Role of quantum interference in thermodynamic equilibrium

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The influence of quantum interference between different decay channels of three-level atoms on various characteristics of matter and radiation is considered for the case of thermodynamic equilibrium of atoms in a photon gas. It is shown that for rare gases when collisions between atoms are not important, the direct consequence of such an interference is a spontaneously induced atomic coherence between two upper levels. Therefore, for an accurate description of such atoms and radiation in thermodynamic equilibrium with each other it is not sufficient to specify only Boltzmann exponents, which correspond to the diagonal elements of the density matrix. The reason for this is that the conventional Boltzmann description is applied to infinitely sharp eigenstates of the unperturbed Hamiltonian. As we show, it becomes essential to take into account nonzero width of the levels if the levels are relatively close to each other. This produces nonzero nondiagonal elements of the density matrix that modify the emissivity of the equilibrium medium and lead to a zero point at a certain frequency as well as to an enhanced intensity in the red wing at the corresponding temperature. It is shown that the occupation number of the photons is not changed and obeys the equilibrium Planck distribution.
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I. INTRODUCTION

Quantum interference between different decay channels [1,2] in three-level atoms has attracted much interest in recent years [3,4]. It was shown that it leads to the appearance of nondiagonal elements in the spontaneous relaxation operator that are responsible for such effects as lasers without inversion [5,6], spectral line narrowing [3,1], line elimination [4], and cancellation of spontaneous emission over the range of frequencies [7].

Depending on the atomic scheme and the environment considered, quantum interference may be of a different origin. It may be due to either the coherent laser field [4,8,9] or spontaneous emission from two closely spaced upper levels [1–3,5] or both [10]. One type of interference is due to coherent properties of the laser field, while the other one is produced by the coupling of the upper levels to the same quantum mode of the radiation field. This interference is known to lead to nondiagonal elements of the relaxation operator [1–3,5,10,11], which may also have an additional contribution due to Lamb shifts [7].

In this paper we consider three-level atoms in thermodynamic equilibrium with the resonant photon gas and investigate the influence of quantum interference due to spontaneous emission from two upper levels on various properties of radiation and matter. The dipole transitions $|2\rangle \rightarrow |1\rangle$ and $|3\rangle \rightarrow |1\rangle$ (see Fig. 1) should be allowed by selection rules for this interference to occur.

We show that the direct consequence of quantum interference in question is the appearance of a nonzero coherence between upper levels even in thermodynamic equilibrium. This is certainly a surprising result since one would expect no processes inducing coherence to be present in equilibrium. However, as will become clear later, coupling of the upper levels to the same quantum mode of the radiation field is such a process, which is an inherent property of the atomic system. Therefore, our conclusion is that to describe the thermodynamic equilibrium of the three-level atoms in question with the photon gas it is not sufficient to specify only Boltzmann exponents, which correspond to the diagonal elements of the density matrix. One needs to compute nondiagonal elements as well, which become nonzero for the atomic system considered. The underlying reason for this is that the conventional Boltzmann description is applied to infinitely sharp eigenstates of the unperturbed Hamiltonian, in our case the Hamiltonian of the atom. As upper levels become broadened through the interaction with zero fluctuations of the radiation field, their overlapping becomes important and produces nonzero nondiagonal elements of the density matrix.

We show that these elements lead to the same Planck distribution equilibrium that photons should obey. We find, however, that the quantum interference modifies the emissivity of the medium, which acquires a dark line at a certain frequency. More importantly, this interference increases sharply with the intensity of the recently predicted [12] and

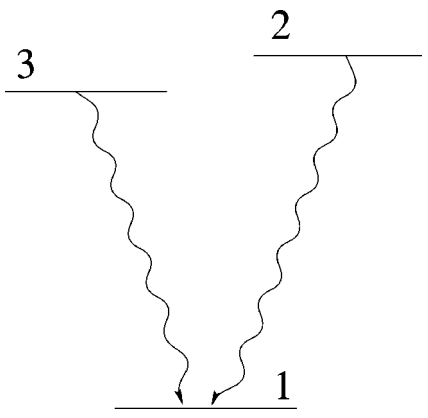

FIG. 1. Three-level atoms.

