Electron Cross-Field Transport in a Miniaturized Cylindrical Hall Thruster

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Invited Paper

Abstract—Conventional annular Hall thrusters become inefficient when scaled to low power. Cylindrical Hall thrusters, which have lower surface-to-volume ratio, are more promising for scaling down. They presently exhibit performance comparable with conventional annular Hall thrusters. The present paper gives a review of the experimental and numerical investigations of electron cross-field transport in the 2.6-cm miniaturized cylindrical Hall thruster (100-W power level). We show that, in order to explain the discharge current observed for the typical operating conditions, the electron anomalous collision frequency \( \nu_B \) has to be on the order of the Bohm value, \( \nu_B \approx \omega_c / 16 \). The contribution of electron-wall collisions to cross-field transport is found to be insignificant. The optimal regimes of thruster operation at low background pressure (below \( 10^{-4} \) torr) in the vacuum tank appear to be different from those at higher pressure (\( \sim 10^{-3} \) torr).

Index Terms—Bohm diffusion, electron transport, Hall discharge, Langmuir probes, plasma propulsion, turbulence.

I. INTRODUCTION

The Hall thruster [1] is a well-studied electric propulsion device at intermediate to high power, but it appears to be promising also for relatively low-power propulsion on near-earth missions [2], such as orbit transfer and repositioning. In a conventional Hall thruster, the plasma discharge is sustained in the axial electric (\( \mathbf{E} \)) and radial magnetic (\( \mathbf{B} \)) fields applied in an annular channel. The magnetic field is large enough to lock the electrons in the azimuthal \( \mathbf{E} \times \mathbf{B} \) drift, but small enough to leave the ion trajectories almost unaffected. A large fraction of the discharge electrons is emitted by an external cathode. Electron cross-field diffusion provides the necessary current to sustain the discharge. The thrust is generated in reaction to the axial electrostatic acceleration of ions. Ions are accelerated in a quasi-neutral plasma, so that no space-charge limitation is imposed on the achievable current and thrust densities. Conventional Hall thrusters designed for operation in 600–1000 W power range have outer channel diameter about 10 cm, maximal value of the magnetic field about 100–200 G, and applied discharge voltage \( U_d = 300 \) V.

The thruster efficiency is defined as \( \eta = T^2 / 2 \mu P \), where \( T \) is the generated thrust, \( \mu \) is the supplied propellant flow rate, and \( P \) is the applied electric power. The efficiency of the state-of-the-art kilowatt and subkilowatt conventional Hall thrusters is about 50%–60% [1], [3]. The efficiency can be conveniently factorized as

\[
\eta \approx \frac{I_i M}{e \mu} \times \frac{I_i}{I_i + I_e} \times \alpha
\]

where \( M \) is a mass of a propellant gas atom, \( e \) is the electron charge, \( I_i \) and \( I_e \) are the electron and ion currents, respectively, and \( \alpha \) is the efficiency of ion acceleration. The first fraction in the right-hand side of (1), the so-called propellant utilization, is a measure of how effectively the supplied propellant gas is ionized in the discharge, whereas the second fraction, the so-called current utilization, determines how effectively the electron transport to the anode is suppressed by the applied magnetic field. With all other parameters held constant, the thruster efficiency decreases with increasing electron current. Understanding of the mechanisms of electron transport in the discharge is, therefore, essential for the development of higher efficiency thrusters.

