Parametric Excitations of Fast Plasma Waves by Counterpropagating Laser Beams

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Short- and long-wavelength plasma waves can become strongly coupled in the presence of two counterpropagating laser pump pulses detuned by twice the cold-plasma frequency ω_p. What makes this four-wave interaction important is that the growth rate of the plasma waves occurs much faster than in the more obvious copropagating geometry.

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An important nonlinear process in plasma physics is the beat wave excitation of the electron plasma wave using high-frequency lasers, with applications including plasma heating and current drive [1,2], studying and controlling the ionosphere [3], and accelerating charged particles [4,5]. In this Letter we demonstrate that a fast plasma wave with phase velocity close to the speed of light can be generated by crossing two counterpropagating laser beams, which are detuned by $|\Delta\omega| \approx 2\omega_p$, where $\omega_p = (4\pi e^2 n_0/m)^{1/2}$ is the plasma frequency, $-e$ and m are the electron charge and mass, and n_0 is the plasma density. The counterpropagating geometry departs from the geometry employed in the traditional plasma beat wave accelerator approach to generating fast plasma waves for particle acceleration, which utilizes copropagating laser pulses [6] detuned by $\Delta\omega = \omega_p$.

That a plasma wave can be driven unstable by the $2\omega_p$ beat wave was originally proposed by Rosenbluth and Liu (RL) [7], who calculate a growth rate of a fast plasma wave $\gamma_{\rm RL} \approx \omega_p a_0 a_1/2$ (copropagating lasers). Note that this decay is high order, with growth rate going as pump amplitude squared. Thus, for pump waves of subrelativistic intensity, i.e., $a_0, a_1 \ll 1$, this decay instability is too slow to be of great practical interest.

What we propose here is that a counterpropagating pump geometry results in a growth rate also second order in the pump amplitude, but strongly enhanced by the factor $2\omega_0^2/\omega_p^2$. We consider the four-wave interaction, in which the four participating waves are the counterpropating lasers and two plasma waves, one (slow) with about twice the laser wave number, and one (fast) with the small wave number (ω_p/c). For the similar reason why Raman backscattering is much faster than Raman forward scattering, here the counterpropagating geometry enhances the growth rate, but now in the much different context of a four-wave interaction in which there is decay to a fast plasma wave capable of particle acceleration.

To proceed, consider then the interaction of two counterpropagating laser beams (labeled by 0 and 1), with the corresponding normalized vector potentials given by $\vec{a}_{0,1} = a_{0,1}[\vec{e}_\pm \exp(i\theta_{0,1}) + \text{c.c.}]$, where $\vec{e}_{+(-)} = (\vec{e}_x \pm i\vec{e}_y)/2$, $\theta_0 = k_0 z - \omega_0 t$, and $\theta_1 = k_1 z + \omega_1 t$. We assume that the duration of the forward-moving laser pulse is short (several plasma periods) and the duration of the backward-moving pulse is twice the length of the plasma. Tenuous plasma $\omega_p \ll \omega_0$ is assumed, ensuring that the lasers propagate almost as in vacuum: $v_{g0} \approx c$ and $|\vec{k}_0 - \vec{k}_1| \approx 2k_0$. The four-wave instability we consider involves a short-wavelength (slow) and a long-wavelength (fast) plasma wave. The wave number of the slow wave is $k_s = 2k_0 - k_p$. The wave number of the fast wave k_p is determined by the group velocity of the short pulse v_g: $k_p = \omega_p/v_g \approx \omega_p/c$.

Just as for the copropagating geometry, the time-averaged ponderomotive force $\vec{F} = -mc^2\nabla(\vec{a}_0 \cdot \vec{a}_1) \approx 2k_0 mc^2 \sin(2k_0 z - \Delta\omega)$ due to the pump lasers drives the plasma waves:

$$\ddot{\xi} + \omega_p^2 \xi = i k_0 c^2 a_0 a_1 e^{i[\Delta\omega t - 2k_0 z]} + \text{c.c.}, \qquad (1)$$

where $\xi \equiv z - z_0$ is the Lagrangian electron displacement. For the copropagating geometry, RL [7] used a single-wave ansatz for the plasma electron displacement $\xi = A(t)\sin[kz - \omega_p t + \phi(t)]$. The single-wave ansatz used by RL, however, is not sufficiently general for the case of counterpropagating lasers. Consider instead then the *two-wave* ansatz:

$$\xi = A_f \sin[k_p z_0 - \omega_p t + \phi_f] + A_s \sin[k_s z_0 - \omega_p t + \phi_s], \qquad (2)$$

where A_f (ϕ_f) and A_s (ϕ_s) are the amplitudes (phases) of the fast and slow plasma waves. For simplicity, we first consider the temporal evolution of the plasma wave amplitudes assuming that the driving laser fields are monochromatic waves. Short-pulse effects are numerically studied later in the paper. Substituting $z = z_0 + \xi$, where ξ is given by Eq. (2), into the right-hand side (rhs) of Eq. (1) yields

$$\frac{\partial^2 \xi}{\partial t^2} + \omega_p^2 \xi = i k_0 c^2 a_0 a_1 \sum_{k,l} (-1)^{k+l} J_k(2k_0 A_f) J_l(2k_0 A_s) \times e^{ik[k_p z_0 - \omega_p t + \phi_f]} e^{il[k_s z_0 - \omega_p t + \phi_s]} \times e^{i[\Delta\omega t - 2k_0 z_0]} + \text{c.c.}, \qquad (3)$$

where $J_{k,l}$ are Bessel functions, and $\Delta\omega = \omega_0 - \omega_1 \equiv 2\omega_p + \delta\omega$. In writing the rhs of Eq. (3), we used the

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identity $e^{i\alpha\sin\phi} = \sum_k J_k(\alpha)e^{ik\phi}$. A set of purely time-dependent equations can now be obtained by separating the z_0 dependent terms on both sides of Eq. (3). Thus, substituting Eq. (2) into the lhs of Eq. (3) and matching the corresponding harmonics of $k_p z_0$ and $k_s z_0$ on both sides of the equation, we can write for the $(k = 0, l = 1)$ and $(k = 1, l = 0)$ terms the following:

$$\frac{\partial\phi}{\partial t} = \delta\omega - \frac{\Omega_B^2}{4}\,\omega_p G(A_f, A_s)\sin\phi\,, \tag{4}$$

$$\frac{\partial(k_0A_f)}{\partial(\omega_p t)} = \frac{\Omega_B^2}{4}\,J_0(2k_0A_f)J_1(2k_0A_s)\cos\phi\,, \tag{5}$$

$$\frac{\partial(k_0A_s)}{\partial(\omega_p t)} = \frac{\Omega_B^2}{4}\,J_1(2k_0A_f)J_0(2k_0A_s)\cos\phi\,, \tag{6}$$

where $\phi = \phi_s + \phi_f + \pi/2 + \delta\omega t$; $\Omega_B^2 = 4a_0a_1\omega_0^2/\omega_p^2$ is the square of the electron bounce frequency in the optical lattice created by the interference of the counterpropagating lasers, and

$$G(A_f, A_s) = \frac{J_0(2k_0A_f)J_1(2k_0A_s)}{k_0A_f} + \frac{J_1(2k_0A_f)J_0(2k_0A_s)}{k_0A_s}.$$

Higher order Bessel terms are of the same order in the plasma wave amplitudes $2k_0A_{f,s}$ and are assumed small.

At resonance $(\delta\omega = 0)$, the relative phase ϕ locks at $\phi = 0$. For small amplitudes of the plasma waves, $2k_0A_{f,s} \ll 1$, Eqs. (5) and (6) can then be linearized, predicting the simultaneous exponential amplification of the fast and slow waves with the growth rate $\Omega_i = \omega_0^2 a_1 a_0/\omega_p$. In a nutshell, this is the principal result of this work: fast plasma waves capable of accelerating relativistic particles can be produced with a high temporal growth rate Ω_i. This growth rate is much higher than that predicted by RL for the copropagating lasers: $\Omega_i/\gamma_{\mathrm{RL}} \approx 2\omega_0^2/\omega_p^2$. The fast wave A_f grows so rapidly because it is parametrically coupled to the slow wave A_s. The coupling mechanism is the ponderomotive force due to the counterpropagating optical mixing of the laser beams.