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observed [13] red wing of the equilibrium spectrum.

We obtained this result under conditions in which atomic vapor is rare enough so that one can neglect collisions between atoms and take into account only the interaction with photons. It is easy to generalize this result to higher densities and include collisions. Then one has to compare the rate of destruction of atomic coherence due to collisions ν_c with the rate of its creation $\sqrt{A_2A_3}$ (see below) due to quantum interference in order to draw a conclusion about the form of the spectrum, density matrix, etc. In the case when $\nu_c < \sqrt{A_2A_3}$ our main conclusions are still valid.

Note that the model atomic scheme of the Fig. 1 should not necessarily be that of an inherent atomic Hamiltonian. In fact, dressed states produced by external electric fields may have the desired relaxation matrix. This observation makes an experimental verification and possible applications of the predicted effects more feasible. In this paper we use the diagrammatic approach of Keldysh [14] and Korenman [15], which enabled earlier workers [12] to analyze correctly equilibrium spectra.

The paper is organized as follows. In Sec. II we evaluate mass operators. In Sec. III we find atomic Green's functions. Secs. IV and V are devoted to the calculation of the polarization operators and the emissivity of the medium. In Sec. VI we present our conclusion.

II. EVALUATION OF MASS OPERATORS

There are two types of Green's functions involved in the calculations according to the Green's function approach of Keldysh [14] and Korenman [15]: retarded (advanced) and kinetic, which describe the dynamics and kinetics of the system, respectively. Correspondingly, one needs to evaluate retarded (advanced) and kinetic mass operators.

Consider a three-level atom (Fig. 1) interacting with the electromagnetic field. It is now well known that if both transitions are allowed, then coupling of both upper levels to the same quantum modes leads to nondiagonal elements in the atomic relaxation operator [1–7,16]. These elements are represented by the nondiagonal mass operators $\Sigma_{32}^{\alpha\alpha'}$, $\Sigma_{23}^{\alpha\alpha'}$ in the Keldysh-Korenman formalism. As was shown in Ref. [16], the real part of these operators, which stems from the Lamb shift contribution, leads to nonexponential decay of atomic populations and coherences. In this paper we omit these terms and restrict ourselves only to the imaginary part of these operators. We also want to remove the Lamb shift contribution to the diagonal terms of retarded (advanced) and kinetic mass operators.

To do this we use a linear connection between $\Sigma^R, \Sigma^A, \Sigma^{\alpha\alpha'}$ and rewrite Σ^R in the form

$$\Sigma^R = -\frac{1}{2}(\Sigma^{--} - \Sigma^{++}) - \frac{1}{2}(\Sigma^{+-} - \Sigma^{-+}). \tag{1}$$

The term in the first set of parentheses is the Lamb shift contribution, while that in the second set represents a relaxation part. Assuming that the Lamb shifts are already included in the corresponding frequencies, we write for retarded mass operator the formula

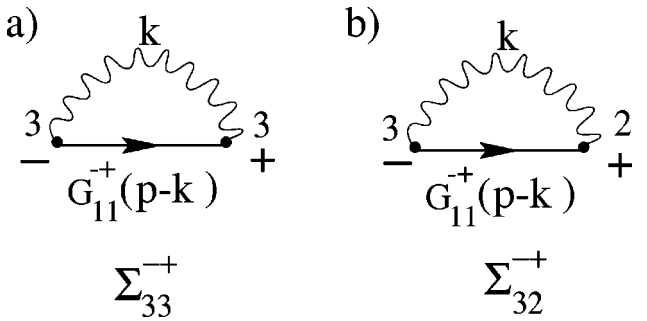

FIG. 2. Mass operators of a three-level atom.

$$\Sigma^R \approx -\frac{1}{2}(\Sigma^{+-} - \Sigma^{-+}). \tag{2}$$

Analogously, from

$$\Sigma^{--} = \Sigma^R - \Sigma^{-+}, \quad \Sigma^{++} = -\Sigma^R - \Sigma^{+-}, \tag{3}$$

and Eq. (2) we find

$$\Sigma^{--} \approx \Sigma^{++} \approx -\frac{1}{2}(\Sigma^{+-} + \Sigma^{-+}). \tag{4}$$