The electrons in Hall thrusters exhibit anomalous cross-field transport. The electron conductivity across the magnetic field is larger than that predicted by the classical electron–atom collision rate [1], [4]. It is believed that two collisional processes contribute to the conductivity enhancement in Hall thrusters: 1) electron scattering in electric field fluctuations (anomalous or “Bohm” diffusion [4]), and 2) the electron-wall collisions (the near-wall conductivity [5], [6]). The electron-wall interaction plays also a very important role by shaping the electron distribution function (EDF) in the thruster channel. In Hall discharge simulations, in order to account for an enhanced electron cross-field transport, the two nonclassical conductivity mechanisms are usually incorporated in models in one or another parametric way. In fluid and hybrid fluid-particle models, some investigators impose the anomalous Bohm conductivity inside the channel [7]–[9], while others use only the near-wall conductivity [10] or a combination of both Bohm transport and wall collisions [8]–[16]. Full particle-in-cell (PIC) simulations [17], [18] reveal turbulence increasing the cross-field transport. Some theoretical studies [19], [20] suggest that due to the non-Maxwellian shape of the EDF in a Hall thruster, electron–wall collisions do not make a significant contribution.
to cross-field transport. In a 2-kW Hall thruster operated at low discharge voltage [21], in the channel region where the magnetic field was the strongest, anomalous fluctuation-enhanced diffusion was identified as the main mechanism of electron cross-field transport. It is important to emphasize here that most of investigations, which addressed the question of the electron conductivity, have been performed for kilowatt and subkilowatt thrusters, where the maximal magnetic field strength in the channel is about 100–200 G.

Scaling to low-power Hall thrusters requires a thruster channel size to be decreased while the magnetic field must be increased inversely to the scaling factor [1]. Thus, in general, the rate of electron cross-field transport required to sustain the discharge in a low-power thruster may be different from that in kilowatt thrusters. In other types of low-temperature magnetized laboratory plasmas, variation of the electron cross-field diffusion rate with applied magnetic field \( B \) occurs indeed. For example, in [22], cross-field diffusion coefficient \( D_{\perp} \) was observed to approach the Bohm value when \( B \) was greater than 2–3 kG, while in \( B < 1 \) kG case \( D_{\perp} \) was much smaller than the Bohm value.

Increasing the magnetic field while the thruster channel sizes are being reduced is technically challenging because of magnetic saturation in the miniaturized inner parts of the magnetic core. A linear scaling down of the magnetic circuit leaves almost no room for magnetic poles or for heat shields, making difficult the achievement of the optimal magnetic fields. Nonoptimal magnetic fields result in enhanced electron transport, power and ion losses, heating and erosion of the thruster parts, particularly the critical inner parts of the coaxial channel and magnetic circuit.

Currently existing low-power Hall thruster laboratory prototypes with channel diameters 2–4 cm operate at 100–300 W power levels with efficiencies in the range of 10%–40% [2]. However, further scaling of the conventional geometry Hall thruster down to sub-centimeter size results in even lower efficiencies, 6% at power level of about 100 W [23]. The low efficiency might arise from a large axial electron current, enhanced by magnetic field degradation due to excessive heating of the thruster magnets, or from a low degree of propellant ionization. Thus, miniaturizing the conventional annular Hall thruster does not appear to be straightforward.

A cylindrical Hall thruster (CHT), illustrated in Fig. 1(a), overcomes these miniaturization problems [24]. It has been studied both experimentally and theoretically [25]–[28]. The thruster consists of a boron-nitride ceramic channel, an annular anode, which serves also as a gas distributor, two electromagnetic coils, and a magnetic core. The axial electron current in a CHT can be reduced by the magnetic field with an enhanced radial component and/or by the strong magnetic mirror in the cylindrical part of the channel. The magnetic field lines intersect the ceramic channel walls. The electron drifts are closed, with the magnetic field lines forming equipotential surfaces, with \( E = -\mathbf{v}_e \times \mathbf{B} \). Ion thrust is generated by the axial component of the Lorentz force, proportional to the radial magnetic field and the azimuthal electron current.

The cylindrical channel features a short annular region and a longer cylindrical region. The length of the annular region is selected to be approximately equal to an ionization mean free path of a neutral atom. Compared to a conventional geometry (annular) Hall thruster, the CHT has lower surface-to-volume ratio and, therefore, potentially smaller wall losses in the channel. Having potentially smaller wall losses in the channel, a CHT should suffer lower erosion and heating of the thruster parts, particularly the critical inner parts of the channel and magnetic circuit. This makes the concept of a CHT promising for low-power applications.

