremsstrahlung the energy fraction

$$q_{ib} = \nu_{ib} L/c. \quad (6)$$

For a plasma of length  $L$ , the pump laser pulse length should be  $2L$ , in order for the counterpropagating seed to meet the pump front upon the seed entering the plasma, and for the pump to end upon the seed exiting the plasma. To keep the inverse bremsstrahlung (6) loss small, the pump duration  $t_{\text{pmp}} = 2L/c$  should satisfy the condition

$$q_{ib} = \nu_{ib} t_{\text{pmp}}/2 \ll 1. \quad (7)$$

Note that the average distance traversed by the pump before meeting the pumped pulse (and consumed by it) is  $L/2$ . For the energy uniformly dissipated, the average fraction of pump energy spent for the inverse bremsstrahlung is  $q_{ib}/2$ .

Note also that the laser pump energy fraction transferred to Langmuir waves constitutes

$$q_L = \omega_e/\omega.$$

Let  $q_{ef}$  ( $q_{ef} \leq 1 - q_L - q_{ib}/2$ ) be the efficiency of laser energy conversion from the pump to a shorter pumped pulse (longitudinal compression). Then, in order to produce a BRA output pulse of fluence  $w$ , the pump fluence  $w/q_{ef}$  is needed, and the pump of intensity (power density)  $I$  should have the duration

$$t_{\text{pmp}} = w/(q_{ef} I). \quad (8)$$

The required BRA output fluence  $w$  depends on the output pulse duration  $t_o$  to be achieved, since  $t_o$  is of the order of the time of pump depletion by the amplified seed pulse. This pump depletion time decreases as the amplified pulse intensity grows. Assuming that the BRA is transient within the time  $t_o$  (which means that the Langmuir wave, mediating energy transfer from the pump to the pumped pulse, is not damped within the output pumped pulse duration), the fluence of the pumped pulse to be compressed to the duration about the electron plasma wave period (4) can be evaluated as

$$w = \frac{q_w}{\lambda} \left( \frac{mc^2}{e} \right)^2 \approx 0.3 \frac{q_w}{\lambda} \frac{\text{J}}{\text{cm}}, \quad (9)$$

where  $q_w \sim 1$  is a slowly varying function of the system parameters.

The pump pulse intensity  $I$  should not significantly exceed the threshold for breaking of the mediating Langmuir wave in the transient BRA regime,

$$I_{\text{thr}} = n_e m c^3 q_L / 16, \quad (10)$$

since both the laser coupling and BRA efficiency decrease for intensities too far beyond the threshold. Thus, in efficient BRA regimes,

$$I = q_I I_{\text{thr}}, \quad q_I \lesssim 1. \quad (11)$$

Substitution of Eqs. (5) and (8)–(11) into Eq. (7) gives

$$q_{\text{ib}} = \frac{16 q_w \nu_{\text{ei}}}{q_{\text{ei}} q_I \omega_e} \ll 1. \quad (12)$$

Interestingly, this condition does not contain explicitly the laser parameters. It also implies that the Langmuir wave collisional damping  $\nu_{L\text{ei}} \approx \nu_{\text{ei}}/4$  is negligible within the pumped pulse duration  $t_o$  of order of the plasma period,

$$\nu_{L\text{ei}} t_o \sim \frac{\pi \nu_{\text{ei}}}{2 \omega_e} \ll 1. \quad (13)$$

For the BRA regime to be essentially transient, Landau damping of Langmuir wave mediating the energy transfer from the pump to the pumped pulse should also be negligible within the duration  $t_o$ . Larger Landau damping may reduce the Langmuir wave amplitude and, hence, the coupling of pump and pumped lasers, which would increase the duration of BRA output pulse (at the same input parameters). To avoid excessive Landau damping, it is necessary to have the phase velocity of the Langmuir wave,

$$v_{\text{ph}} \approx c q_L / 2, \quad (14)$$

exceeding significantly the thermal electron velocity  $\sqrt{T_e/m}$ . This is possible for not too high electron plasma temperatures  $T_e$ ,

$$T_e \ll m c^2 q_L^2 / 4 \equiv T_M. \quad (15)$$

This condition is necessary to avoid too large Landau damping as long as it is a standard Langmuir wave. Nonlinear kinetic electron modes, such as BGK waves considered, for instance, in [14], or other modes as ones proposed in [15], might have negligible Landau damping even at wavelengths shorter than Debye length. However, these modes have not yet been sufficiently analyzed, particularly in the context of BRA.

