UTILITY OF EXTRACTING α PARTICLE ENERGY BY WAVES

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ABSTRACT. The utility of extracting α particle power, and then diverting this power to fast fuel ions, is investigated. As power is diverted to fast ions and then to ions, a number of effects come into play, as the relative amounts of pressure taken up by electrons, fuel ions and fast α particles shift. In addition, if the α particle power is diverted to fast fuel ions, there is an enhanced fusion reactivity because of the non-thermal component of the ion distribution. Some useful expressions for describing these effects are derived, and it is shown that fusion reactors with power density about twice what otherwise might be obtained can be contemplated, so long as a substantial amount of the α particle power can be diverted. Interestingly, in this mode of operation, once the electron heat is sufficiently confined, further improvement in confinement is actually not desirable. A similar improvement in fusion power density can be obtained for advanced fuel mixtures such as D–^{3}He, where the power of both the energetic α particles and the energetic protons might be diverted advantageously.

1. INTRODUCTION

If the energy from energetic α particles could be extracted by waves and diverted to the tail of the fuel distribution in a tokamak reactor, there are a number of benefits: first, the energetic α particle pressure is suppressed, allowing for more fuel ion pressure. Second, the electron temperature is suppressed while the ion temperature is enhanced, possibly giving rise to the so-called 'hot ion mode'. Third, there is a non-thermal fuel ion component that may lead to increased reactivity at a given pressure. On the other hand, there are costs: to divert α particle power may require external catalytic heating, and, in any event, the increased reactivity leads to more α particle pressure, which also must be taken into account. What this paper attempts to do is to quantify these benefits and costs.

It has been recognized that there are advantages in attempting to operate fusion reactors in regimes in which there is a significant hot, non-Maxwellian component to the fuel ions [1–3] or in which the fuel ion temperature can be much greater than the electron temperature [4, 5]. Noting a number of experiments [6–8] exhibiting the hot ion mode, Clarke [5] pointed out that the hot ion mode regime could be reached if the ion energy confinement time exceeds the electron energy confinement time, assuming velocity space instabilities that diverted α particle power to the fuel ions. Such instabilities have been considered [9–12] in the context envisioned by Clarke, but the amount of free energy is limited. Recently, it has been recognized that the free energy in the α particles might be more completely tapped by injecting waves that diffuse the α particles both in space and in energy, rather than just in energy [13–15]. In fact, it appears that, at least in principle, eventually all of the α particle power could be diverted to the ions.

In view of the added element that there are now at hand definite ways [13–17] of tapping the α particle power by waves, and that these waves might then damp resonantly on the fast energy tail of the fuel ions, this paper builds upon the work of Clarke. Thus, not only is the hot ion mode realized through the diversion of α particle power, as envisioned by Clarke, but a significant non-Maxwellian fusion component is realized simultaneously, as envisioned by Furth, Dawson and co-workers.

The approach adopted in this paper is to solve self-consistently the heat balance equations in 0-D for electrons and ions, including the heating from both the Maxwellian and the non-Maxwellian contributions to the fusion power. This simple 0-D model demonstrates the possible improvement in power density, although 1-D considerations would be required to quantify the benefits of channelling the α particle power with realistic plasma profiles. In 0-D, heat is lost from electrons or ions either through collisional equilibration with another plasma species or to the outside. Heat lost to the outside, whether by radiation, conduction or convection, is lumped into an electron or ion heat confinement time. Importantly, the electron heat and the ion heat confinement times are distinguished as separate quantities. This is a point worth brief elaboration, in view of the importance of this distinction in achieving the hot ion mode.

In order to attain ion temperatures that are far in excess of electron temperatures, it is not only important to channel α particle power into ions; it is also important to remove the heat from the reactor through the electrons. It may be advantageous, in fact, to spoil the electron heat containment deliberately. To see that this must be so, at least in some regime of tokamak operation, consider the limit in which the ion heat confinement time is finite, but the electron heat is perfectly contained. In such a case, even if all the α particle power were channelled to the ions, the electrons would reach a collisional equilibrium at the ion temperature, thus excluding the hot ion mode. At least in this limit, it would be desirable if electron heat were more poorly contained, so that the electron pressure could be decreased. Of course, if the electron heat were too poorly confined, a self-sustained fusion reactor would not be attainable either.

An aspect of this work is to treat the confinement times as essentially independent parameters. Thus, rather than to embrace at the outset a particular scaling law that solves for confinement times in terms of other parameters, a feature of the present work is to survey exhaustively possible electron and ion heat confinement times that yield self-consistent fusion production. This approach will yield combinations of confinement times that optimize the fusion power production. Of course, these confinement times depend upon plasma temperature, density, impurity concentration, magnetic field strength and other quantities. Hence, in designing a reactor, it would be necessary to arrange, through adjusting these other quantities, for the optimal or near optimal combination of confinement times.

This approach to the problem departs somewhat from other approaches that employ particular confinement scalings. These confinement scalings are, however, a matter of considerable debate (for recent references, see, e.g., Refs [18–22]). Plasma heat confinement, for example, sometimes deteriorates and sometimes improves with increasing temperature [18, 22]. Moreover, confinement appears to vary in present experiments for ohmic, RF or neutral beam heated discharges, so that such data may not extrapolate to α particle heated reactor plasmas. In treating the confinement times as independent parameters, we manage to postpone the choice of confinement laws which might further restrict the range of reactor operation that we consider. Among other things, this allows us to focus the discussion on what confinement times are useful operating points. It may, in fact, turn out that these times may be directly controllable, at least insofar as purposefully degrading confinement.

This paper does not address the utility in diverting α particle power for the purpose of amplifying the current drive effect. The possibilities for significantly less circulating power in accomplishing the current drive effect have been discussed elsewhere [13]. In principle, both enhanced reactivity and enhanced current drive efficiency could be obtained at once, if not necessarily optimized at once. Here, our concern will be the benefits of the hot ion mode, and how it might be attained.

The paper is organized as follows: in Section 2, the effect of diverting a small amount of power at constant plasma pressure is examined. This incremental posing of the problem is useful, among other reasons, for isolating and understanding the different effects that come into play upon diverting power. In Section 3, 4 and 5, these various effects are quantified and discussed. In Section 6, a global approach is taken to find operating points that give self-consistent burn. In Sections 7 and 8, analytic expressions are given for ignition or self-consistent burn in the hot ion mode. Some useful limiting cases are examined in which progress can be made analytically. In Section 9, a number of examples of self-consistent burn parameters are calculated numerically. In Section 10, contour plots are introduced that depict how optimized operating points can be found. These plots are very useful in navigating through parameter space to reach optimum reactor performance, whether defined by doubling the fusion power density, or by optimizing in some other way, such as by reducing the necessary heat confinement times. The point is made in Section 11 that similar increases in fusion power density are available through diverting the charged fusion products in a D–^{3}He reactor. A summary of our findings is presented in Section 12.

2. INCREMENTAL DIVERSION OF α PARTICLE POWER

One way of quantifying the benefits and costs of diverting α particle power is to calculate the net effect of diverting some small amount of α particle power Δp, that normally would have gone to electron heating and is now to go to superthermal fuel ion heating. (Note that the quantity Δp is not quite the diverted power, since some of the diverted power might in any event have been absorbed by ions, and that part would not count in Δp.) This is the so-called 'incremental' posing of the problem. Presumably, in the absence of the diversion, the reactor is designed to operate at ignition and at maximally allowable pressure.

This posing of the problem should quantify the utility of an auxiliary system to an operable reactor, where that auxiliary system extracts extra fusion power from the reactor without changing its operating design, particularly with respect to maintaining the total plasma pressure. There are, however, a number of subtleties here. How exactly is the plasma pressure to be maintained, if extra fusion power, together with the associated extra plasma heating, is the result of diverting this small amount of power? This is intimately related to the question of burn control, which in any event must be a part of the reactor design.

In order to pose the incremental problem sensibly, without going into the details of a specific reactor design, let us assume that burn control is essentially exercised by the prompt loss of some α particles. These α particles then do not contribute to the plasma heating, nor do they contribute to the plasma pressure, nor is the power here available for diversion to the ions. By adjusting the rate of these prompt losses, a steady state plasma burn at constant plasma pressure can be maintained.

Thus, in quantifying the effect of enhanced reactivity, we shall not consider, in the incremental problem, the effect of the enhanced reactivity on the plasma operating regime; specifically, we shall assume that any extra fusion power produced is somehow promptly lost so that it neither further heats the plasma nor contributes to the energetic α particle pressure, with the provision that, in order to maintain the plasma at constant pressure, the precise amount of α particle power available to indeed heat the plasma and to contribute to the plasma pressure may be adjusted through the burn control. In this regard, i.e. to remain at constant pressure, we imagine too that upon diverting power it may even be necessary to adjust the base level (excluding the enhanced production) of α particle power that is deposited within the plasma.

This posing of the incremental problem is not unique. For example, an alternative posing of the problem might be to imagine a subignited plasma, with flexibility maintained over the external heat source. A second example might be to allow for an adjustment in the energy confinement times of the fuel ions and electrons. Indeed, with the flexibility to tamper individually with these confinement times, somewhat more optimistic results could be obtained. The present posing, however, appears to be both pristine and simple, while capturing the essential physics. The key question to be answered here is how many extra watts of fusion power can be captured for every watt of α particle power that is diverted. An add-on system will be economical if this number is large, assuming that the cost to divert power is small.