An important practical issue is the sensitivity of the instability to the deviation from the exact two-plasmon resonance $\delta\omega$. For the finite frequency detuning from resonance $\delta\omega \neq 0$, there is an intensity threshold: phase locking takes place only if $\Omega_B^2/2 > \delta\omega/\omega_p$. Here, again, the counterpropagating geometry offers an advantage over the copropagating case: the intensity threshold is given by

$$\sqrt{I_0I_1}\,[\mathrm{W/cm^2}] = 1.4 \times 10^{-3}(\delta\omega/\omega_p)n_0[\mathrm{cm^{-3}}]. \tag{7}$$

For example, if the laser wavelengths are $\lambda_0 = 0.8$ and $\lambda_1 = 1.0\ \mu$m, and plasma density is $n_0 = 10^{19}\ \mathrm{cm^{-3}}$ (corresponding to $\omega_0 - \omega_p = 2.5\omega_p$), the geometric mean of the laser intensities should exceed the threshold value of $8.0 \times 10^{15}\ \mathrm{W/cm^2}$. Since this threshold is not too high, the instability is quite robust to the plasma inhomogeneity and detuning errors.

Equations (4)–(6) can be simplified by noting that, from the last two equations, $J_0(2k_0A_f)/J_0(2k_0A_s) = \mathrm{const}$. If both waves start out negligibly small, the constant is equal to unity, and one can assume that $A_f = A_s$ at all times. This assumption is meaningful only when the instability significantly amplifies both A_s and A_f, so that the small absolute difference of the initial amplitudes is unimportant. The equations for the phase and the normalized amplitude $u = 2k_0A_s = 2k_0A_f$ become

$$\dot\phi = (\delta\omega/\omega_p) - \Omega_B^2\,\frac{J_0(u)J_1(u)}{u}\sin\phi\,, \tag{8}$$

$$\dot u = \frac{\Omega_B^2}{2}\,J_0(u)J_1(u)\cos\phi\,, \tag{9}$$

where the dot indicates a derivative with respect to $\omega_p t$. The conserved invariant of Eqs. (8) and (9) is $\mathcal{H} = \Omega_B^2 u^2\sin\phi - 2(\delta\omega/\omega_p)F(u)$, where

$$F(u) = \int_0^u dx\,\frac{x^2}{J_0(x)J_1(x)}\,.$$

Note that $F(u)$ diverges for $u \to \mu_0$, where $\mu_0 = 2.405$ is the first zero of J_0.

For the excitation which starts out infinitesimally small $\mathcal{H} \approx 0$, and the $\sin\phi$ can be expressed in terms of the amplitude u. The expression for the $\cos\phi$ is then substituted into Eq. (9):

$$\dot u = \frac{J_0(u)J_1(u)}{2}\left[\Omega_B^4 - \frac{4F^2(u)\,(\delta\omega)^2}{u^4\omega_p^2}\right]^{1/2}. \tag{10}$$

Equation (10) gives the trajectory of the wave amplitude as a function of time. The plus (minus) sign corresponds to the increasing (decaying) portions of the trajectory. For a finite detuning $\delta\omega$, the "motion" of u is periodic between its initial starting value u_0 and the maximum value $u_{\max}$.

For a perfect laser detuning $\delta\omega = 0$, the mode amplitude has a stable attractor at $u = \mu_0$. Since $\mu_0 > 1$, Eq. (3) no longer holds because of the breaking of the slow wave [8]. For $0 < (\delta\omega/\omega_p) < \Omega_B^2/2$ the amplitude u oscillates periodically between its initially small value u_0 and $u_{\max} < \mu_0$ which is found by solving the equation $F(u_{\max})/u_{\max} = \Omega_B^2\omega_p/2\delta\omega$. This equation has no solutions for $(\delta\omega/\omega_p) < \Omega_B^2/2$, i.e., there is no instability. Defining $\delta\omega$ according to $\Omega_B^2/2 = (1 + \epsilon)(\delta\omega/\omega_p)$, we plotted in Fig. 1 the temporal evolution of u for a fixed $\Omega_B = 1$ and three different detunings corresponding to $\epsilon = 0.2$, 0.1, and 0.05.

Analytic progress can be made in the limit of $u < 1$, which is, in any case, the applicability limit of Eq. (3). Then $F(u) = u^2 + 3/16u^4 + \cdots$, and the maximum amplitude can be evaluated as $u_{\max} = 4\sqrt{\epsilon/3}$. The oscillation period is given by $(\delta\omega)T = 8\sqrt{2/3}\ln[2u_{\max}/u_0]/u_{\max}$, where $u_0 \ll u_{\max}$ is the initial mode amplitude. Figure 1 confirms that the smaller the peak amplitude of the wave, the longer the oscillation period.