From similar considerations we arrive at the following formula for Σ^A:

$$\Sigma^A \approx \frac{1}{2}(\Sigma^{+-} - \Sigma^{-+}). \tag{5}$$

As can be seen from Eqs. (2), (5), and (4) we need to find only the Σ^{-+} and Σ^{+-} operators. We now evaluate the first nonvanishing diagrams, which correspond to the operators depicted in Fig. 2.

As can be seen from Fig. 2, diagonal terms of the atomic relaxation operator can be obtained from the mass operator in Fig. 2(a), while the interference between two decay channels $2 \to 1$ and $3 \to 1$ is taken into account by the operator in Fig. 2(b). Other mass operators Σ_{22}^{-+}, Σ_{23}^{-+}, Σ_{22}^{+-}, Σ_{23}^{+-}, are represented in similar diagrams.

Analytical expressions, corresponding to the diagrams in Fig. 2, can be written as

$$\Sigma_{33}^{-+}(\omega, \mathbf{p}) = \sum_{i,j} \int d_{31}^i d_{31}^{j*} D_{ij}^{-+}(\omega^{\rm ph}, \mathbf{k}) \times \mathbf{G}_{11}^{-+}(\mathbf{p}-\mathbf{k}, \omega - \omega^{\rm ph}) \frac{d\mathbf{k}\, d\omega^{\rm ph}}{(2\pi)^4},$$

$$\Sigma_{32}^{-+}(\omega, \mathbf{p}) = \sum_{i,j} \int d_{31}^i d_{21}^{j*} D_{ij}^{-+}(\omega^{\rm ph}, \mathbf{k}) \times \mathbf{G}_{11}^{-+}(\mathbf{p}-\mathbf{k}, \omega - \omega^{\rm ph}) \frac{d\mathbf{k}\, d\omega^{\rm ph}}{(2\pi)^4},$$

where we use the photon Green's function D_{ij}^{-+} in free space

$$D_{ij}^{-+}(\omega^{\rm ph}, \mathbf{k}) = \mathbf{J}_{ij}(2\pi)^2 \mathbf{h}\omega_{\mathbf{k}}[n_{\mathbf{k}}\delta(\omega^{\rm ph} - \omega_k) + (1 + n_{-\mathbf{k}})\delta(\omega^{\rm ph} + \omega_k)], \tag{6}$$

with $J_{ij}=\delta_{ij}-k_ik_j/k^2$, $\omega_k=kc$, and $n_{\mathbf{k}}$ being the photon number. Assuming for simplicity that atoms are motionless. Carrying out integrations, we find in the resonance approximation

$$\Sigma^{-+}_{uu'}(\omega_p,\mathbf{p})=i\frac{4}{3}\frac{d_{u1}d_{u'1}h(\omega_p-\epsilon_p)^3}{c^3}n(\omega_p-\epsilon_p)N_1(\mathbf{p}), \tag{7}$$

where $\epsilon_{\mathbf{p}}=p^2/2m$ is the kinetic energy of the atom, d_{u1} is the reduced dipole moment, $N_1(\mathbf{p})$ is the distribution function of atomic electrons in the ground level, and u and u' stand for the upper levels 2 and 3. Mass operators $\Sigma^{+-}_{uu'}$ can be obtained from $\Sigma^{-+}_{uu'}$ by the substitutions $n(\omega)\to 1+n(\omega)$ and $N(\mathbf{p})\to 1-N(\mathbf{p})$, reflecting the fact that we use Bose statistics for photons and Fermi statistics for atoms. The latter choice is not important since we consider the nondegenerate situation.