The incremental problem addresses the question of net power amplification, but not how much extra fusion power is in fact available. If by diverting α particle power more α particle power is produced, this further power might also be diverted to advantage. The 'maximal' rather than 'incremental' posing of the problem addresses self-consistently exactly how large an effect is possible. The incremental problem is considered first, and is useful for understanding in detail the competing effects that occur upon diverting power.

3. ENHANCED ION PRESSURE FROM DIVERTING α PARTICLE POWER

Suppose that a quantity of α particle power Δp is diverted from electron heating to ion heating. To calculate the increase in reactivity, we use a 0-D model of the heat flow,

$$\frac{d}{dt} u_e = \nu(\zeta u_i - u_e) + p_e - u_e/\tau_e \tag{1}$$

$$\frac{d}{dt} u_i = \nu(u_e - \zeta u_i) + p_i - u_i/\tau_i \tag{2}$$

where $u_e = 3n_eT_e/2$ is the electron energy density, u_i is the ion energy density, ν is the energy equilibration rate, τ_e and τ_i are the electron and ion energy confinement times, respectively, and p_e and p_i are the external heating powers to electrons and ions, respectively, including α particle power. Here, we have defined

$$\zeta \equiv \sum_j n_j Z_j \Big/ \sum_j n_j$$

which is the ratio of electron to ion densities, with Z_j defined as the ion charge state for the jth species. Suppose, for simplicity, a pure hydrogen plasma, so that $\zeta = 1$. To find the steady state operating energy densities, solve Eqs (1) and (2) with $d/dt = 0$ to obtain

$$u_i = \frac{p_e + (1 + 1/\nu\tau_e)p_i}{D} \tag{3}$$

$$u_e = \frac{p_i + (1 + 1/\nu\tau_i)p_e}{D} \tag{4}$$

where

$$D = \frac{1}{\tau_i} + \frac{1}{\tau_e} + \frac{1}{\nu\tau_e\tau_i} \tag{5}$$

If a quantity of α particle power Δp is diverted from electron heating to ion heating, then it may be seen from Eqs (3) and (4) that the total plasma energy density can change if the plasma is not equally able to contain electron heat and ion heat. Assuming operation at the maximum pressure for any Δp, power diversion at constant pressure or energy density must be accompanied by incremental changes either in the confinement times or in the total power input. The power input might be controlled, as discussed in Section 2, for example, by designing the operating point at somewhat different α particle heating such that

$$p_i = (1 - \theta)p_{i0} + \Delta p \tag{6}$$

$$p_e = (1 - \theta)p_{e0} - \Delta p \tag{7}$$

where θ is the incremental fraction change in alpha heating for finite Δp which assures that operation is at constant total energy density, i.e.

$$u \equiv u_e + u_i = u_{e0} + u_{i0} \tag{8}$$

For simplicity, we assume in linearizing these equations that the confinement times are constants (or at least depend only on the total energy density).

Substituting Eqs (6) and (7) into Eqs (3) and (4), and using Eq. (8), results in three linear equations in the three unknowns u_i, u_e and θ, with solutions such that the plasma achieves a new operating point at

$$\theta = \frac{\Delta p}{\nu D}\left(\frac{1/\tau_e - 1/\tau_i}{u_{e0} + u_{i0}}\right) \tag{9}$$

where u_{e0} and u_{i0} are the equilibrium energy densities in the absence of any power diversion. Note that if $\tau_i > \tau_e$, then $\theta > 0$, indicating that less α particle power maintains the plasma at the maximum energy density. If we write

$$u_i = u_{i0} + \Delta u_i$$

$$u_e = u_{e0} + \Delta u_e$$

then we have $\Delta u_i = -\Delta u_e \equiv \Delta u$ with

$$\Delta u = \frac{\Delta p}{\nu D}\left(\frac{u_{e0}/\tau_e + u_{i0}/\tau_i}{u_{e0} + u_{i0}}\right) \tag{10}$$

The ratio $\Delta u/\Delta p$ can be thought of as the incremental efficiency in diverting power to increase u_i. Note that constant plasma energy density is maintained by adjusting the α particle heating power. Although this keeps the sum of the fuel ion and electron pressure constant, the change in the number of energetic α particles present to maintain the plasma pressure does affect the α particle pressure as addressed in Section 4.

4. ENHANCED ION ENERGY DENSITY FROM REDUCING α PARTICLE ENERGY DENSITY

When the α particle power is diverted into fast fuel ions, the fast fuel ions at say 100 keV slow down quickly compared with the 3.5 MeV α particles. In the maximal posing of the problem, addressed in later sections, the energetic fuel ion pressure is taken into account. For simplicity here, however, we neglect the fast ion pressure (see Appendix). The extra pressure available to the plasma, which must be shared between electrons and ions, is then just the amount lost by the α particles. Let the total α particle energy density be $u_{\alpha H}$, let P_α be the α particle power, and define the α particle slowing down time as $\tau_\alpha = u_{\alpha H}/P_\alpha$. If Δp is diverted from the α particles into fast ions, the change in the α particle energy density is $\Delta u_{\alpha H} = -\Delta p \tau_\alpha$. Thus, if the total fixed plasma energy density is

$$u_0 \equiv u_e + u_i + u_{\alpha H} = u_i(1 + u_e/u_i) + u_{\alpha H}$$

then for a fixed ratio $u_e/u_i = u_{e0}/u_{i0}$, one recovers the change in u_i due to the decreased α particle energy density upon diverting power Δp as

$$\Delta u_i^{(1)} = -\frac{\Delta u_{\alpha H}}{1 + u_e/u_i} = \frac{\Delta p \tau_\alpha}{1 + u_{e0}/u_{i0}} \tag{11a}$$

The enhanced fusion reactivity leads to more α particles and hence more fast α particle pressure, but the number of α particles retained, in the incremental model, is only sufficient to maintain the total plasma pressure. The α particle power available to heat the plasma is, from Eqs (6) and (7), changed by an amount $\Delta P_\alpha = \theta P_\alpha$. Thus, in addition to the decrement in $u_{\alpha H}$ upon diverting power Δp, there may be an additional difference, if $\theta \neq 0$. This results from the altered operating regime, since a smaller or larger amount of α particle power needs to be absorbed to maintain the plasma pressure, this difference being $\Delta P_\alpha = -\theta P_\alpha$. Using Eq. (9), the increase in the available fuel ion energy density is then

$$\Delta u_i^{(2)} = \frac{\theta P_\alpha \tau_\alpha}{1 + u_{e0}/u_{i0}} = \frac{\Delta p}{\nu D}\left(\frac{1/\tau_e - 1/\tau_i}{u_{e0} + u_{i0}}\right)\frac{P_\alpha \tau_\alpha}{1 + u_{e0}/u_{i0}} \tag{11b}$$

which is in addition to the term in Eq. (11a).

5. INCREMENTAL ENHANCED FUSION PRODUCTION

The enhanced fusion resulting from the production of fast non-thermal ions can be written as $\Delta p \chi_\alpha$, where χ_α can be treated, for simplicity here, as a constant (see Appendix). The enhanced fusion resulting from an increase in the thermal ion energy density can be written, assuming ion temperatures that optimize the fusion power at constant energy density, as $P_\alpha \sim c u_i^2$, where c is a constant. Thus, the incremental power produced upon realizing an incremental increase in ion energy density is

$$\frac{\Delta P_\alpha}{P_\alpha} \simeq 2\frac{\Delta u_i}{u_i} \tag{12}$$

Let us specialize to the case where the only heating is from α particles with

$$p_e = (1 - \eta)P_\alpha \tag{13a}$$

$$p_i = \eta P_\alpha \tag{13b}$$

so that from Eq. (3) we have

$$\frac{P_\alpha}{u_i} = \frac{D}{1 + \eta/\nu\tau_e} \tag{14}$$

Using now Eqs (10) and (11) for Δu_i, we get from Eq. (12)

$$\Delta P_\alpha \simeq \Delta p \left\{\chi_\alpha + \frac{2}{\nu + \eta/\tau_e}\left(\frac{u_{e0}/\tau_e + u_{i0}/\tau_i}{u_{e0} + u_{i0}}\right) + \left(\frac{2D}{1 + \eta/\nu\tau_e}\right)\left(\frac{\tau_\alpha}{1 + u_{e0}/u_{i0}}\right) \times \left[1 + \frac{P_\alpha}{\nu D}\left(\frac{1/\tau_e - 1/\tau_i}{u_{e0} + u_{i0}}\right)\right]\right\} \tag{15}$$

where the first term on the right is the incremental fast ion fusion, the second term is the enhanced u_i due to the diversion of power from electrons to ions, and the third term, with two parts, is due to the availability of extra fuel pressure because of the decrease in α pressure. The first part arises from the direct diverting of fast α particle power and the second part arises from the decreased amount of α particle heating needed to maintain the plasma pressure upon diverting α particle power should τ_i be greater than τ_e.

To determine the power multiplication available upon an incremental power diversion, consider a DT reactor operating such that $\tau_i \simeq \tau_e \equiv \tau$, $u_{e0} \simeq u_{i0}$, and such that $\nu\tau \gg 1$. Then Eq. (15) simplifies to

$$\Delta P_\alpha \simeq \Delta p\left(\chi_\alpha + \frac{2}{\nu\tau} + 2\frac{\tau_\alpha}{\tau}\right) \tag{16}$$

Note that, for better confinement times, the incremental advantage in diverting power is less. This is because, for good confinement, it is relatively more difficult to achieve the non-Maxwellian features that enhance the reactivity. On the other hand, poor confinement now has certain advantages.

