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Inverse Faraday effect in a relativistic laser channel

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Abstract

Interaction between energetic electrons and a circularly polarized laser pulse in a relativistic plasma channel is studied. Laser radiation can be resonantly absorbed by electrons executing betatron oscillations in the channel and absorbing angular momentum from the laser. The absorbed angular momentum manifests itself as a strong axial magnetic field (inverse Faraday effect). The magnitude of this magnetic field is calculated and related to the amount of the absorbed energy.

One of the most important processes that accompany laser–matter interaction is the magnetic field generation. Magnetic fields could have a significant effect on the overall nonlinear plasma dynamics. Particularly, extremely high (megagauss) magnetic fields play an essential role in propagation of laser pulses, laser beam self-focusing, and penetration of laser radiation into overdense plasma. Recently, ultrahigh self-generated magnetic fields have been revealed in experiments (Najmudin *et al.*, 2000) and in numerical simulation (Sheng *et al.*, 1998).

Among the various mechanisms (Stamper, 1991) which are responsible for the magnetic field generation, we will consider the inverse Faraday effect (Steiger & Woods, 1972; Gorbunov *et al.*, 1996) resulting from the electron motion in the circularly polarized electromagnetic wave. During interaction with a circularly polarized laser pulse, plasma electrons absorb not only the laser energy but also the amount of the total angular momentum of the laser pulse. This angular momentum transfer leads to the electron rotation and the generation of the axial magnetic field by the azimuthal electron current.

Here we will consider another mechanism for laser–plasma angular momentum exchange. Both experiments (Key *et al.*, 1998) and Particle-In-Cell (PIC) simulations (Pukhov & Meyer-ter-Vehn, 1997) demonstrate relativistically strong laser plasma channels or filaments in a near-critical plasma. A circularly polarized Gaussian laser pulse propagates along the z axis with vector potential

$$\mathbf{A}(\mathbf{r}, t) = A_0 \exp(-r^2/R^2 - \xi_2^2/T^2)[\mathbf{e}_x \sin(\xi_1) + \mathbf{e}_y \cos(\xi_1)], \quad (1)$$

where $\xi_1 = \omega t - \omega z/v_{ph}$, $\xi_2 = z/v_{gr} - t$, v_{ph} and v_{gr} are the phase and group velocity of the pulse, respectively, $cT \gg R$. For simplicity, we will consider $v_{ph} \simeq v_{gr} \simeq c$. General case $v_{ph} \neq c$ and $v_{gr} \neq c$ is considered by Kostyukov *et al.* (2001). The ponderomotive force

$$F_{pond} = -\frac{r}{R^2} \frac{A_0^2 \exp(-2r^2/R^2 - 2\xi_2^2/T^2)}{4\gamma mc^2} \quad (2)$$

pushes out electrons from the high intensity region. Here e and m are the charge and the rest mass of electron, c is the speed of light, $\gamma = \sqrt{1 - v^2/c^2}$ is the relativistic gamma factor of the electron. The ion channel along the z axis is formed with the electron expelling. It is seen From Eq. (2) that the ponderomotive force is essentially reduced for hot electrons with $\gamma \gg 1$. They may not be pushed out from the ion channel and can oscillate across the z axis ("betatron" oscillation). Laser–electron energy exchange occurs at the resonance between electron betatron oscillation and laser field $\Omega/\sqrt{\gamma} = \omega - kv_z$, where Ω is the betatron frequency (Pukhov, 1999). It was recently observed in simulations (Pukhov, 1999; Tsakiris *et al.*, 2000) that electrons effectively absorb laser energy at this resonance. In the case of a circularly polarized laser pulse, the electrons can resonantly absorb the significant amount of the angular momentum too. The main objective of the paper is to calculate the generated magnetic field.