III. EQUATIONS FOR ATOMIC GREEN'S FUNCTIONS

The expressions for the mass operators obtained in Sec. II describe the interaction of three-level atoms with the gas of resonant photons. They determine the evolution of atomic Green's functions and hence enter into the corresponding equations. In the quasiclassic approximation equations for the retarded and kinetic Green's functions of the atoms can be written as

$$\left[i\left(\frac{\partial}{\partial\tau}+\frac{\mathbf{p}}{m}\frac{\partial}{\partial\mathbf{R}}\right)-\omega_u+\omega_{u'}\right]G^{-+}_{uu'}(\mathbf{R},\tau;\mathbf{p},\omega)$$
$$=\Sigma^{-\alpha}_{uu_1}G^{\alpha+}_{u_1u}+G^{-\alpha}_{uu_1}\Sigma^{\alpha+}_{u_1u'}, \tag{8}$$

$$\left[\frac{i}{2}\frac{\partial}{\partial\tau}+\omega-\omega_u+\frac{1}{8m}\frac{\partial^2}{\partial\mathbf{R}^2}+\frac{i\mathbf{p}}{2m}\cdot\frac{\partial}{\partial\mathbf{R}}-\epsilon_{\mathbf{p}}\right]G^R_{uu'}$$
$$=\delta_{uu'}+\Sigma^R_{uu_1}G^R_{u_1u'}, \tag{9}$$

where it is assumed that the summation is taken over repeated indices.

In this paper we are interested in the case of thermodynamic equilibrium, therefore, we consider the homogeneous space, time-independent case. This simplifies Eqs. (8) and (9) considerably. For the retarded Green's functions Eq. (9) gives

$$(\omega-\omega_2-\epsilon_{\mathbf{p}})G^R_{22}=1+\Sigma^R_{22}G^R_{22}+\Sigma^R_{23}G^R_{32}, \tag{10}$$

$$(\omega-\omega_3-\epsilon_{\mathbf{p}})G^R_{33}=1+\Sigma^R_{33}G^R_{33}+\Sigma^R_{32}G^R_{23}, \tag{11}$$

$$(\omega-\omega_3-\epsilon_{\mathbf{p}})G^R_{32}=\Sigma^R_{33}G^R_{32}+\Sigma^R_{32}G^R_{22}, \tag{12}$$

$$(\omega-\omega_2-\epsilon_{\mathbf{p}})G^R_{23}=\Sigma^R_{22}G^R_{23}+\Sigma^R_{23}G^R_{33}. \tag{13}$$

This system can be easily solved and leads to the following answer for $G_{uu'}$:

$$G^R_{22}=(\Omega_3-\Sigma^R_{33})/\Delta_{23},\quad G^R_{33}=(\Omega_2-\Sigma^R_{22})/\Delta_{23}, \tag{14}$$

$$G^R_{32}=\Sigma^R_{32}/\Delta_{23},\quad G^R_{23}=\Sigma^R_{23}/\Delta_{23}, \tag{15}$$

$$\Delta_{23}=(\Omega_2-\Sigma^R_{22})(\Omega_3-\Sigma^R_{33})-\Sigma^R_{32}\Sigma^R_{23}, \tag{16}$$

where we introduced the notation $\Omega_u=\omega-\omega_u-\epsilon_p$.

In the same way we write out by components equations for $G^{-+}_{uu'}$, using Eq. (8) and linear relations for mass operators and the Green's functions:

$$G^{-+}_{33}(\Sigma^{+-}_{33}-\Sigma^{-+}_{33})+\frac{1}{2}(\Sigma^{-+}_{32}-\Sigma^{-+}_{32})$$
$$=\Sigma^{-+}_{33}(G^R_{33}-G^A_{33})+\Sigma^{-+}_{32}(G^R_{32}-G^A_{23}), \tag{17}$$

$$G^{-+}_{22}(\Sigma^{+-}_{22}-\Sigma^{-+}_{22})+\frac{1}{2}(\Sigma^{-+}_{32}-\Sigma^{-+}_{32})$$
$$=\Sigma^{-+}_{22}(G^R_{22}-G^A_{22})+\Sigma^{-+}_{32}(G^R_{23}-G^A_{32}), \tag{18}$$