For a 50:50 DT mixture, one can expect $\chi_\alpha \simeq 1/4$. For a reactor regime (e.g., similar to that in Table II), one has $\tau_\alpha/\tau \simeq 1/6$ and $\nu\tau \simeq 3$. Thus, one can expect about $1/4 + 2/3 + 1/3 = 5/4$ watts of α particle power back for every watt of α particle energy diverted in such a reactor, or 25/4 fusion watts per diverted watt, excluding extra fusion reactions in the blanket.

What remains to be calculated is how many watts of external power it takes to divert a watt of α particle power. In principle, the α particle free energy could be tapped without any external source of power, but that is very unlikely to happen. Suppose, for example, it takes M watts to divert one watt, with the heating power amplified by the diverted watt as it heats the energetic fuel ions. Suppose further that there are enhanced losses of electron heat to accommodate the increase in ion heating so that the reactor operating regime remains at constant pressure. Then the heating power itself contributes to both enhanced fast ion reactivity and enhanced ion temperature relative to electron temperature. Any extra α particle power produced, in the incremental posing of the problem, is assumed lost. Thus, in the limit in which Eq. (16) is valid, the extra production of α particle power due to heating power $p_H = M\Delta p$ is

$$\Delta P_\alpha \simeq p_H\left(\chi_\alpha + \frac{2}{\nu\tau}\right) \tag{17}$$

Suppose a ratio of fusion to α particle power of $\epsilon_f/\epsilon_\alpha$. Then applying heating power p_H results in extra heating power

$$\Delta P \simeq p_H\left[1 + \frac{\epsilon_f}{\epsilon_\alpha}(\chi_\alpha + 2/\nu\tau) + \left(\frac{\epsilon_f/\epsilon_\alpha}{M}\right)\left(\chi_\alpha + \frac{2}{\nu\tau} + 2\frac{\tau_\alpha}{\tau}\right)\right] \tag{18}$$

where the first term is the external heating power itself recovered in electron heat.

To continue this example for a reactor design, suppose $M = 1$, i.e. it takes one watt to divert a watt. This means that the heating watt results in about 11/12 W of increased α particle power, or 4.6 total fusion watts. In addition, there is the extra watt of injected heat removed from electrons. Thus, using one heating watt in this manner results in $6.3 + 4.6 + 1 = 11.9$ W of heating power. This incremental 'Q' of 11.9 at $M = 1$ is large enough to make a very worthwhile piggyback system for diverting power off an operating reactor. Of course, if one watt could be diverted with just one half watt of external heating ($M = 1/2$), then Q rises to 18.1. Note that just heating the ions alone, without diverting α particle power, results merely in a piggyback Q of 5.6.

Caution should be exercised, however, in using the numbers in this example. These numbers are not expected to be precise. They are derived only in the limit where $\nu\tau \gg 1$, a limit that is only very marginally satisfied in the case here. Also, the neglect of the additional fast α pressure tends to overestimate Q, and this pressure is likely to be present unless substantial α power is diverted.

6. OPERATING POINT FOR HOT ION MODE

Significant incremental power gain in diverting α particle power suggests that an 'add-on system' is highly desirable. However, if it produces only a small fraction of the total reactor output, its impact on the cost of electricity will be marginal. This section addresses the 'maximal' problem, supposing that essentially all of the α particle power can be diverted to fast ions. The 'hot ion mode', where $T_i > T_e$, as envisioned by Clarke, can be attained, most easily by means of this diversion when $\tau_i \gg \tau_e$.

For reference, consider a generic reactor that operates with $T_e \simeq T_i$, and in which there is a certain percentage of the reactor pressure taken up by the energetic α particles. The advantage gained by diverting α particle power may be measured by the significantly higher fusion power density attained at the same pressure confined as in the reference reactor. Such a design, which may have comparable ion heat confinement times, but small electron confinement times, should achieve ignition in the hot ion mode, ideally with essentially no pressure taken up by energetic α particles. (Although this is a useful measurement, in practice, the economical advantages of diverting α particle power may be realized in other ways, e.g., by lowering the magnetic field but keeping the fusion power density constant.)

To find a self-consistent set of parameters, begin, again, with Eqs (3), (4) and (8), retaining the generalization to ions with arbitrary charge state ($\zeta \neq 1$), and use Eqs (13a, b) to write

$$u_e = \frac{\nu\tau_e}{1 + \nu\tau_e}\left(\zeta u_i + \frac{1-\eta}{\nu} P_D\right) \tag{19}$$

$$\zeta u_i = \frac{\nu\zeta\tau_i}{1 + \nu\zeta\tau_i}\left(u_e + \frac{\eta}{\nu} P_D\right) \tag{20}$$

$$u_e + u_i = u \tag{21}$$

where P_D is the total power that is deposited in the plasma, and where η represents the fraction of this power absorbed by the ions.

Equations (19), (20) and (21) can be solved for the three unknowns u_i, u_e and P_D, resulting in

$$\frac{u_i}{u} = \frac{(1-\eta)\rho_i\rho_e + \eta\rho_i}{K} \tag{22}$$

where, for convenience, we have defined

$$\rho_i \equiv \frac{\nu\tau_i\zeta}{1 + \nu\tau_i\zeta}$$

$$\rho_e \equiv \frac{\nu\tau_e}{1 + \nu\tau_e}$$

$$K \equiv (1-\eta)\rho_e(\zeta + \rho_i) + \eta\rho_i(1 + \rho_e\zeta)$$

Note from Eq. (22) that u_i is maximized, i.e. $u_i \to u$, for $\eta \to 1$ and for $\rho_e \sim \tau_e \to 0$, irrespective of τ_i; in the event that $\eta \neq 1$, then u_i is maximized for $\tau_i \to \infty$, although, in this case, the maximum is less. Maximizing u_i means maximizing the thermal component of the fusion power. Similarly, one can find u_e as

$$\frac{u_e}{u} = \frac{(1-\eta)\rho_e + \eta\rho_e\rho_i}{K/\zeta} \tag{23}$$

The power necessary to maintain these plasma pressures is

$$\frac{P_D}{\nu u} = \frac{1 - \rho_e\rho_i}{K/\zeta} \tag{24}$$

Note that P_D increases as τ_e or τ_i decreases, since greater external power is needed if confinement is poor. However, for $\eta \to 1$, and $\rho_e \sim \tau_e \ll 1$, then $u_i \to u$, and the amount of external power necessary to maintain the ion and electron pressure, $P_D \to (\nu + \tau_i^{-1})u$, is essentially determined by the smaller of τ_i and $1/\nu$.

7. IGNITION IN THE HOT ION MODE

The power absorbed in the plasma is

$$P_D = P_\alpha + P_H \tag{25}$$

if all the external heating power, P_H, and all the α particle power, P_α, is absorbed in the plasma. The α particle power might either be absorbed directly in the plasma, or some fraction, $\eta_w P_\alpha$, might be diverted by waves to the fast fuel ion population, drawing out a non-thermal high energy tail to this distribution, which then heats the bulk plasma. Similarly, a fraction η_{Hi} of the external heating, or $\eta_{Hi}P_H$, is first deposited in fast ions.

The α particle power produced by the plasma can then be written as

$$P_\alpha = c(T_i)u_i^2 f_D f_T + \eta_w P_\alpha \chi_\alpha + \eta_{Hi} P_H \chi_\alpha \tag{26}$$

where χ_α is the ratio of extra α particle power produced by the non-thermal fast fuel ions to the amount of power diverted into these ions, and where f_D and f_T are the fractions of total ions taken up by deuterium and tritium, respectively.

Accompanying the non-thermal fusion power available is the cost of fast fuel ion pressure. Suppose the total available plasma energy density is u_0, of which $u_{\alpha H}$ is taken up by the hot energetic α particles and u_{fi} by fast fuel ions. The available bulk plasma energy density is then

$$u = u_0 - u_{\alpha H} - u_{fi} \tag{27}$$

where

$$\begin{aligned} u_{\alpha H} &= (1-\eta_w)\tau_\alpha(P_D - P_H) \\ &= (1-\eta_w)\tau_\alpha P_D/(1+\phi) \end{aligned} \tag{28}$$

and

$$u_{fi} = \eta_w \tau_s (P_D - P_H) + \eta_{Hi} P_H \tau_s$$
$$= \eta_w \tau_s P_D/(1 + \phi) + \eta_{Hi} \tau_s P_D \phi/(1 + \phi) \quad (29)$$

where τ_α is the slowing down time of fast α particles, where τ_s is the slowing down time of fast fuel ions (see Appendix), and where, for convenience, we have defined $\phi \equiv P_H/P_\alpha$, so $P_D = (1 + \phi)P_\alpha$.