The physics of the amplitude oscillation can be understood as follows. Initially, u is very small, and since the

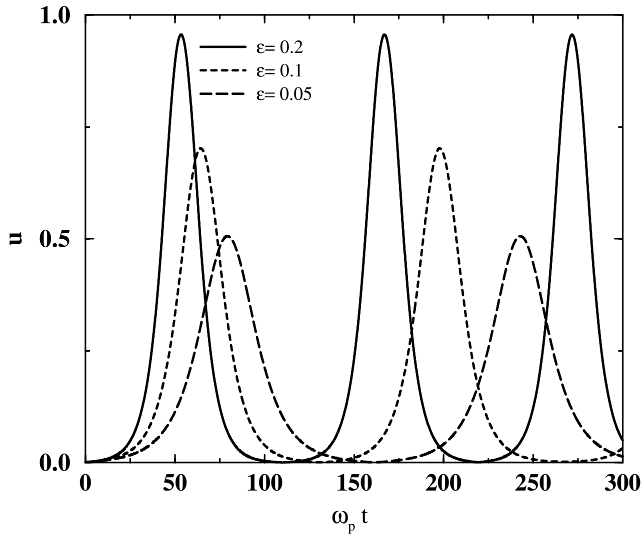

FIG. 1. Fast and slow wave amplitudes $u = 2k_0 A_f$ as functions of time for three detunings: $\Omega_B^2/2 = (1 + \epsilon)(\delta\omega/\omega_p)$. Initial excitation $u_0 = 10^{-3}$, $\Omega_B^2 = 1$.

ratio $F(u)/u^2$ is approximately a constant, the relative phase is locked at a constant $\phi = \sin^{-1} 2(\delta\omega/\omega_p)/\Omega_B^2$. As u undergoes an exponential growth, the phase "unlocks" and drifts towards $\phi = \pi/2$, at which time the amplitude peaks at $u = u_{\max}$ and starts dropping. After the amplitude drops to its initial value u_0, the phase locks again, and the process repeats.

While the fastest instability corresponds to $\Delta\omega = \omega_s + \omega_f$, it is instructive to understand qualitatively how two plasma oscillations ω_s and ω_f (fast and slow) can become strongly coupled by a beat wave which has a frequency $\Delta\omega \neq \omega_s + \omega_f$. It turns out that both ω_f and ω_s are shifted from ω_p in the presence of the lasers. The simplified description of the instability, expressed by Eqs. (4)–(6), predicts that the frequency shifts are proportional to $(a_0 a_1)^2$. Indeed, consider the small-intensity regime $(\delta\omega/\omega_p) \gg \Omega_B^2/2$. Then using $\phi \approx (\delta\omega)t$ and expanding Bessel functions to the lowest order in $A_{s,f}$, it can be shown that both ϕ_s and ϕ_f acquire a time-averaged drift $\dot{\phi}_{s,f} = -\delta\Omega_{s,f}$, where $\delta\Omega_s = \delta\Omega_f = \Omega_B^4/(32\delta\omega/\omega_p)$. Therefore, in the presence of the nonresonant beat wave the frequencies of both modes are shifted in the direction of $(\delta\omega)$. A rough estimate of the instability threshold can be obtained by requiring that $\delta\Omega_s + \delta\Omega_f = (\delta\omega/\omega_p)$. This results in $\Omega_B^2 = 4(\delta\omega/\omega_p)$, overestimating the earlier obtained expression for the intensity threshold by a factor of 2. As shown below, there is an additional mechanism of shifting the frequency of the slow plasma wave via backscattering the short laser pulse. This frequency shifting can significantly modify the threshold intensity.

Since multiple plasma and laser waves are involved, Eqs. (4)–(6) describe the instability only approximately. Some of the missed effects are (i) plasma perturbation driven at frequency $\Delta\omega$; (ii) modification of a_1 by the backscattering of a_0 off this driven density perturbation; (iii) the renormalization of the slow wave frequency due to its interaction with the short laser pulse. Therefore, we supplement the above calculation by a more rigorous two-scale particle simulation, which takes advantage of the scale separation between the short period of the slow plasma wave and a much longer period of the fast wave. We also assume for simplicity that the forward propagating laser pulse a_0 is much shorter than a_1.