$$G^{-+}_{33}\Sigma^A_{32}-G^{-+}_{22}\Sigma^R_{32}+(\omega_{23}+\Sigma^A_{22}-\Sigma^R_{33})G^{-+}_{32}$$
$$=\Sigma^{-+}_{22}G^R_{32}-\Sigma^{-+}_{33}G^A_{32}+\Sigma^{-+}_{32}(G^R_{33}-G^A_{22}), \tag{19}$$

$$-G^{-+}_{33}\Sigma^R_{32}+G^{-+}_{22}\Sigma^A_{23}+(\omega_{32}-\Sigma^R_{22}+\Sigma^A_{33})G^{-+}_{23}$$
$$=-\Sigma^{-+}_{22}G^A_{23}+\Sigma^{-+}_{33}G^R_{23}+\Sigma^{-+}_{23}(G^R_{22}-G^A_{33}). \tag{20}$$

This system is a system of linear algebraic equations and therefore can be solved easily. To simplify the answer we will use the following relations for the mass operators, which follow from Eqs. (2), (5), and (7):

$$\Sigma^R=-\Sigma^A, \tag{21}$$

$$\Sigma^A_{22}\Sigma^A_{33}=(\Sigma^A_{23})^2. \tag{22}$$

Also note from Eqs. (7) and (15) that the atomic Green's functions obey the equalities

$$G^R_{32}=G^R_{23},\quad G^A_{32}=G^A_{23}. \tag{23}$$

As we mentioned in Sec. II, we made use of the fact that the occupation number of photons is very small, i.e., $n_{\mathbf{k}}\ll 1$, and evaluated mass operators in a one-loop approximation with accuracy $O(n_{\mathbf{k}})$. It is easy to prove that the following identity holds in this approximation:

$$\Sigma^{-+}_{32}\Sigma^A_{32}\Sigma^A_{22}\approx\Sigma^{-+}_{22}\Sigma^A_{32}\Sigma^A_{32}. \tag{24}$$

Note that this identity still holds if we interchange any two lower indices, which may belong even to different Σ. We will use this fact in simplifying the final answers for $G^{-+}_{uu'}$.

Now, from Eqs. (17)–(20) with the help of Eqs. (21)–(24) and plugging in expressions for $G^R_{uu'}$ from Eqs. (14)–(16) we arrive at the following answer for $G^{-+}_{uu'}$ after a few pages of algebra:

$$G^{-+}_{33}=-\Sigma^{-+}_{33}\frac{\Omega_2^2}{(\Omega_2\Omega_3)^2+|\Omega_2\Sigma^A_{33}+\Omega_3\Sigma^A_{22}|^2}, \tag{25}$$

$$G^{-+}_{22}=-\Sigma^{-+}_{22}\frac{\Omega_3^2}{(\Omega_2\Omega_3)^2+|\Omega_2\Sigma^A_{33}+\Omega_3\Sigma^A_{22}|^2}, \tag{26}$$

$$G_{32}^{-+}=-\Sigma_{32}^{-+}\frac{\Omega_2\Omega_3}{(\Omega_2\Omega_3)^2+|\Omega_2\Sigma_{33}^A+\Omega_3\Sigma_{22}^A|^2}. \quad (27)$$

Note the following property of these equations: Once we know the answer for G_{33}^{-+}, expressions for all other functions can be obtained simply by the appropriate interchanges of indices $3\rightarrow 2$ and $2\rightarrow 3$.

One can see from Eqs. (25)–(27) that the Green's functions have a similar structure: The corresponding mass operators are multiplied by the line contours, which give the line shape of probability of the atom to emit the photon of a certain frequency. However, the line shapes in Eqs. (25)–(27) are not normalized. To maintain consistency with the interference-free case and to clarify the physical meaning we rewrite these equations in such a way as to have normalized contours on the right-hand side. Carrying out integration over ω we find that the normalized contours are

$$a_{uu'}(\omega)=\frac{\Omega_u\Omega_{u'}\Sigma_{uu'}^A}{(\Omega_2\Omega_3)^2+|\Omega_2\Sigma_{33}^A+\Omega_3\Sigma_{22}^A|^2}. \quad (28)$$