Let η_f be the fraction of the power absorbed first by fast ions that is then absorbed by the bulk ions. Let η_0 be the fraction of α particle power absorbed collisionally by ions in the absence of wave effects. Then one can write η, the ratio of power absorbed by the ions to power absorbed in the plasma, as

$$\eta \equiv p_i/P_D = [\eta_w \eta_f P_\alpha + (1 - \eta_w)\eta_0 P_\alpha$$
$$+ \eta_{Hi}\eta_f P_H]/P_D \quad (30)$$

Using Eqs (24), (27), (28) and (29), we find a reduction in the available pressure from

$$u = u_0/\{1 + [(1 - \eta_w)\tau_\alpha + \eta_w \tau_s + \phi\eta_{Hi}\tau_s]$$
$$\times (1 + \phi)\nu\zeta(1 - \rho_e\rho_i)/K\} \equiv u_0/G \quad (31)$$

Operation of what we call self-consistent burn is possible if $P_\alpha > P_D$; if $P_H = \phi = 0$, then this condition implies ignition. Write Eq. (26) in the form

$$cu_i^2 f_D f_T = (1 - \eta_w \chi_\alpha - \phi\eta_{Hi}\chi_\alpha)\frac{P_D}{1 + \phi} \quad (32)$$

and now use Eqs (22), (24), (31) and (32) to write the self-consistent burn condition as

$$\frac{c_0 u_0}{\nu} = \frac{G\zeta(1 - \rho_e\rho_i)K}{[(1 - \eta)\rho_e + \eta]^2 \rho_i^2 (1 + \phi)}$$
$$\times (1 - \eta_w \chi_\alpha - \phi\eta_{Hi}\chi_\alpha) \quad (33)$$

where the quantity G is defined from Eq. (31), and where we have defined $c_0 \equiv c(T_i)f_D f_T$, which will be fairly insensitive to T_i in the range of interest.

One can take a number of useful and simplifying limits. First of all, in a pure DT plasma, $\zeta = 1$. Let all the α particle power be diverted, η, $\eta_w \rightarrow 1$, assume $\eta_f \rightarrow 1$, and assume no external heating, $P_H = \phi = 0$, i.e. ignition. Assume also negligible fast ion pressure, i.e assume $\tau_s \rightarrow 0$. Then Eq. (33) simplifies to

$$c_0 u_0 \tau_i > (1 + \nu\tau_i)(1 - \chi_\alpha)(1 + \rho_e)(1 - \rho_e\rho_i) \quad (34)$$

One interesting question to ask is what τ_e (or ρ_e) minimizes the right hand side of Eq. (34), and so makes ignition easier to achieve. With ν fixed, the right hand side of Eq. (34) is quadratic in ρ_e, and the maximum, as a function of ρ_e, occurs at $\rho_e = (1 - \rho_i)/2\rho_i$, and the minima occur for $\rho_e \rightarrow \pm\infty$. With the restriction that $0 < \rho_e, \rho_i < 1$, it is clear that the minimum must occur at either of the end points of this region, namely, either at $\rho_e = 1$ or at $\rho_e = 0$. Moreover, for $\rho_i < 1/2$, namely poor ion heat confinement, the minimum occurs at $\rho_e = 0$ (poor electron heat confinement); and for $\rho_i > 1/2$, namely good ion heat confinement, the minimum occurs at $\rho_e = 1$ (good electron heat confinement). (Note, however, that actually $\nu\tau_e \rightarrow 0$ must occur for $\rho_e = 0$, but, with $\eta \rightarrow 1$ and $\tau_e \rightarrow 0$, then ν may be larger for fixed T_i.)

That ignition occurs most easily also with good electron heat confinement, when the ion heat confinement is very good, is intuitive. In the opposite limit, however, the result is not intuitive; here, when ion confinement is not very good, ignition is actually more easily achieved with poorer electron confinement, so long as all the α particle power is diverted! The reason for this somewhat odd looking result is that although ion heat confinement is poor, poor electron heat confinement assures that at least $u_i > u_e$, i.e. operation in the hot ion mode.

Consider three limits of Eq. (34): first, for $\rho_e \rightarrow 1$, the electron heat is very well confined and the ignition criterion reduces to

$$c_0 u_0 \tau_i > 2(1 - \chi_\alpha) \quad (35)$$

Second, suppose $\tau_i = \tau_e \equiv \tau$. Then, Eq. (34) reduces to

$$c_0 u_0 \tau > (1 - \chi_\alpha)\left(\frac{1 + 2\nu\tau}{1 + \nu\tau}\right)^2 \rightarrow 4(1 - \chi_\alpha) \quad (36)$$

where the limit taken is for $\nu\tau \gg 1$. In this limit, where the confinement of both ion heat and electron heat is very good, the advantage in the ignition margin of diverting α particle power is (see Eq. (40) below) left to just two terms, the factor $1 - \chi_\alpha$, since the only non-thermal feature is the hot ion population, and the lack of fast α particle pressure.

Consider now a third case of very poor electron heat confinement, ρ_e, $\tau_e \rightarrow 0$. The ignition criterion reduces to

$$c_0 u_0 \tau_i > (1 + \nu\tau_i)(1 - \chi_\alpha) \quad (37)$$

which, interestingly, gives a more relaxed ignition criterion than does the limit of good electron confinement (Eq. (35)), provided that $\nu\tau_i < 1$, or, equivalently, $\rho_i < 1/2$. This is interesting because poorer confinement of the ion heat makes ignition easier. Additionally, apart from the purely mathematical considerations that lead to the more relaxed ignition condition, it may in practice be hard to achieve the limit of very good electron heat confinement, whereas poor confinement can be arranged in a variety of ways.

These limiting cases can be compared with the case of undiverted α particle power $\eta_w = 0$. Take the limit $\eta = 0$ (true for small T_e), in which limit, again with the assumptions of a DT mixture ($\zeta = 1$) and ignition ($\phi = 0$), Eq. (33) reduces to

$$\frac{c_0 u_0}{\nu} > \frac{1 - \rho_e\rho_i}{\rho_e^2\rho_i^2}[\rho_e(1 + \rho_i) + \nu\tau_\alpha(1 - \rho_e\rho_i)] \qquad (38)$$

First note a qualitative difference: here, in contrast to the case of diverting all the α particle power, in the limit $\tau_e \to 0$ and when no α particle power is diverted, there is no ignition possible.

Consider now the other cases: in the limit of very good electron heat confinement, $\tau_e \to \infty$, then $\rho_e \to 1$, and we require

$$c_0 u_0 \tau_i > 2 + \frac{1 + \nu\tau_\alpha}{\nu\tau_i} \qquad (39)$$

which is more stringent than the condition for $\eta \to 1$, Eq. (35). For the third case, namely the limit $\tau_e = \tau_i \equiv \tau$, Eq. (38) reduces to the usual '$nT\tau$' ignition criteria in the form

$$c_0 u_0 \tau > \left(2 + \frac{1}{\nu\tau}\right)^2 \left(1 + \frac{\tau_\alpha}{\tau}\right) \to 4(1 + \tau_\alpha/\tau) \qquad (40)$$

where the limit is taken for $\nu\tau \gg 1$ and this 'normal' case may be compared with that obtained upon diverting power, under good confinement conditions, (see Eq. (36)).

Thus, whereas the diverting of the α particle power always produces some advantage, it is in the case of very poor electron heat confinement that a qualitative difference emerges, making for a very different mode of operation, the so-called 'hot ion mode', which is explored in the next three sections.

8. OPTIMIZING OPERATION WITH THE HOT ION MODE

A worthy goal in reactor design would be to find those operating conditions that maximize the fusion power density, yet keep the plasma ignited at constant plasma pressure. In this section we consider this optimization problem in the context of two limiting cases: normal operation with $T_e \simeq T_i$ and hot ion operation with $T_e \ll T_i$.

Thus, to consider normal operation first, in the limit $T_e \simeq T_i$, with $u_e \simeq u_i = u/2$, maximize

$$P_\alpha = \epsilon_\alpha \frac{n^2\langle\sigma v\rangle}{4} = \epsilon_\alpha \left(\frac{2}{3}\right)^2 \frac{u^2\langle\sigma v\rangle}{16T_i^2} \equiv \frac{u^2}{16} g(T_i) \qquad (41)$$

subject to the ignition constraint of Eq. (33). If the constraint were met for any ion temperature, then P_α would simply be maximized when $g(T_i)$ is maximized with respect to T_i. This function has a well known maximum at $T_i \simeq 15$ keV. Then, the density at optimal operation is found from $n = u/3T_i$.

The optimization in the presence of diverting α particle power is considerably more complicated, since large temperature differences maximize the fusion power density but are hard to maintain. To proceed, use Eq. (26) with $P_D = P_\alpha$ at ignition, to write

$$P_\alpha = \frac{u^2}{16}\frac{1}{1 - \eta_w\chi_\alpha} g(T_i) \frac{4}{(1 + r)^2} \qquad (42)$$

where $r = r(\rho_e, \rho_i, \eta)$ is the ratio of electron to ion temperature, which can be found from Eqs (22) and (23) as

$$r \equiv \frac{T_e}{T_i} = \frac{\rho_e}{\rho_i}\left(\frac{1 - \eta + \eta\rho_i}{(1 - \eta)\rho_e + \eta}\right) \qquad (43)$$

Note the critical role played by diverting α particle power: for $\eta \to 0$, $r \to 1/\rho_i > 1$, meaning that a hot ion mode is not obtainable; on the other hand, note that for $\eta \to 1$, $r \to \rho_e < 1$, meaning that a hot ion mode is not only obtainable, but can be made arbitrarily large, in principle, simply by spoiling the electron heat confinement.