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The Hamiltonian of the electron motion in the ion channel and in the laser field is

$$H = c\sqrt{mc^2 + (\mathbf{p}_\perp + e\mathbf{A}/\mathbf{c})^2 + p_z^2} + \frac{m\Omega^2(x^2+y^2)}{2}, \qquad (3)$$

where $\Omega = \omega_{pi}$ is the effective ion frequency in the channel. For simplicity, we neglect the self-generated magnetic field of hot electrons. From here on we will use the unities ω^{-1} for the time, c/ω for the length, the momentum and vector potential are normalized by mc and mc^2/e, respectively. Assuming that $p_z^2 \gg 1 + (\mathbf{p}_\perp + \mathbf{A})^2$ and using canonical transformation we can derive the Hamiltonian for the electron near "betatron resonance"

$$H = p_z + \frac{M^2}{2p_z} + \frac{\epsilon I}{\sqrt{p_z}} + A_0 \exp\left(-\frac{\xi_2^2}{T^2}\right)\sqrt{\frac{\epsilon(I+L)}{2p_z^{3/2}}} \times \sin[\xi_1 - \theta_L - \theta], \qquad (4)$$

where $\epsilon = \Omega/\omega$, $M^2 = 1 + A^2$ is the electromagnetic mass of an electron in the circularly polarized electromagnetic wave, $\epsilon I/\sqrt{p_z} = p_\perp^2/(2p_z) + \epsilon^2(x^2+y^2)/2$ is the transversal electron energy in ion channel, L is the angular momentum of the electron, θ and θ_L are the angle variables canonically conjugated to I and L, respectively.

Let us now consider in more detail the transversal electron dynamics in the ion channel at the absence of the laser pulse. It follows from Eq. (4) that the Hamiltonian of the transversal motion is

$$H_\perp = \frac{p_\perp^2}{2p_z} + \frac{\epsilon^2 r^2}{2} = \frac{\epsilon I}{\sqrt{\gamma}}. \qquad (5)$$

Integrating the Hamiltonian equations we obtain

$$r^2 = \frac{I}{\epsilon\sqrt{\gamma}} + \frac{\sqrt{I^2 - L^2}}{\sqrt{\gamma}\epsilon}\sin(2\theta_L), \qquad \theta_L = \frac{\epsilon}{\sqrt{\gamma}}\, t. \qquad (6)$$

Equation (6) describes the all electron trajectories defined by the constant of motion I and L ($-I < L < I$). If angular momentum, L, is equal to zero, then the electron motion is just one-dimensional oscillations. If L is equal to I or $-I$, then the electron performs circular motion with radius

$$r_0 = \sqrt{I/\sqrt{\gamma\epsilon^2}}.$$

In the general case (an arbitrary value of L) the electron trajectory is confined between the maximal radius,

$$r_{\max}(I,L) = \sqrt{(I + \sqrt{I^2 - L^2})/\sqrt{\gamma\epsilon^2}},$$

and minimal radius,

$$r_{\min}(I,L) = \sqrt{(I - \sqrt{I^2 - L^2})/\sqrt{\gamma\epsilon^2}}.$$

We assume that all electrons have the same value of L_0 and I_0 and evenly distributed over the angles φ and θ and along the z direction. Then electron distribution function is

$$F(I,L,r) = \frac{N}{2\pi}\int_0^{2\pi}\int_0^{2\pi} d\varphi\, d\theta\, \frac{1}{r}\,\delta(I - I_0)\delta(L - L_0) \times \delta[r - r(I,L,\theta)]\delta[\varphi - \varphi(I,L,\theta)], \qquad (7)$$

where N is the linear density of the electrons along the channel.

After integrating over the angles, we obtain the expression for the electron distribution function:

$$F(I,L,r) = \frac{2N}{\pi}\frac{\delta(I - I_0)\delta(L - L_0)}{\sqrt{[r_{\max}^2(I,L) - r^2][r^2 - r_{\min}^2(I,L)]}}, \qquad (8)$$

where $r_{\min}(I,L) < r < r_{\max}(I,L)$. If electrons are evenly distributed over angular momentum $-I_0 < L_0 < I_0$, then integrating the electron distribution function (8) over L, L_0, and I, we obtain the electron density in the channel

$$n(r,\varphi) = \begin{cases} \dfrac{N(\epsilon\sqrt{\gamma})}{2\pi I_0}, & r^2 < \dfrac{2I_0}{\epsilon\sqrt{\gamma}}, \\ 0, & r^2 > \dfrac{2I_0}{\epsilon\sqrt{\gamma}}. \end{cases} \qquad (9)$$

Using Eq. (8) the expression for φ component of the electron current density in the channel can be derived:

$$j_\varphi = \frac{1}{\pi^2}\, eN\, \frac{L}{r\gamma}\, \frac{\delta(I - I_0)\delta(L - L_0)}{\sqrt{[r_{\max}^2(I,L) - r^2][r^2 - r_{\min}^2(I,L)]}}. \qquad (10)$$