Using Eqs. (28) and (5) we rewrite Eqs. (25)–(27) as

$$G_{uu'}^{-+}(\omega,\mathbf{p})=-\frac{\Sigma_{uu'}^{-+}}{\Sigma_{uu'}^{+-}-\Sigma_{uu'}^{-+}}2\pi i a_{uu'}(\omega). \quad (29)$$

Finally, we substitute instead of $n(\omega)$ and $N_1(\omega)$ the Planck and Fermi distributions correspondingly and arrive at the following answer, omitting terms of order $n(\omega)$ and $N_1(\epsilon_{\mathbf{p}})$ in the expression for the contour $a_{uu'}(\omega)$:

$$G_{uu'}^{-+}(\omega)=-\frac{1}{e^{\omega/T}+1}2\pi i a_{uu'}(\omega), \quad (30)$$

$$a_{uu'}(\omega)=\frac{\sqrt{A_uA_{u'}}\Omega_u\Omega_{u'}}{(\Omega_2\Omega_3)^2+\frac{1}{4}(\Omega_2A_3+\Omega_3A_2)^2}. \quad (31)$$

The most unexpected result that follows from Eqs. (30) and (31) is that the nondiagonal atomic Green's function is not zero in thermodynamic equilibrium. It is known that atoms acquire coherence between levels as a result of the interaction with a monochromatic external classical electromagnetic field. Since an electromagnetic field in thermodynamic equilibrium with atoms does not possess any of these properties, the obvious conclusion would be that atoms cannot have any nonzero coherence between levels in thermodynamic equilibrium, even when collisions between atoms are neglected. Besides this objection, we do not know any general theorem that would prevent coherence from developing in thermodynamic equilibrium.

The physical reason behind our result, obtained through the diagrammatic approach, is that we take into account the interaction of both upper levels with the same quantum mode of the field. As we see later, an electromagnetic field is described by the Planck distribution for the occupation numbers and hence does not possess a nonzero electric field. Nevertheless, the simultaneous interaction of both upper levels with the same quantum mode is responsible for inducing coherence between atomic levels. An indication that this may be true was found in our earlier paper [7,16], where we investigated the spontaneous emission of an initially excited atom with interfering decay channels. As was explained in Ref. [16] for the case of a free atom, this coherence is a result of a reabsorption of the virtual photon by different atomic levels. The same mechanism is responsible for the atomic coherence in our case of equilibrium.

It is important to realize that such coherence does not lead to a nonzero quadrupole moment of the medium in the absence of magnetic fields, as it may seem from the equation

$$Q^{ij}=\sum_{u\neq u'}q_{uu'}^{ij}\rho_{uu'}, \quad (32)$$

where the atomic density matrix $\rho_{uu'}$ is obtained from the Green's function through integration over ω:

$$\rho_{uu'}=\int G_{uu'}^{-+}(\omega,\mathbf{p})\frac{d\omega}{2\pi}. \quad (33)$$

To calculate Q^{ij} correctly one needs to carry out calculations for the real levels characterized by the angular momentum and its projection $|J,M_j\rangle$. Since the relaxation operator, as obtained in Ref. [16], has a $\delta_{M_aM_b}$ dependence, it can only lead to $G_{\alpha\beta}\propto\delta_{M_aM_b}$. This coherence, however, leads to a zero quadrupole moment.

Note that Eqs. (30) and (31) are valid for the two-level case if we set $A_3=0$ and $\omega_{32}=0$. As was rigorously shown in Ref. [17], only if the Fermi exponent depends on the fluent photon frequency, rather than on the frequency of atomic transition, does such a Green's function lead to the correct Planck distribution for the photons. As we shall see in Sec. IV, this dependence, which is contained in our result (30) and (31), leads to the same Planck distribution, despite the nonzero coherence of Eqs. (30) and (31).

In Sec. II we assumed that the gas of photons is described by the occupation number $n(\omega)$, which has a Bose-Einstein distribution. In order to be consistent, we have to obtain the solution for the photonic Green's functions, using the atomic Green's functions (30) and (31), to see that our choice is consistent. This consistency check is the subject of the next section.