The electron–ion energy density equilibration rate ν can be written in the form

$$\nu = au \frac{1}{T_i^{5/2}(1 + r)r^{3/2}} = \nu(r, T_i) \qquad (44)$$

where the constant a depends on the impurity content. Maximizing P_α is thus reduced to a maximization over the parameters $(T_i, \rho_i, \rho_e, \eta)$, constrained only by Eq. (33). Each of these parameters may be treated independently; for example, ρ_e is monotonic in τ_e, which is considered here as a free parameter. The optimization of P_α over η and ρ_i (or τ_i) is straightforward, since it is always preferred to divert more energy into the fast ions and to contain the ion heat longer. Thus, to maximize P_α, separately maximize η and ρ_i, i.e. take η and ρ_i at the maximum practically obtainable values. (Note, however, that in the limit $\eta \to 1$ the optimization is sensitive to η, but only weakly sensitive to ρ_i.) Then, the ignition condition can be used to write, for example, ρ_e and hence r in terms of T_i, so that P_α can be written as a function of T_i only. Note, however, that maximizing P_α with respect to T_i may now occur at temperatures other than 15 keV. The foregoing procedure demonstrates that the optimization problem in the limit of the hot ion mode is well posed and will yield a definite set of optimized parameters.

To illustrate this procedure in a limit of interest, consider the case ζ, η, $\eta_w \to 1$. For simplicity, also choose

$\tau_s = 0$. In this limit,

$$u \rightarrow u_0$$

$$u_e/u_i = r \rightarrow \rho_e$$

and

$$K \rightarrow \rho_i(1 + \rho_e) = \rho_i(1 + r)$$

The ignition condition (34) can then be written as

$$c_0 u_0/\nu = (c_0/a) T_i^{5/2}(1 + r) r^{3/2}$$
$$= (1 - \chi_\alpha)(1 + r)(1 - r\rho_i)/\rho_i \quad (45)$$

from which one has $r = r(T_i; \rho_i)$ as a monotonically decreasing function of T_i, with ρ_i entering, not particularly sensitively, as a parameter. Note, in Eq. (42), that for $T_i > 15$ keV, $g(T_i)$ is monotonically decreasing, whereas $(1 + r)^{-2}$ is monotonically increasing in T_i. Hence, as a function of T_i, there is a single maximum to P_α satisfying ignition, *although not necessarily at* $T_e = T_i = 15$ keV. For example, if the factor $1 - \chi_\alpha$ in Eq. (45) becomes small, it is clear that r can become small and then P_α will be maximized (at considerably greater power density than for normal operation) at $T_i = 15$ keV but $T_e \ll 15$ keV.

9. EXAMPLES OF IGNITION PARAMETERS

In this section, we present examples in which the fusion power density is significantly increased by diverting α particle power. To establish a comparison, let us consider a reference reactor similar to the ARIES-I design [23], which delivers about 2 GW fusion power with no α particle power diverted. If a substantial fraction of the α particle power can be diverted, it turns out that a reactor with about twice the fusion power density is possible.

Table I establishes the comparison, where we solved self-consistently for the plasma parameters in a self-sustained burn by choosing T_i and T_e, and then finding all the other parameters at fixed total pressure of the plasma. Here, the total plasma pressure and the electron and ion temperatures correspond roughly to the ARIES-I reactor, but without impurities ($Z_{eff} = 1.65$ in ARIES-I) and without any external power (there is 100 MW of current drive power in ARIES-I). The fraction of the total power deposited in ions, η, is not identically zero because α particles do slow down somewhat on ions. The self-consistent solution to the 0-D equations gives a P_f of 4.7 W$\cdot$cm^{-3} without blanket reactions. In order to arrive at electrons and ions at the same temperature, the electron confinement time must be half the ion confinement time. Note that this is consistent with α particle heating on the electrons twice that on the ions. The fast α particle pressure accounts for 18% of the total plasma pressure. Recall that, departing somewhat from convention, τ_e lumps both the effects of radiation by synchrotron motion or bremsstrahlung and the effects of heat conduction or convection.

TABLE I. OPERATING POINT BASED ON THE ARIES-I DESIGN

u_0 (10^{14} keV$\cdot$cm^{-3})	91.0	τ_e (s)	0.95
T_i (keV)	20.0	n_i (10^{14} cm^{-3})	1.24
T_e (keV)	20.0	n_e (10^{14} cm^{-3})	1.24
η	0.33	P_f (W$\cdot$cm^{-3})	4.67
η_w	0	$u_{\alpha H}/u_0$	0.18
ρ_i	0.66	u_i/u_0	0.41
ρ_e	0.48	u_e/u_0	0.41
ν (s^{-1})	0.99	τ_α (s)	0.28
τ_i (s)	1.95		

TABLE II. OPERATING POINT BASED ON THE ARIES-I DESIGN, BUT WITH $T_i = T_e = 15$ keV

u_0 (10^{14} keV$\cdot$cm^{-3})	91.0	τ_e (s)	0.71
T_i (keV)	15.0	n_i (10^{14} cm^{-3})	1.79
T_e (keV)	15.0	n_e (10^{14} cm^{-3})	1.79
η	0.26	P_f (W$\cdot$cm^{-3})	6.11
η_w	0	$u_{\alpha H}/u_0$	0.12
ρ_i	0.81	u_i/u_0	0.44
ρ_e	0.61	u_e/u_0	0.44
ν (s^{-1})	2.19	τ_α (s)	0.14
τ_i (s)	1.99		

The ARIES-I temperature was chosen to be 20 keV, somewhat higher than optimum for the fusion power density, to accommodate high current drive efficiency. A reactor design at the same pressure as ARIES-I that would optimize for fusion power density is shown in Table II. Here, $T_i = T_e = 15$ keV, which is close to the maximum reactivity per unit pressure of the plasma, and, in addition, the α particles slow down more rapidly on the colder denser plasma, so that the fusion power density, P_f, increases by 30%. Note that $\tau_e < \tau_i$ is necessary to give $T_i = T_e$.

If three quarters of the α particle power can be diverted to the fast ions, Table III shows that a very different regime of operation is possible, where T_i is nearly twice T_e, $\tau_e \ll \tau_i$, and P_f is 2.3 times higher in this case than in Table I. This increase is due to the increase in the ion pressure that is available in the hot ion mode of operation, to the reduction in the fast α particle pressure and to an increase in the reactivity arising from the creation of a

TABLE III. OPERATING POINT BASED ON THE ARIES-I DESIGN, EXCEPT FOR 75% OF THE α PARTICLE POWER DIVERTED TO FAST DEUTERIUM IONS AT 70 keV

u_0 (10^{14} keV·cm^{-3})	91.0	τ_e (s)	0.29
T_i (keV)	20.0	n_i (10^{14} cm^{-3})	1.75
T_e (keV)	11.9	n_e (10^{14} cm^{-3})	1.75
η	0.69	P_f (W·cm^{-3})	10.65
η_w	0.75	$u_{\alpha H}/u_0$	0.04
χ_α	0.18	u_{fi}/u_0	0.04
ρ_i	0.86	u_i/u_0	0.58
ρ_e	0.47	u_e/u_0	0.34
ν (s^{-1})	3.06	τ_α (s)	0.11
τ_i (s)	1.94		

TABLE IV. OPERATING POINT BASED ON THE ARIES-I DESIGN, EXCEPT FOR 75% OF THE α PARTICLE POWER DIVERTED TO FAST DEUTERIUM IONS AT 70 keV AND $\tau_i/\tau_e \simeq 2$

u_0 (10^{14} keV·cm^{-3})	91.0	τ_e (s)	0.54
T_i (keV)	15.0	n_i (10^{14} cm^{-3})	2.11
T_e (keV)	12.0	n_e (10^{14} cm^{-3})	2.11
η	0.70	P_f (W·cm^{-3})	9.73
η_w	0.75	$u_{\alpha H}/u_0$	0.03
χ_α	0.16	u_{fi}/u_0	0.03
ρ_i	0.77	u_i/u_0	0.52
ρ_e	0.66	u_e/u_0	0.42
ν (s^{-1})	3.62	τ_α (s)	0.09
τ_i (s)	0.93		

fast deuterium tail. Note that the reduction in fast α pressure arises from two effects: first, there is the instantaneous and direct diversion of three quarters of the α particle energy to ions, and, second, the quarter of the α particles that are not directly affected now slows down much faster because the electron temperature is halved. Note also that, interestingly, the electron confinement time is almost 3 times shorter than for the reference case and is only a sixth of the ion heat confinement time.

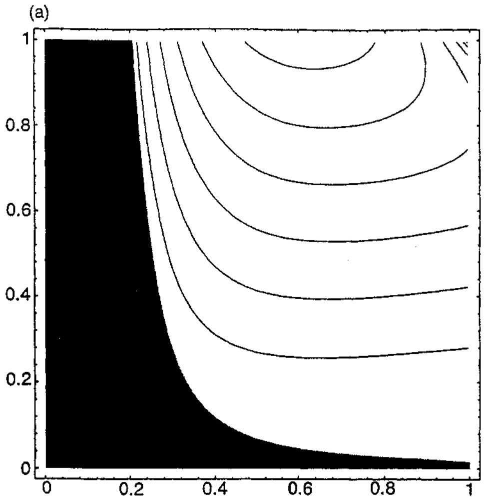
Table IV shows a somewhat different ignition regime also made possible by diverting the α particle power. Although the temperature disparity between electrons and ions is less than that shown in Table III, the fusion power density is about the same. While the relative pressure taken up by the ions is necessarily less than that in Table III, operation at 15 keV for the ions is a more efficient use of the available ion pressure. This scenario accommodates a lower ion heat confinement time; although τ_e is longer, τ_i is almost half the τ_i in Table III. In fact, in Table III, $\tau_i/\tau_e = 6.7$, while, in Table IV, $\tau_i/\tau_e = 1.7$.