Solving Maxwell's equation $\nabla \times \mathbf{B} = 4\pi\mathbf{j}$, the axial component of the static magnetic field can be found

$$B_z(r) = \begin{cases} \mathrm{sign}(L)\,\dfrac{2eN\epsilon}{\sqrt{\gamma}}\,\delta(I - I_0)\delta(L - L_0), & 0 < r < r_{\min} \\ \mathrm{sign}(L)\,\dfrac{eN\epsilon}{\sqrt{\gamma}}\,\delta(I - I_0)\delta(L - L_0) \times \left(1 - \dfrac{2}{\pi}\arcsin\dfrac{Ir^2\epsilon\sqrt{\gamma} - L^2}{\epsilon\sqrt{\gamma}r^2\sqrt{I^2 - L^2}}\right), & r_{\min} \le r \le r_{\max} \\ 0, & r > r_{max}. \end{cases} \qquad (11)$$

The magnetic flux generated by the hot electrons along the channel is

$$\Phi = \iint \mathbf{B}\cdot d\mathbf{s} = \frac{2\pi eNL}{\gamma}\,\delta(I - I_0)\delta(L - L_0) \qquad (12)$$

and the mean axial magnetic field in the ion channel is

$$\langle B\rangle = \frac{\Phi}{\pi r_{\max}^2} = \frac{2eN\epsilon}{\sqrt{\gamma}}\,\frac{L}{I+\sqrt{I^2-L^2}}\,\delta(I-I_0)\delta(L-L_0). \quad (13)$$

It follows from Hamiltonian (4) that the interaction can be described by the one action variable $\gamma \simeq p_z$ and by the conjugated angle variable $\Psi = \xi_1 - \theta_L - \theta$. Hamiltonian (4) takes a form in new variables

$$H = H_0 + H\sim, \qquad H_0 = \frac{M^2}{2\gamma} + \frac{\epsilon(\gamma+\gamma_1)}{\sqrt{\gamma}},$$

$$H\sim = A_0 \exp\left(-\frac{t^2}{d^2}\right)\sqrt{\frac{\epsilon(\gamma+\gamma_2)}{\gamma^{3/2}}}\sin\Psi, \quad (14)$$

where $d = 2\lambda(M^2\gamma^{-2} + 2\epsilon\gamma^{-1/2} + 2\epsilon\gamma_1\gamma^{-3/2})^{-1}$ is the laser pulse duration in the coordinate system of the electron, $\lambda = T\omega$ where γ_1 and γ_2 are the constants of the motion.

The resonance condition is

$$\dot{\Psi} = \frac{\partial H_0}{\partial\gamma} = -\frac{M^2}{2\gamma^2} + \frac{\epsilon}{2\sqrt{\gamma}} + \frac{\epsilon\gamma_1}{2\gamma^{3/2}} = 0. \quad (15)$$

From virial theorem we can write

$$\epsilon(\gamma_0+\gamma_1)\sqrt{\gamma_0} = \epsilon I_0\sqrt{\gamma_0} = p_{\perp,0}^2/2 + \epsilon^2 r_0^2/2 \approx p_{\perp,0}^{\,2}, \quad (16)$$

where $p_{\perp,0}$ is the transverse momentum at the initial moment of time and r_0 is the initial radius of the electron location. At the resonance, $\gamma_{res} = \gamma_0$ and $p_{\perp,res} = p_{\perp,0}$ and the resonance energy is

$$\gamma_{res} \simeq \left(\frac{M^2 + p_{\perp,0}^2}{2\epsilon}\right)^{2/3}. \quad (17)$$

In the limit $M = 1$ and $p_{\perp,0} = 0$, $v_{ph} = c$, the expression for resonance electron energy coincides with one calculated by Pukhov (1999).