On the other hand, the electron plasma temperature should be large enough to suppress the inverse bremsstrahlung of the x-ray pulses in the plasma by satisfying the condition (12). The rate of electron-ion collisions  $\nu_{\text{ei}}$  in this condition could be evaluated for a nearly ideal and classical plasma with singly charged ions as

$$\nu_{\text{ei}} \approx \frac{4}{3} \sqrt{\frac{2\pi \Lambda n_e e^4}{m T_e^{3/2}}}, \quad (16)$$

where  $\Lambda$  is the Coulomb logarithm.

By substituting Eq. (16) and

$$T_e \equiv q_T T_M \quad (q_T \ll 1) \quad (17)$$

into formula (12) for  $q_{\text{ib}}$ , condition (12) can be rewritten in the form

$$q_{\text{ib}} = \frac{256 \sqrt{2\pi} \Lambda q_w r_e}{3 q_T^{3/2} q_{\text{ei}} q_I q_L^2 \lambda} \ll 1, \quad (18)$$

where

$$r_e = \frac{e^2}{m c^2} = 2.818 \text{ fm}.$$

Taking into account physical ranges of parameters ( $q_L < 1/2$ ,  $q_T \ll 1$ , etc.), condition (17) could possibly be satisfied just for laser wavelengths  $\lambda$  larger than angstrom.

For the BRA to be in the favorable transient regime, there is yet another requirement associated with the plasma heating via the inverse bremsstrahlung of laser energy. Under conditions when the electron cooling by thermoconductivity and ions are negligible, the inverse bremsstrahlung increases the electron plasma temperature by  $\delta T_e = \nu_{\text{ib}} t_h I / (C_e n_e c)$  where  $t_h$  is the time of heating, and  $C_e = 3/2$  is the specific heat per electron. The largest heating occurs at the edge where the pump enters the plasma. There,  $t_h = 2L/c$  and

$$\delta T_e = \frac{2 q_{\text{ib}} I}{C_e n_e c}. \quad (19)$$

By using Eqs. (7), (8), (10), (11), (15), and (17), this temperature increase can be presented in the form

$$\delta T_e = T_M \frac{q_{\text{ib}} q_I}{2 q_L C_e}. \quad (20)$$

Since  $\delta T_e \leq T_e$ , it follows that

$$q_{\text{ib}} \leq 2 q_L C_e q_T / q_I. \quad (21)$$

When the right-hand side is much smaller than unity (which is true for not too small  $q_I$ ), this condition is stricter than Eq. (12). In this case, the shortest allowed laser wavelength is somewhat longer than according to Eq. (18) and is given by

$$\frac{256 \sqrt{2\pi} \Lambda q_w r_e}{9 q_T^{5/2} q_{\text{ei}} q_L^3 \lambda} \leq 1. \quad (22)$$

Note that the laser pump energy per plasma electron can be evaluated as

$$T_p = \frac{2I}{n_e c} = m c^2 q_I q_L / 8. \quad (23)$$

Electron plasmas can be treated as nearly classical at temperatures exceeding the electron Fermi energy,

$$T_e \gg F = \frac{\pi^2 \hbar^2}{2m} \left( \frac{3n_e}{\pi} \right)^{2/3}. \quad (24)$$

Generally speaking, care should be taken to prevent premature Raman backscattering of the pump by noise (see, for instance, [5]), since the BRA length may exceed significantly the linear BRA  $e$ -folding length. The linear BRA growth rate is given by

$$\Gamma_R = \sqrt{\frac{q_I \omega_e^2}{2 \cdot 4 \omega_e}}. \quad (25)$$

Interestingly, the parasitic Raman instability may be sig-

TABLE III. Numerical examples of system parameters.

Name	Value 1	Value 2	Units
Laser wavelength $\lambda$	7	15	Å
Laser frequency $\omega$	2.69	1.26	$10^{18} \text{ sec}^{-1}$
Single photon energy $\hbar\omega$	1.77	0.83	keV
Laser pump pulse energy $\varepsilon$	2	2	mJ
$\varepsilon/\varepsilon_c$	0.75	0.08	
Electron plasma concentration $n_e$	1	0.3	$10^{26} \text{ cm}^{-3}$
Electron plasma frequency $\omega_e$	5.64	3.09	$10^{17} \text{ sec}^{-1}$
Pump energy fraction going to Langmuir waves $q_L = \omega_e/\omega$	0.21	0.25	
Electron Fermi energy $F$	0.79	0.35	keV
Laser pump energy per plasma electron $T_p$	13.4	16	keV
Electron energy $T_M$ corresponding to Langmuir wave phase velocity	5.6	7.7	keV
Electron plasma temperature $T_e$	2.8	2.6	keV
Plasma edge electron heating by the pump inverse bremsstrahlung $\delta T_e$	2.0	1.8	keV
Electron-ion collision time $\nu_{ei}^{-1}$	0.25	0.6	fsec
Langmuir wave collisional damping time $\nu_{Lei}^{-1}$	1	2.4	fsec
Inverse bremsstrahlung time $\nu_{ib}^{-1}$	5.8	10	fsec
Laser pump pulse duration $t_{\text{pmp}}$	2.6	3.4	fsec
Pump energy fraction lost for the inverse bremsstrahlung $q_{ib}/2$	0.11	0.086	
Thermal electron free-flight time across the pump pulse diameter	16	24	fsec
Pump pulse diffraction time	460	450	fsec
The pump pulse diameter $D$	350	510	nm
Laser pump intensity $I$	3.2	1.1	$10^{21} \text{ W/cm}^2$
Linear BRA growth rate $\Gamma_R$	2	1.3	$10^{16} \text{ sec}^{-1}$
Time of linear Landau damping of Langmuir wave	2.7	4	asec
BRA output duration $t_o$	11	20	asec
BRA output power $P$	90	50	TW
Intensity of BRA output focused to spot of diameter $\lambda/2$	9	1	$10^{28} \text{ W/cm}^2$