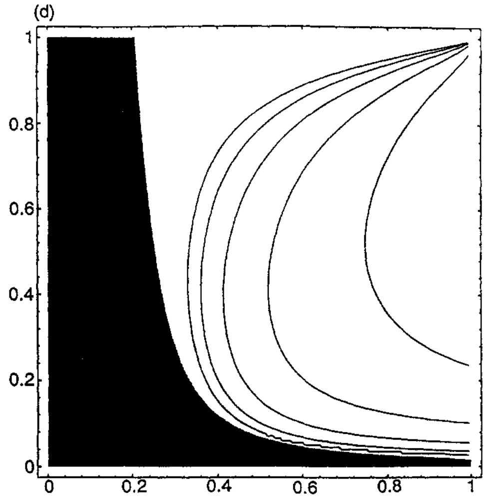
What these tables show is that fusion power densities in excess of twice the reference design for ARIES-I (Table I) are clearly attainable. Power densities about 1.7 the optimized reference design (Table II) are also attainable, but, in the optimized reference case, the optimization is just for power density, without any provision for current drive. Here, three quarters of the α particle power is to be diverted, namely, by immediately slowing down three quarters of the α particles. Somewhat greater power densities would be obtainable if the same three quarters of the α particle power came from all of the α particles slowing down immediately to one quarter of their birth energy, since the last quarter of the energy is to a large extent collisionally absorbed by ions in any event.

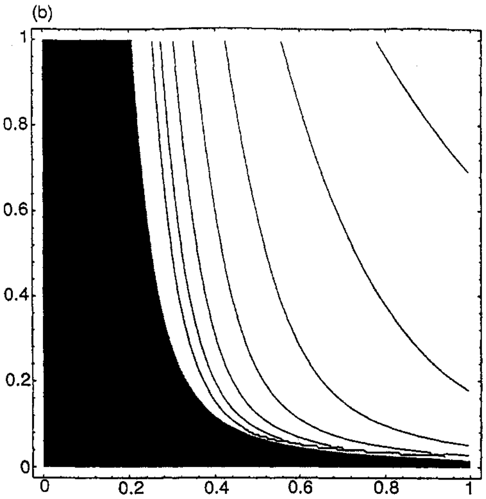
10. SELF-CONSISTENT BURN PLOTS

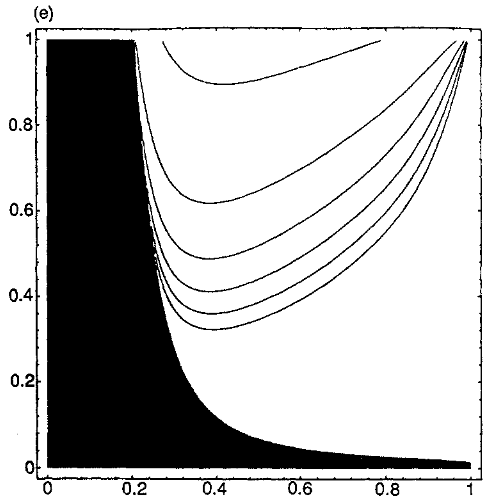
By plotting contours of various plasma parameters as a function of ρ_e and ρ_i, it can be depicted graphically how the operating point at ignition can be chosen to optimize the fusion power density. The ignition condition, Eq. (33), with the help of Eq. (44), can be used to find T_i in terms of ρ_e and ρ_i, if η_w, u_0 and E_d (which, from Eq. (48), gives χ_α) are specified. Then, given T_i, ρ_e and ρ_i, it is possible to solve for quantities such as P_f, T_e, τ_i and τ_e. Incidentally, it is by no means assured that a solution, i.e. a set of self-consistent burn parameters, exists for the complete range of ρ_e and ρ_i; in fact, it turns out that ignition is generally not possible for $T_i > 70$ keV; in the following figures this area is shaded.

Figure 1(a) shows contours of the fusion power density, P_f, for the case of ARIES-I-like parameters, with no diversion of α particle power. Note that the maximum fusion power density is in the range of 7 W·cm^{-3}, and it occurs for $\rho_i \rightarrow 1$, but for ρ_e values considerably different from 1. In Figs 1(b–e), additional parameters are plotted in terms of ρ_e and ρ_i. Fig. 1(b) shows the existence of ignition solutions in the full region $T_i < 70$ keV, although some solutions may correspond to extremely low fusion power density. The maximum fusion power density occurs for T_i between 10 and 15 keV, as one might expect. From Fig. 1(c) it is clear that this maximum is also characterized by T_e between 10 and 15 keV, which is not an unexpected result. However, from Figs 1(d) and (e), one sees that the maximum fusion power density requires $\tau_i \rightarrow \infty$, with $\tau_e \rightarrow 0$. This indicates that even in a conventional fusion









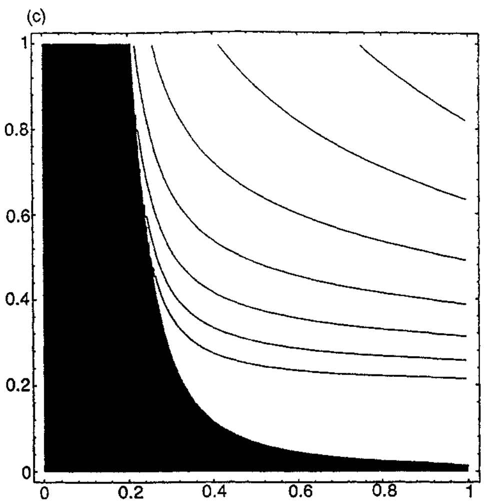
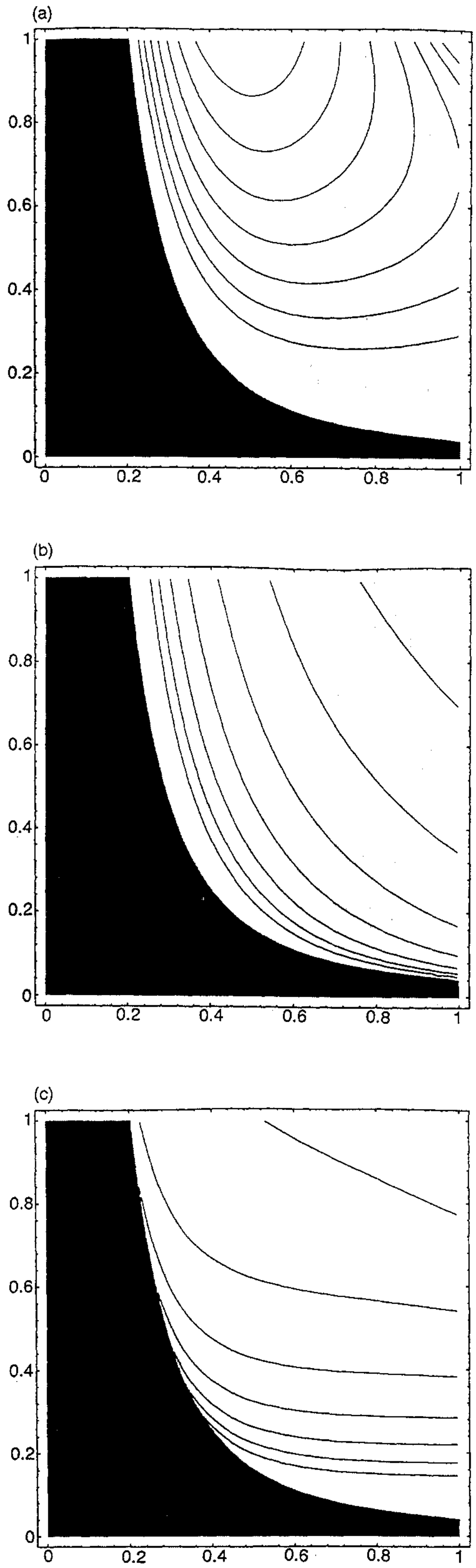


FIG. 1. (a) Contours of P_f versus ρ_e and ρ_i, with $u_0 = 91$. Contours from bottom to top are P_f $(W \cdot cm^{-3})$ = 2, 3, 4, 5, 6, 7. (b) Contours of T_i versus ρ_e and ρ_i, with $u_0 = 91$. Contours from left to right are T_i (keV) = 40, 35, 30, 25, 20, 15, 10. (c) Contours of T_e versus ρ_e and ρ_i, with $u_0 = 91$. Contours from left to right are T_e (keV) = 40, 35, 30, 25, 20, 15, 10. (d) Contours of τ_i versus ρ_e and ρ_i, with $u_0 = 91$. Contours from left to right are τ_i (s) = 3.0, 2.5, 2.0, 1.5, 1.0. (e) Contours of τ_e versus ρ_e and ρ_i, with $u_0 = 91$. Contours from bottom to top are τ_e (s) = 3.0, 2.5, 2.0, 1.5, 1.0, 0.5.