The small-scale dynamics of the plasma electrons is characterized by their location (or phase) $\theta_j = \theta_0 + \theta_1 \approx 2k_0 z_j$ inside the optical lattice produced by the interference of the two lasers. Equations of motion for the j's electron in a reference frame moving with the short pulse are described in Refs. [9,10]:

$$\ddot{\theta}_j + \Omega_B^2 \sin(\theta_j - \Delta_0 \zeta) = -\sum_{l=1}^{\infty} \hat{n}_l e^{il\theta_j} - \tilde{e}_z + \text{c.c.}, \tag{11}$$

where a dot denotes a derivative with respect to $\zeta = \omega_p(t - z/c)$, $\hat{n}_l = i\langle e^{-il\theta_j}/l\rangle_{\lambda_0/2}$ is the lth harmonic of the small-scale electron plasma wave averaged over one lattice period, and $\Delta_0 = \Delta\omega_0/\omega_p$. The global electric field $\tilde{e}_z = 2\omega_0 e E_z/mc\omega_p^2$ is generated owing to the average momentum deposition from the lasers into the plasma [11]. In normalized units, equations for $\tilde{e}_z$ and a_1 can be written as

$$\frac{\partial \tilde{e}_z}{\partial \zeta} = \langle \dot{\theta}_j \rangle_{\lambda_0/2}, \qquad \frac{\partial a_1}{\partial \zeta} = -i\,\frac{\omega_p a_0^*}{4\omega_0} \langle e^{-i\theta_j}\rangle_{\lambda_0/2}. \tag{12}$$

Equations (11) and (12), supplemented by the initial conditions at $\zeta = -\infty$, are numerically solved using macroparticles. As an initial condition, we assume that at $\zeta = -\infty$ plasma is uniform ($\hat{n}_l = 0$ for all l) and stationary ($\dot{\theta}_j = 0$

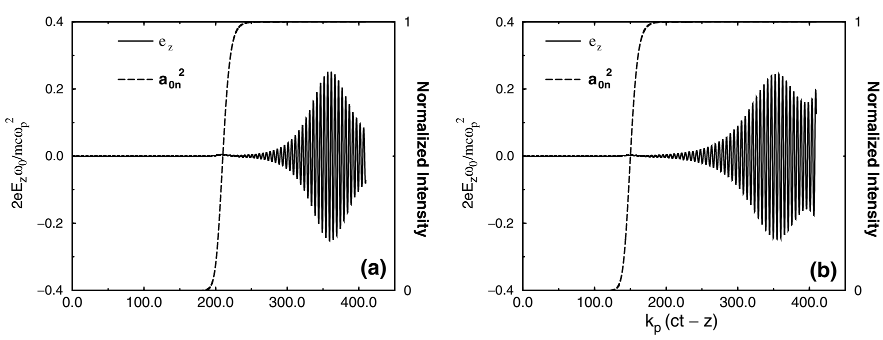

FIG. 2. Solid line: fast electric field $\tilde{e}_z$, dashed line: normalized intensity of short pulse a_{0n}^2. (a) $a_0 = a_1 = 0.06$, fixed a_1; (b) $a_0 = 0.19$, $a_1(\zeta = 0) = 0.015$, and a_1 is solved from Eq. (12).

for all j), and that a small initial fast plasma wave is present ($\tilde{e}_z = \tilde{e}_0$). The presence of a much larger plasma wave inside the short pulse (taken here in the form $a_0 = 0.5\bar{a}_0[\tanh(-\zeta/\tau_L) + 1]$) indicates an instability.

The fast electric field E_z obtained by integrating Eqs. (11) and (12) is shown in Fig. 2 for two sets of laser field amplitudes a_0 and a_1. Simulation parameters are $\omega_0/\omega_p = 10$, $\omega_0 - \omega_1 = 2.5\omega_p$, and $\tilde{e}_0 = 10^{-3}$. In Fig. 2(a) $a_0 = a_1 = 0.06$ were assumed fixed. Evolving a_1 according to the second equation in (12) did not result in any significant change of E_z. We also simulated the case of the fixed $a_0 = 0.19$ and $a_1 = 0.015$, which did not show any instability since in this case Ω_B^2 is smaller than in Fig. 2(a). However, when a_1 was self-consistently evolved, a large electric field was excited, as shown in Fig. 2(b). This result is a manifestation of the physics which was not included in the above two-wave analysis which predicted that the threshold for the instability is determined by the frequency detuning $\delta\omega$ and $\Omega_B^2 = 4a_0a_1\omega_0^2/\omega_p^2$, which depends only on the *product* of the laser amplitudes, not on the individual amplitudes.