nificantly suppressed by collisional and Landau damping of the thermal Langmuir waves, while the useful BRA survives in a narrow domain trailing the short seed, where ultimately the output pulse is built.

The linear Landau damping rate for Maxwellian electron distribution is

$$\Gamma_{\text{Lnd}} = \frac{\omega_e \sqrt{\pi}}{(2q_T)^{3/2}} \exp\left(-\frac{1}{2q_T}\right). \quad (26)$$

The condition for BRA be transient within the time  $t_o$ ,

$$1 \gg \Gamma_{\text{Lnd}} t_o = \frac{\pi^{3/2}}{\sqrt{2} q_T^{3/2}} \exp\left(-\frac{1}{2q_T}\right),$$

could be satisfied just for  $q_T < 0.1$ . However, nonlinear saturation of Landau damping can allow transient BRA at larger  $q_T$  and, respectively, smaller  $\lambda$  [Eqs. (18) and (22)].

Note that for a highly efficient BRA, the seed pulse front should be short enough, namely, as short as the desired duration of compressed output pulse  $t_o$ .

Note also the BRA length  $L$  must be shorter than diffraction lengths of lasers  $L_D$ . A laser pulse of fluence  $w$  given by Eq. (9) and energy  $\varepsilon$  can be accommodated within the cross section  $S = \varepsilon/w$ , so that the respective diffraction length is

$$L_D \approx \frac{S}{\lambda} \approx \frac{3\varepsilon}{q_w} \frac{\text{cm}}{\text{J}}. \quad (27)$$

Interestingly, it does not depend explicitly of laser wavelength.

The transverse thermoconductivity is small for

$$S \gg \frac{24T_e}{mc^2} \frac{L^2}{1 + 2\nu_{ei}L/c}. \quad (28)$$

To better outline the BRA applicability limits for com-

pression of powerful x-ray pulses in plasmas, consider two extreme numerical examples of system parameters calculated according to the above formulas and presented in Table III. Both cases use parameter values  $q_l=q_w=1$  and  $q_{ef}=0.5$ . These examples rely on nonlinear saturation of Langmuir wave Landau damping, thereby allowing transient BRA at larger  $q_T$ , like 1/2 or 1/3. As seen from the table, the plasma edge electron heating by the pump inverse bremsstrahlung is close to allowed limit, while the rest applicability conditions are satisfied with reasonable safety factors. It must be noted that producing the pump and seed pulses and mediating plasma required in these examples is in itself a technological challenge. It also must be noted that the tough focusing requirement restricts the allowed focal length and could make challenging keeping good vacuum in the focal spot where the breakdown is expected. A rough sketch of the experimental geometry and wave-front shapes could be found in [5].

In summary, we found the short wavelength theoretical limit for compression and focusing of powerful x-ray pulses by Raman backscattering in plasmas. The shortest wavelength appears to be about 1 nm, at which mJ x-ray pulses might be, in principle compressed and focused to intensities sufficient for the vacuum breakdown. Reaching these highest

intensities, however, requires the nonlinear suppression of the Landau damping of the Langmuir waves mediating the energy transfer from the pump to the pumped laser pulse.

In general, Raman backscattering in plasmas reduces the problem of focusing intense x-ray pulses to a simpler problem of focusing less intense seed x-ray pulses which consume then intense pumps while remaining well-focused. Beam cleaning of noisy pump pulses is also thus accomplished, while the entropy is taken by plasma waves. The pulse focusing can be accomplished either simultaneously with the longitudinal compression, or as an extra step of an x-ray pulse processing by means of Raman backscattering in plasmas.

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