IV. POLARIZATION OPERATORS AND PHOTONIC GREEN'S FUNCTIONS

Now we need to evaluate polarization operators in order to write down equations for the kinetic Green's functions for photons. In the one-loop approximation polarization operators Π^{-+} are represented by the sum of diagrams, similar to the one shown in Fig. 3.

An analytical expression for the operator Π^{-+}, based on Fig. 3, can be written as

$$\Pi^{-+}(\mathbf{k},\omega^{\mathrm{ph}})=\frac{i}{3}\int\frac{d\omega\,d\mathbf{p}}{(2\pi)^4}\sum_{u,u'}G_{uu'}^{-+}(p+k)\times G_{11}^{+-}(p)d_{u1}d_{1u'}. \quad (34)$$

Analogously, we can write down expression for the operator Π^{+-}:

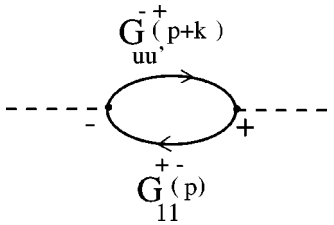

FIG. 3. Polarization operators of the medium, consisting of three-level atoms.

$$\Pi^{+-}(\mathbf{k},\omega^{\mathrm{ph}})=\frac{i}{3}\int\frac{d\omega\,d\mathbf{p}}{(2\pi)^4}\sum_{u,u'}G^{+-}_{uu'}(p+k)\times G^{-+}_{11}(p)d_{u1}d_{1u'}. \tag{35}$$

In Eqs. (34) and (35) the atomic Green's function should be taken from Eqs. (30) and (31). To find the kinetic photon Green's function D^{-+} we need to know the ratio $\Pi^{+-}:\Pi^{-+}$ since D^{-+} is equal to

$$D^{-+}=\frac{\Pi^{-+}}{\Pi^{+-}-\Pi^{-+}}(D^R-D^A). \tag{36}$$

By comparing Eqs. (34) and (35) we find that

$$\Pi^{-+}=e^{\omega^{\mathrm{ph}}/T}\Pi^{+-}. \tag{37}$$

Then the function D^{-+} can be written as

$$D^{-+}=\frac{1}{e^{\omega^{\mathrm{ph}}/T}-1}(D^R-D^A). \tag{38}$$

The first multiplier gives the photon statistics, while D^R-D^A determines the line contour of propagation of photons with different frequencies. We see that the nonzero coherence of Eqs. (30) and (31) does not influence the photon statistics. As is obvious from Eq. (38), the photon occupation number is governed by the Bose-Einstein distribution, which proves the consistency of our earlier assumption.

V. EMISSIVITY OF THE MEDIUM

There is one more important characteristic of the medium in question, namely, its emissivity. It is related to the polarization operator through the formula

$$\varepsilon(\omega)=-\frac{2i\omega^2}{c\lambda^2}\Pi^{-+}(\omega). \tag{39}$$

Plugging in an expression for Π^{-+} from Eq. (34) and assuming that the lower level has a zero width, we find

$$\varepsilon(\omega)=\frac{4}{3}\frac{\omega^4}{4\pi c^3}[1-N(0)]N(\omega)\sum_{u,u'}d_{u1}d_{u'1}a_{uu'}(\omega), \tag{40}$$

where $N(\omega)=1/(1+e^{\omega/T})$ is the Fermi distribution function.

In order to see what role is played by interference, we plot $\epsilon(\omega)/\omega$ from Eq. (40) with and without interference in Fig. 4. The case of no interference can be obtained from Eq. (40) by omitting nondiagonal terms and making the $\Sigma_{ii}\to 0$ transition for each contour $a_{uu}(\omega)$.