As was explained earlier, the instability threshold arises because the finite Ω_B^2 is needed to shift the frequencies of the fast and slow plasma waves to compensate for the frequency detuning $\delta\omega$. However, there may be other mechanisms of frequency shifting unaccounted for by the two-wave treatment. In particular, it follows from Eq. (12) that a slow wave with amplitude $\hat{n}_1 \sim e^{-i\zeta}$ excites a backward wave $\delta a_1 = \omega_p a_0^* \hat{n}/4\omega_0(\Delta_0 - 1)$, which then forms a beat wave with a_0 and acts back on the plasma electrons. Substituting δa_1 into Eq. (11), we obtain an additional frequency shift of the slow plasma wave $\delta\Omega_s^+ = \omega_p^2|a_0|^2/4\omega_0(\Delta\omega - \omega_p)$. For the simulation parameters of Fig. 2(b), this additional frequency shift, *independent* of a_1, effectively reduces the $\delta\omega = 0.5\omega_p$ frequency mismatch. Hence, Ω_B^2 required to bridge the remaining gap is reduced as well. For the simulation parameters of Fig. 2(a) this reduction was negligible because of the smallness of a_0^2.

The relatively modest intensity threshold, given by Eq. (7), can be further lowered by employing a chirped laser pulse. Frequency chirp $\delta\omega(\zeta)$ also provides the benefit of suppressing the Raman backscattering of the more intense short pulse which can evolve from noise [12]. In Fig. 3 we plotted the amplitudes of the fast and slow plasma waves, $\tilde{e}_z$ and $\langle\cos\theta_j\rangle$, for a linearly chirped Gaussian pulse. Assuming that $\lambda_1 = 1\ \mu$m, the central frequency of the laser $\omega_0 = \omega_1 + 2.35\omega_p$ corresponds to $\lambda_0 = 810$ nm, and the plasma frequency $\omega_p/\omega_1 = 0.1$ corresponds to $n_0 = 10^{19}$ cm^{-3}, the pulse profile is as follows: $a_0 = 0.15\exp[-\zeta^2/2\tau_L^2]$ with $\tau_L = 25$ (160 fs FWHM) and $d\delta\omega/d\zeta = -9.5 \times 10^{-3}\omega_p$ (3% bandwidth). The initial fast plasma wave $\tilde{e}_0 = 10^{-3}$ and $a_1 = 0.0165$ have been assumed. In this example an accelerating plasma field of up to 9 GeV/m is generated.

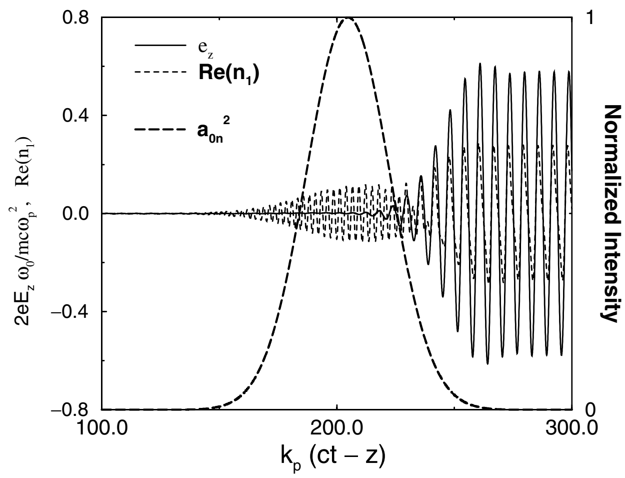

FIG. 3. Solid line: fast electric field $\tilde{e}_z$, long-dashed line: normalized intensity of short pulse a_{0n}^2, dashed line: density bunching of the slow plasma wave $\mathrm{Re}(\hat{n}_1) = \langle\cos\theta_j\rangle$. The rapidly varying part of $\hat{n}_1$ is the driven plasma response inside the laser pulse.

In conclusion, we showed that large-amplitude fast plasma waves might be very effectively excited by two counterpropagating laser pulses detuned by approximately two plasma frequencies. In this arrangement, a slow plasma wave is incidently excited, which is very effective in coupling the laser energy to the very useful for particle acceleration fast plasma waves.

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