In contrast to the conventional annular geometry, in the cylindrical geometry, the axial potential distribution is critical for electron confinement. This is because there is now a large axial gradient to the magnetic field over the cylindrical part of the channel, which means that electrons drift outwards through the \( \mu_e \nabla B \) force, even as they drift azimuthally around the cylinder axis. In the absence of an axial potential, the electrons would simply mirror out of the region of high magnetic field. The axial potential that accelerates ions outwards, now also plays an important role in confining electrons within the thruster.

A relatively large 9-cm-diameter version of the cylindrical thruster, operated in the subkilowatt power range [24], and miniaturized 2.6-cm- [25] and 3-cm-diameter CHTs [29], [30], operated in the power range 50–300 W, exhibit performance comparable with that of the conventional state-of-the-art annular Hall thrusters of the same size. In [27], the plasma...
potential, electron temperature, and plasma density distributions were measured inside the 2.6-cm CHT. It was found that even though the radial component of the magnetic field has a maximum inside the annular part of the CHT, the larger fraction of the applied voltage is localized in the cylindrical region. A significant potential drop was also observed in the plume. Ion acceleration in the CHT is expected to occur predominantly in the longitudinal direction and toward the thruster axis. Therefore, the CHT, having lower surface-to-volume ratio as compared with conventional Hall thrusters, may suffer lower erosion of the channel walls and have a longer lifetime.

In recent work [28], electron cross-field transport in a 2.6-cm miniaturized cylindrical Hall thruster was studied through the analysis of experimental data and Monte Carlo (MC) simulations of electron dynamics in the thruster channel. The numerical model takes into account elastic and inelastic electron collisions with atoms, electron–wall collisions, including secondary electron emission, and Bohm diffusion. It was shown that in the typical operating regime, the electron anomalous collision frequency $\nu_B$ was of the order of the Bohm value, $\nu_B \approx \omega_e/16$. The contribution of electron–wall collisions to cross-field transport was found to be insignificant.

The present paper gives a review of the experimental and numerical investigations of electron cross-field transport in the 2.6-cm CHT and reports a few recent experimental results that suggest directions for further studies.

This article is organized as follows. In Section II, the main features of the 2.6-cm CHT are presented and the experimental results, obtained in the vacuum facility with a relatively high background pressure, are reviewed. Section III gives a description of the developed MC code and outlines the key results of the numerical simulations. We discuss the electron cross-field transport in Section IV. In Section V, a few recent experimental results, obtained at low background pressure, are presented, and their implications are discussed. In Section VI, we summarize our main conclusions.

II. EXPERIMENTS

The results of comprehensive experimental investigations of the 2.6-cm CHT are given in [25]–[29]. Experiments described in this section were performed in the Small Hall Thruster facility at Princeton Plasma Physics Laboratory (PPPL), Princeton, NJ.

The 2.6-cm CHT, shown in Fig. 1(b), was scaled down from the 9-cm CHT to operate at about 200-W power level. The total length of the channel is 2.2 cm, the annular region is approximately 0.6 cm long. The outer and the inner diameters of the channel are 2.6 and 1.4 cm, respectively. The overall diameter and the thruster length are both 7 cm.

The magnetic field profiles in the 2.6-cm CHT are shown in Fig. 2(a). The radial component $B_r$ of the magnetic field reaches its maximum near the anode and then reduces toward the channel exit. Although the axial component $B_z$ is also strong, the magnetic field in the annular part of the channel is predominantly radial, the average angle between the field line and the normal to the walls is about $30^\circ$ [see Fig. 2(b)]. Magnetic field has a mirror-type structure near the thruster axis, with the maximum $B \approx 1.400$ G at the central ceramic piece wall. Due to the mirroring effect of the magnetic field in the cylindrical part of the channel [see Fig. 2(b)], most of the electrons injected from the cathode are reflected from the region of strong $B$ field, and move in the downstream direction. Upon crossing the thruster exit plane and entering the plume plasma, the electrons become unmagnetized and face the potential drop of about 100 V, which reflects them back into the thruster. Thus, most of the electrons injected from the cathode to the CHT appear to be confined in a hybrid trap formed by the magnetic mirror and by the plume potential drop. Diffusion of these electrons across the magnetic field occurs on a time scale much larger than the bounce time in the trap [28].