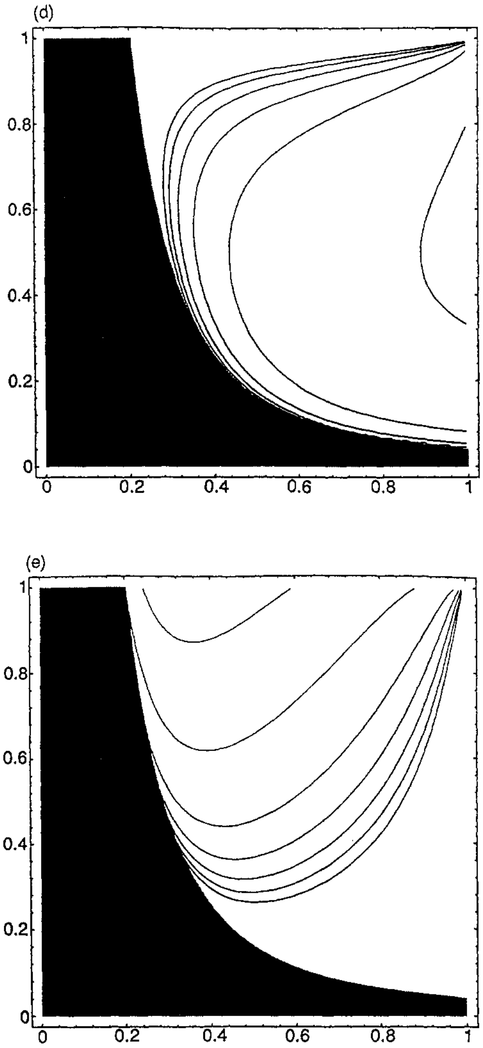


FIG. 2. (a) Contours of P_f *versus* ρ_e *and* ρ_i*, with* $u_0 = 91$ *and* $\eta_w = 0.75$*. Diverted* α *particle power is applied to deuterium ions at 70 keV. Contours from bottom to top are* P_f $(W\cdot cm^{-3}) = 5, 6, 7, 8, 9, 10, 11$*. (b) Contours of* T_i *versus* ρ_e *and* ρ_i*, with* $u_0 = 91$ *and* $\eta_w = 0.75$*. Diverted* α *particle power is applied to deuterium ions at 70 keV. Contours from left to right are* T_i *(keV) = 40, 35, 30, 25, 20, 15, 10. (c) Contours of* T_e *versus* ρ_e *and* ρ_i*, with* $u_0 = 91$ *and* $\eta_w = 0.75$*. Diverted* α *particle power is applied to deuterium ions at 70 keV. Contours from left to right are* T_e *(keV) = 40, 35, 30, 25, 20, 15, 10. (d) Contours of* τ_i *versus* ρ_e *and* ρ_i*, with* $u_0 = 91$ *and* $\eta_w = 0.75$*. Diverted* α *particle power is applied to deuterium ions at 70 keV. Contours from left to right are* τ_i *(s) = 3.0, 2.5, 2.0, 1.5, 1.0, 0.5. (e) Contours of* τ_e *versus* ρ_e *and* ρ_i*, with* $u_0 = 91$ *and* $\eta_w = 0.75$*. Diverted* α *particle power is applied to deuterium ions at 70 keV. Contours from bottom to top are* τ_e *(s) = 3.0, 2.5, 2.0, 1.5, 1.0, 0.5, 0.25.*

reactor, i.e. with no diverted α particle power, there is an advantage in small electron heat confinement times.

Note that the fusion power is maximized here for T_i about equal to or somewhat greater than T_e. These hot ion modes of operation, although not very pronounced, are available even though no α particle power is diverted. What enables these modes of operation is short electron heat confinement times, something that was foreseen above through analytic considerations. In these ignition modes, there is little sensitivity to the precise value of the ion heat confinement, since most of the ion heat is lost through the electron channel rather than by direct means. As the electron heat confinement times increase, at constant ion heat confinement times, the fusion power density decreases as more of the plasma pressure is taken up by the electrons. The sensitivity on the electron confinement time is quite dramatic; for example, consider operating points in the vicinity of ion heat confinement times of about 2 s and electron heat confinement times of about 1 s. Changing the electron heat confinement times moves the operating point roughly perpendicular to contours of constant fusion power density, whereas changing the ion heat confinement times results in less dramatic changes to the fusion power density. For example, $\tau_i = 2.5$ s and $\tau_e = 1$ s results in ignition at $P_f = 4\ \mathrm{W \cdot cm^{-3}}$, but the same τ_i with $\tau_e = 0.6$ s results in ignition at $P_f = 7\ \mathrm{W \cdot cm^{-3}}$. Of course, if the electron heat confinement time is too small, then there may be no ignition point at all. Hence, in optimizing reactor performance, one must design for adequate electron heat confinement. However, more than the adequate amount for ignition is actually deleterious to the reactor performance.

Note that self-sustained burn becomes impossible in the lower left corner of Figs 1(a–e), where the ion and electron temperatures are large, but densities are very small. As this corner is approached, ignition is possible but at low power density. Self-sustained burn is possible near the lower right corner, but at very low fusion power density. This corner corresponds to large T_e, large T_e/T_i, and (unrealistically) large τ_e. Here the electrons are so hot that they are essentially collisionally decoupled from the ions, and so do not lose heat to the ions. These cases illustrate the fact that ignition at very low power density is possible if heat containment times are good enough; indeed, even a glass of water can be considered to be ignited (absent evaporation) in the sense that the fusion power though uninterestingly small can exceed the power required to confine the fuel.

Somewhat more interesting is the upper right corner, where both T_i and T_e are modest and about equal, and which can be reached at modest values of τ_e and τ_i. (This is close to the present ignition scenario on the ITER tokamak, where confinement times are several seconds, and ion and electron temperatures are about equal and in the range of 10 keV.) Note, however, that this regime, while not at the very low power densities characteristic of operation in the lower corners of the ρ_e–ρ_i domain, still tends to be off the maximum fusion power density, even in the case here in which no power is diverted.

In Fig. 2(a) we show that much higher fusion power densities are possible when 75% of the α particle power is diverted to deuterium ions at 70 keV. The highest contour level shown is 11 $\mathrm{MW \cdot cm^{-3}}$, which can be compared with the contour level at 7 $\mathrm{MW \cdot cm^{-3}}$ in Fig. 1(a), when no power is diverted by waves. Immediately evident from Fig. 2(a), in comparison with Fig. 1(a), is that there is a shift of the fusion power density contours to lower ρ_e. Similarly, the maximum power density occurs at lower ρ_e, which corresponds, as we expect now, to lower electron heat containment. From Fig. 2(b), in comparison to Fig. 1(b), it is evident that the fusion power maximum occurs at higher ion temperature, between 15 and 20 keV, in the case of diverting α particle power. One can see from Fig. 2(c) why there is that shift to higher ion temperature; it is only the higher ion temperatures that allow higher electron temperatures, while still retaining the preponderance of the pressure in ions. The higher electron temperatures are necessary to achieve smaller collisional coupling between the ions and electrons.

If one were to overlay Fig. 2(d) and Fig. 2(e), it would become clear what pairs of τ_e and τ_i are consistent with ignition. Note in particular that at a constant τ_i there is a minimum τ_e necessary for there to be overlap anywhere. However, for τ_e greater than this minimum value, there is, in general, a monotonic decrease in fusion power density with increasing τ_e.

Note that Figs 1 and 2 result from solving simultaneously Eqs (19), (20), (21), (26), (27), (28), (29), (30) and (48). These equations are all scale invariant under the transformations $u_j \to \lambda u_j$, $P_j \to \lambda^2 P_j$, $\nu \to \lambda\nu$, $\tau_j \to \tau_j/\lambda$, $\chi_\alpha \to \chi_\alpha$ and $\eta_j \to \eta_j$, where, for example, u_j includes all of the energy densities u_i, u_e, u_0, u, u_{fi} and $u_{\alpha H}$. Thus, under these scalings, the contour shapes in Figs 1 and 2 remain invariant, indicating, for example, that the fusion maxima are obtained at the same T_e and T_i, even for very different total confined energy densities.

11. DIVERTING ENERGETIC α PARTICLE AND PROTON POWER IN D–^{3}He REACTORS

In advanced fuel reactors such as D–^{3}He, where the fusion power density tends to be unacceptably low, operation at higher fusion power density by diverting power

TABLE V. OPERATING POINT BASED ON THE ARIES-III DESIGN

u_0 (10^{14} keV·cm^{-3})	514.0	n_i (10^{14} cm^{-3})	2.11
T_i (keV)	55.0	n_e (10^{14} cm^{-3})	3.16
T_e (keV)	55.0	P_f (W·cm^{-3})	2.24
η	0.21	$u_{\alpha H}/u_0$	0.01
η_w	0	u_{pH}/u_0	0.14
ρ_i	0.89	u_i/u_0	0.34
ρ_e	0.67	u_e/u_0	0.51
ν (s^{-1})	0.86	τ_α (s)	0.24
τ_i (s)	6.04	τ_p (s)	0.65
τ_e (s)	2.34		

TABLE VI. OPERATING POINT BASED ON THE ARIES-III DESIGN, EXCEPT FOR 75% OF THE FUSION PRODUCT POWER DIVERTED TO 350 keV DEUTERIUM IONS

u_0 (10^{14} keV·cm^{-3})	514.0	n_i (10^{14} cm^{-3})	2.29
T_i (keV)	67.0	n_e (10^{14} cm^{-3})	3.44
T_e (keV)	42.5	P_f (W·cm^{-3})	5.10
η	0.67	$u_{\alpha H}/u_0$	0.01
η_w	0.75	u_{pH}/u_0	0.05
χ_α	0.32	u_{fi}/u_0	0.07
ρ_i	0.92	u_i/u_0	0.44
ρ_e	0.52	u_e/u_0	0.43
ν (s^{-1})	1.38	τ_α (s)	0.18
τ_i (s)	5.85	τ_p (s)	0.41
τ_e (s)	0.79		

TABLE VII. OPERATING POINT BASED ON THE ARIES-III DESIGN, EXCEPT FOR 75% OF THE FUSION PRODUCT POWER DIVERTED TO BULK FUEL IONS

u_0 (10^{14} keV·cm^{-3})	514.0	n_i (10^{14} cm^{-3})	2.46
T_i (keV)	67.0	n_e (10^{14} cm^{-3})	3.68
T_e (keV)	43.9	P_f (W·cm^{-3})	4.45
η	0.79	$u_{\alpha H}/u_0$	0.01
η_w	0.75	u_{pH}/u_0	0.04
χ_α	0	u_{fi}/u_0	0.00
ρ_i	0.93	u_i/u_0	0.48
ρ_e	0.59	u_e/u_0	0.47
ν (s^{-1})	1.40	τ_α (s)	0.17
τ_i (s)	5.88	τ_p (s)	0.40
τ_e (s)	1.02		

from energetic fusion charged by-products could be critical. Consider, for example, the ARIES-III reactor, operating at about $T_e = T_i = 55$ keV [24]. Here 70% of the fusion power output is promptly radiated by electrons. The salient parameters for this reference reactor are shown in Table V, where we have assumed no impurities and no external heating. The proton pressure is significantly more important than the α particle pressure, both because the protons are born with more energy and because the protons slow down more slowly.