Let us now calculate the laser pulse energy absorbed by hot electrons. In this case, we can consider the last term in the Hamiltonian (14) as a perturbation and use the perturbation theory. One of the simplest ways to derive averaged change in γ to the second order in A_0 is to use Madey's theorem (Madey, 1979). According to the theorem, the second-order change in γ is

$$\Delta\gamma = \langle\gamma-\gamma_0\rangle|_{\Psi_0} = \frac{1}{2}\frac{\partial}{\partial\gamma_0}\langle\gamma_{\sim}^2(\gamma_0)\rangle,$$

$$\gamma_{\sim} = \int \frac{\partial H_{\sim}[\gamma_0, \Psi^{(0)}(\gamma_0,\tau,\Psi_0), t]}{\partial\Psi}\,dt. \quad (18)$$

The obtained expression is similar to one for the Landay damping: $\Gamma_L \sim \int P_k[k(\partial f/\partial v)]$, where P_k is the Cerenkov emission power. In our case $\langle\gamma_{\sim}^2(\mathbf{I}_0)\rangle$ is the betatron emission power. Then the absorbed energy per electron can be calculated with the averaging over the electron distribution function at the initial moment of time, $F(\mathbf{I}_0)$,

$$Q = \int \Delta\gamma F(\mathbf{I}_0)\,d\mathbf{I}_0 = -\frac{1}{2}\int\langle\gamma_{\sim}^2(\mathbf{I}_0)\rangle\hat{G}F(\mathbf{I}_0)\,d\mathbf{I}_0,$$

$$\hat{G} \equiv \left(\frac{\partial}{\partial\gamma_0} + \frac{\partial}{\partial I_0} + \frac{\partial}{\partial L_0}\right). \quad (19)$$

To calculate the absorption energy we consider the cold electron beam with the distribution function $F(p_z, p_\perp, r) = \delta(p_z - \gamma_b)\delta[H_\perp(p_z, p_\perp, r) - W_\perp]$, where $H_\perp = \epsilon I/\sqrt{\gamma} \simeq p_\perp^2/\gamma$ is the effective transversal energy of electron, $W_\perp \gg M/\gamma$.

Introducing $X = \epsilon\sqrt{\gamma_b}/W_\perp$, the absorbed power is

$$Q \simeq \frac{\pi A_0^2}{2\gamma_b}\lambda^2 X^2(1-2X)(X-1)\exp[-\Phi(X,\lambda)],$$

$$\Phi(x,\lambda) = \lambda^2(1-2X)^2, \quad (20)$$

The physical meaning of $\Phi(X,\lambda)$ is the detuning between the phases of laser wave and electron. If $\Phi(X,\lambda) = 0$, the resonance between laser wave and electrons takes place. For $\lambda \gg 1$, the electrons absorb the maximum of laser energy at $X \simeq 1/2 + 1/(2\sqrt{2}/\lambda)$ and the electrons transfer the maximum of energy to the laser at $X \simeq 1/2 - 1/(2\sqrt{2}/\lambda)$. The maximum of the absorbed energy as a function of the normalized pulse duration $\lambda = \omega T \gg 1$ is

$$Q \simeq \frac{\pi A_0^2}{16\gamma_b}\frac{\lambda}{\sqrt{2}}\exp\left[-\frac{1}{2}\right]. \quad (21)$$

Relation (21) is similar to one describing the gain of free electron laser in small signal regime (Madey, 1979).

According to Eq. (13), the electrons with distribution function $F(\gamma, I, L)$ induce the magnetic field

$$B_{ind} = \int\langle B(\mathbf{I})\rangle F(\mathbf{I})\,d\mathbf{I} = \int\frac{2eN\epsilon L}{I+\sqrt{I^2-L^2}}\,\frac{F(\mathbf{I})}{\sqrt{\gamma}}\,d\mathbf{I}. \quad (22)$$

The electrons have symmetrical distribution over the angular momentum $L = L_0$ if $-I_0 \le L \le I_0$ and $L = 0$ if $L < -I_0$ or $L > I_0$ at the initial moment of the ion channel formation. Then there is no generated quasistatic magnetic field at this moment there. After interaction with the laser pulse, the distribution over γ, I, L is modified and the hot electrons can generate a magnetic field. In this case, the induced quasistatic field is

$$B_{ind} = \int\langle B(\mathbf{I})\rangle F(\mathbf{I})\,d\mathbf{I} = \int\langle B[\mathbf{I}(\mathbf{I}_0)]\rangle F(\mathbf{I}_0)\,d\mathbf{I}_0. \quad (23)$$

$B[\mathbf{I}(\mathbf{I}_0)]$ can be derived with Taylor's expansion using the fact that

$$B[\mathbf{I}(\mathbf{I}_0)] = B(\mathbf{I}_0) + \Delta\gamma \hat{G} B(\mathbf{I}_0) + \frac{\langle \gamma_{\sim}^2 \rangle}{2} \hat{G}^2 B(\mathbf{I}_0). \qquad (24)$$