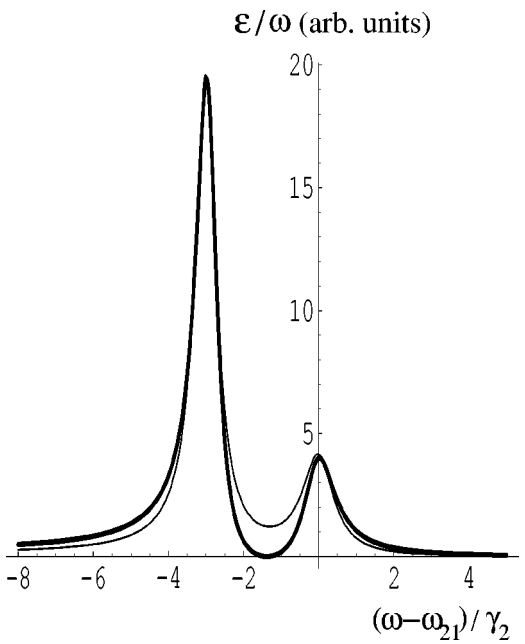

FIG. 4. Zero point in the emission spectrum. Frequency is normalized to the width γ_2 of the level 2. The atomic parameters are $\omega_{32}/\gamma_2=-3$, $\omega_{21}/\gamma_2=100$, $T/\gamma_2=5$, $d_{31}/d_{21}=\sqrt{2}$, and $\omega_{31}/\omega_{21}=0.7$. The thick and thin lines correspond to spectra with and without interference.

We plot $\epsilon(\omega)/\omega$ in Figs. 4 and 5 in different frequency ranges in order to outline the influence of the interference on different parts of the spectrum. The atomic parameters and the temperature of the medium are normalized to γ_2.

As we can see from Fig. 4, quantum interference leads to the appearance of a zero point in the emission spectrum of the medium, as well as to enhancement of the red wing intensity (see Fig. 5). These graphs are plotted for the gas of atoms, which is in equilibrium with the resonant photon gas.

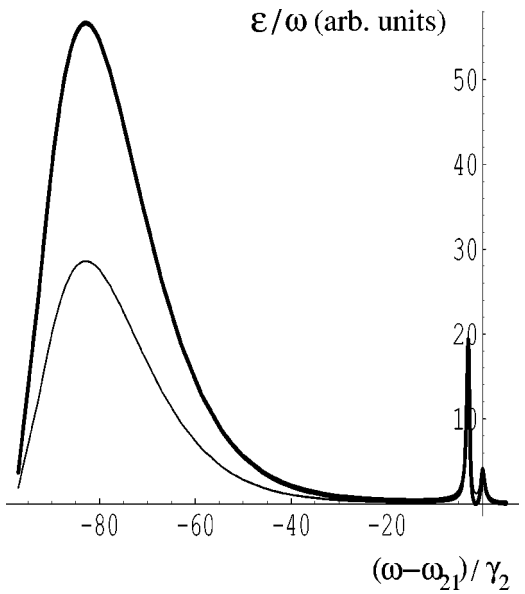

FIG. 5. Enhancement of the red wing due to quantum interference. All conventions are the same as in Fig. 4.

Recall that when a similar medium of three-level atoms interacts with the laser field, the emission spectrum also has a zero point, the so-called dark line [1,4]. Note, however, that these two cases are different since the properties of the photon fields are fundamentally different.

VI. CONCLUSION

In summary, we considered the influence of quantum interference between two decay channels of three-level atoms on various characteristics of an atomic gas and a radiation field in thermodynamic equilibrium with each other. We found that in the absence of collisions between atoms, this interference does not change the populations of the atomic electrons, which are governed by Boltzmann statistics. It leads, however, to nonzero coherences between upper levels, which is due to nonzero nondiagonal elements of the atomic Green's functions (30) and (31). This coherence does not change the distribution of photons in equilibrium, which obeys the conventional Planck formula (38). It changes, however, the emissivity of the medium. Unlike the case of no interference, the emissivity has a zero point at a certain frequency. More importantly, atomic coherence between upper levels leads to enhancement of the red wing intensity as compared to the case of zero nondiagonal elements of the density matrix. Experimental verification of the predicted effects seems feasible owing to recent experiments with sodium dimers [18].

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