Table VI shows the result of diverting 75% of the fast ion power to deuterium at 350 keV. This power is diverted both from the α particles and from the proton by-products of the D–^{3}He fusion. Through diverting this power, a temperature difference between the ions and the electrons can be sustained, so that more than double the fusion power density is then obtained. The increased power density is due in a large part to the decrease in the fast proton pressure, which is decreased both because of the power diversion and because of the increased collisionality. The increase in the collisionality arises from both the reduction in the electron temperature and the background density increase in ions and electrons that is now possible under constant pressure operation. In order to show the effect of diverting power to the bulk of the ion distribution rather than to the energetic tail, we explore in Table VII the result of diverting 75% of the charged fusion by-product power to the bulk ions rather than to the tail deuterium ions. This reduces the fusion power by about 10%.

In examining the implications for D–^{3}He reactors, bear in mind that there is considerable doubt at present about how the desired operating parameters might be achieved in these devices. For example, there are assumptions in the ARIES-III design that very high plasma pressures can be contained within the tokamak. There are also assumptions about the radiation of the very hot electrons, including how this radiation might be reflected back into the plasma. It may turn out that radiative transport of electron heat dominates the electron heat losses so that, effectively, very low τ_e cannot be avoided in the conventional designs. If that turns out to be the case, then diverting energetic α particle and proton power to ions will be even more important, because any power going into electrons will be effectively lost. If τ_e is very small, then unless power can be diverted there may be no ignition possible at all. By diverting power, not only is the fusion power density increased to sustain the self-consistent burn, but the tokamak may be operated at lower electron temperatures where radiation and radiative transport will be manageable.

12. SUMMARY AND CONCLUSIONS

In posing the question of the utility of extracting α particle power, and diverting this power to fast ions, it was useful to pose separately an incremental and a maximal problem. In the former, the diversion of a small amount of power was shown to increase the fusion power by several times the diverted power. In the latter form of the problem, power densities of about twice that achievable in normal operation were shown for a variety of cases. By means of contour plots in ρ_e–ρ_i space, it can be seen how one might vary plasma parameters to try to optimize the power density at constant pressure.

One effect that has not been incorporated in the considerations here is the depletion of cold α particle ash, i.e. the thermalized α particles. The depletion of the ash would arise because, in extracting the free energy of the α particles, waves tend to diffuse α particles to the tokamak periphery [13]. To the extent that the thermal ash is removed from the reactor by means of diverting the α particle power, the reactor power density would be improved even further than the factor of 2 reported in this paper. This is an effect that is particularly important for the case of advanced fuels, such as D–^{3}He, where there is both a tendency for greater accumulation of thermal ash and a greater urgency to make use of all the available plasma pressure.

This work concludes that a reactor operating at much higher power densities is possible, particularly as the electron energy confinement time decreases. Such a reactor is far more interesting economically than could be contemplated in the absence of diverting α particle power. It could be smaller, the magnetic field could be reduced, and, in principle, since there is less free energy in the energetic α particle distribution, the plasma is less prone to deleterious instabilities or disruptions that might have been destabilized by the energetic α particles. The additional power required to divert the α particle power could also secure the burn control. In principle, augmented by the diverted α particle power, even relatively inefficient methods of current drive through ion heating [25, 26] might provide an adequate toroidal plasma current.

The enhanced fusion power density is also available upon diverting energetic charged fusion by-products in D–^{3}He; in fact, the possibilities in diverting power are particularly important in fuel mixtures such as D–D, or D–^{3}He where serious economic considerations will depend upon the attainment of higher fusion power densities.

Thus, the calculations here suggest an eventual much more attractive reactor, which departs considerably from conventional designs. With substantially all the α particle power diverted to waves, the envisioned reactor is very much driven by RF waves; there may be several hundred megawatts of RF power flowing through the tokamak. Part of this power is injected (perhaps up to 100–200 MW), and the remainder arises from amplification by the α particles (perhaps up to 400–800 MW). The RF waves increase the fusion power density, accomplish current drive and tend to expel the α particles in the process of extracting energy, thus accomplishing ash removal.

Appendix

NON-THERMAL REACTIVITY AND PRESSURE

Here we compute χ_α, the fraction of power diverted to superthermal fuel ions that is recovered as α particle power through the enhanced tail fusion reactivity. Apart from the power cost in producing the non-thermal distribution of fuel ions, there is also a pressure cost, since these fast ions (and the electrons required to neutralize them — but that is a small term) take up a certain amount of the plasma pressure that is then not available to thermal ions and electrons.

Consider a non-thermal distribution of ions at a given energy in addition to a thermal distribution of ions and electrons. Assume that the number of particles in this non-thermal distribution is small compared with the number of ions. The α particle power produced by these ions is

$$P_\alpha = \epsilon_\alpha n_f n_T \int \sigma(\boldsymbol{v}_T - \boldsymbol{v}_f)|\boldsymbol{v}_T - \boldsymbol{v}_f| f_T(\boldsymbol{v}_T)\, d^3 v_T \tag{46}$$

where n_f is the number density of fast ions, $\boldsymbol{v}_f$ is the velocity of the fast ions and ϵ_α is 3.5 MeV. In this paper, the fast ions are chosen to be deuterium ions rather than tritium ions, for which a slightly larger χ_α should be available.

The amount of power necessary to maintain a non-thermal distribution of ions at a given energy E_d can be written as

$$P_{\text{input}} = n_f \nu_\epsilon(v) E_d \tag{47}$$

where ν_ϵ is the energy slowing down rate for the fast ions. Hence, χ_α can be written as

$$\chi_\alpha \equiv \frac{P_\alpha}{P_{\text{input}}} = \frac{\epsilon_\alpha n_T \int \sigma(\boldsymbol{v}_T - \boldsymbol{v}_f)|\boldsymbol{v}_T - \boldsymbol{v}_f| f_T(\boldsymbol{v}_T)\, d^3 v_T}{\nu_e(v) E_d} \tag{48}$$

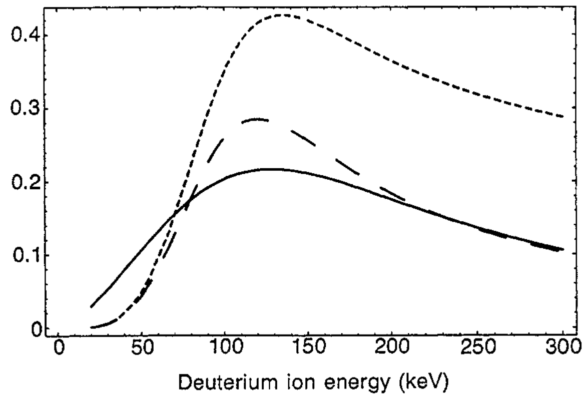


FIG. 3. χ_α versus deuterium ion energy (keV) for a 50:50 DT mixture, for $T_e \to \infty$, $T_T = 0$ keV (short dashed line); for $T_e = 10$ keV, $T_T = 0$ keV (long dashed line); and for $T_e = 10$ keV, $T_T = 20$ keV (solid line).

In Fig. 3 we show how χ_α depends on both the energy of the fast ions and the background ion and electron temperatures; note, however, that χ_α is independent of the background density. The values given in this figure are for a 50:50 DT mixture; for a tritium-rich mixture, these values could be about doubled.

The extra pressure taken up by the fast ion distribution is just $u_{fi} = n_f E_d$, or, in terms of the power diverted, $u_{fi} = P_{input}/\nu_e(v)$. Note that the fast ions represent added deuterium to the plasma, so that it is only the ratio now of thermal deuterium to thermal tritium that is 50:50. Together with the added fast deuterium, to maintain charge neutrality, there must also be additional electrons added. These additional electrons, maintained at the electron temperature, cost in plasma pressure. This added electron pressure has been neglected in our calculations, since it is small compared with the extra fast ion pressure which, in turn, is small compared with the overall pressure in the reactor.

It is worth pointing out that in practice it is not a δ function distribution of particles that is maintained, rather there is a slowing down distribution that is maintained. The calculation of χ_α can also be posed in an incremental way [15], in which the incremental effect of heating on the slowing down distribution is calculated. However, it turns out that, because this distribution arises from the constant heating of ions at a specified energy that then slow down, both calculations yield the same result.

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