Here we use the averaging over Ψ_0 and the relation $\Delta\gamma = \Delta I = \Delta L$, which follows from Hamiltonian (4). Using Madey's theorem (18), we can reduce Eq. (24) to a more convenient form

$$B_{ind}(\mathbf{I}_0) = B(\mathbf{I}_0) + \tfrac{1}{2}\hat{G}[\langle \gamma_{\sim}^2(\mathbf{I}_0)\rangle \hat{G} B(\mathbf{I}_0)]. \qquad (25)$$

Averaging with distribution function $F_0(\mathbf{I}_0)$ and integrating by part we obtain

$$B_{ind} = -\int \frac{\langle \gamma_{\sim}^2(\mathbf{I}_0)\rangle \hat{G} B(\mathbf{I}_0)}{2} \hat{G} F(\mathbf{I}_0)\, d\mathbf{I}_0. \qquad (26)$$

The difference from expression (19) for the absorbed energy is an additional factor $\hat{G}B(\mathbf{I}_0)$:

$$\hat{G}B(\mathbf{I}_0) = \frac{2eN}{I_0 + \sqrt{I_0^2 - L_0^2}} \frac{\epsilon}{\sqrt{\gamma_0}} \left(-\frac{L_0}{2\gamma_0} + \sqrt{\frac{I_0 - L_0}{I_0 + L_0}} \right). \qquad (27)$$

Integrating by part, we can rewrite Eq. (24) in the form

$$B_{ind} = \int \frac{F(\mathbf{I}_0)}{2} \hat{G}[\langle \gamma_{\sim}^2(\mathbf{I}_0)\rangle \hat{G} B(\mathbf{I}_0)]\, d\mathbf{I}_0. \qquad (28)$$

In the same way we calculated the absorbed energy, we find for cold electron beam

$$B_{ind} \simeq \left(1 - \frac{\pi}{4}\right)(2 - X) Q \left(\frac{2eN\epsilon^2}{W_\perp \gamma_b} \right). \qquad (29)$$

Introducing the density of the electron beam in the channel after the laser pulse propagation $n_b \simeq N/(\pi r_{\max}^2) \simeq N\epsilon\sqrt{\gamma}/(2\pi I_0) \simeq N\epsilon^2/(2\pi W_\perp)$, Eq. (29) can be rewritten in the form

$$\frac{\omega_{ce}}{\omega} \simeq 0.2(2 - X) f \epsilon^2 \left(\frac{Q}{\gamma_b mc^2} \right), \qquad (30)$$

where $\omega_{be}^2 \simeq f\,\Omega^2$ and $f = n_b/n_i$ is the effective neutralization factor in the channel. The maximum of the magnetic field is $\omega_{ce}/\omega \simeq 0.21 f\epsilon^2$. For experimental condition $\epsilon^2 \approx \frac{1}{4}$ (Gahn *et al.*, 1999), $f \simeq 0.5$ (Borghesi *et al.*, 1997), the intensity of the azimuthal magnetic field can be up to 5 MG.

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REFERENCES

Borghesi, M. *et al.* (1997). *Phys. Rev. Lett.* **78**, 879.
Gahn, C. *et al.* (1999). *Phys. Rev. Lett.* **83**, 4772.
Gorbunov, L.M. *et al.* (1996). *Phys. Rev. Lett.* **76**, 2495.
Key, M.H. *et al.* (1998). *Phys. Plasmas* **5**, 1966.
Kostyukov, I.Yu. *et al.* (2000). *Phys. Plasmas* (submitted).
Madey, J.M.J. (1979). *Nuovo Cimento B* **50**, 64.
Najmudin, Z. *et al.* (2000). *Bulletin of APS* **45**, 129.
Pukhov, A. (1999). *Phys. Plasmas* **6**, 2847.
Pukhov, A. & Meyer-ter-Vehn J. (1997). *Phys. Rev. Lett.* **79**, 2686.
Sheng, Z.M. *et al.* (1998). *Phys. Plasmas* **5**, 3764.
Stamper, J.A. (1991). *Laser Part. Beams* **9**, 841.
Steiger, A.D. & Woods, C.H. (1972). *Phys. Rev. A* **5**, 1467.
Tsakiris, G.D. *et al.* (2000). *Phys. Plasmas* **7**, 3017.