The typical discharge parameters for the 2.6-cm CHT are: Xe flow rate $\mu = 0.4$ mg/s, discharge voltage $U_d = 250$ V, discharge current $I_d \approx 0.6$ A. Under such conditions, the background gas pressure in the PPPL Small Hall Thruster facility is about $7 \times 10^{-5}$ torr, the propellant utilization in the 2.6-cm CHT is about 1, and the current utilization is approximately equal to 0.5 [25]. In practice, for the given propellant flow rate, discharge voltage, and background gas pressure, the discharge current is minimized by varying the currents in the magnetic coils. This procedure, which appears to be customary for the annular thrusters, is based on the assumption that near the discharge...
current minimum, the variation of the magnetic field affects mainly the electron current to the anode but not the ion current. Thus, the thruster efficiency is maximized by decreasing the discharge current while keeping the generated thrust nearly constant. As shown in Section V, this approach is valid for the CHTs operated at a low background gas pressure (in the $10^{-6}$ torr range). However, in the relatively high background pressure of the Small Hall Thruster facility ($\sim 10^{-4}$ torr), the reduction of the discharge current in certain magnetic field configurations may be due to the suppression of the background gas ionization. Nonetheless, the operating regime considered in Section II–IV is a typical one for the vacuum environment of the Small Hall Thruster facility.

The distribution of plasma potential $\phi$, electron temperature $T_e$, and plasma density $N_e$ inside the 2.6-cm CHT was studied by means of stationary and movable floating emissive and biased Langmuir probes [28]. The probe setup used in the experiments is shown in Fig. 2(b). Measurements were done at the outer channel wall (at four axial locations: $z = 5$, 10.3, 13.5, and 22 mm), as well as at the thruster axis. The results of the probe measurements are shown in Fig. 3. The potential drop in the 2.6-cm CHT is localized mainly in the cylindrical part of the channel and beyond the thruster exit, in the plume. The potential variation along the thruster axis between the central ceramic piece and the channel exit is insignificant. Its maximum possible value is within the data spread of the measurements, which is about 25 V. Much larger potential drops along the magnetic field lines were observed in the end-Hall ion source [31], which has a mirror-type magnetic field distribution similar to that in the central part of the CHT.

Due to a rather large uncertainty of the plasma density measurements, it was possible to determine only the interval, in which the real value of $N_e$ was located. The variation bars in Fig. 3(c) span between the upper and the lower estimates of $N_e$ obtained in the experiments. Due to the reasons discussed in detail in [27], the real values of the plasma density are believed to be closer to the upper bounds of the corresponding intervals. The plasma density in the 2.6-cm CHT has a prominent peak at the thruster axis: $N_e$ at the axis is 4–8 times larger than in the annular part of the channel. This density elevation at the thruster axis should be, in fact, common to all scaled down Hall thrusters and might lead to the enhanced erosion of the tip of the central ceramic piece.

III. NUMERICAL SIMULATIONS

The MC code, developed to study the electron dynamics in the thruster channel, is described in detail elsewhere [28]. In the present paper, we only outline the main code’s features and show the major results of the numerical simulations.

A. Modeling Approach and Assumptions

The MC code in the present realization is used to simulate the charged particles dynamics in the channel of the 2.6-cm CHT. The particle trajectories are traced in the given electric and magnetic fields, which are assumed to be azimuthally symmetric. The magnetic field distribution for a given arrangement of the magnetic circuit is simulated using the commercially available Field Precision software [32].

The electric field distribution is obtained from the experiments assuming that the magnetic field surfaces are equipotential. We assign the measured potential values to the magnetic field lines sampled by the corresponding probes [see Fig. 2(b)]. Between the locations of the probes plasma potential $\phi(z, R)$ is assumed to vary linearly with magnetic flux function $\psi(z, R)$, $\phi(z, R) \propto \psi(z, R)$. The anode’s surface is equipotential with...
$\phi = 250$ V. As suggested by the measurements, the magnetic field line at the thruster axis is assumed to be equipotential as well, and is assigned the potential of 100 V. The sheath potential drop is assumed to be concentrated in the infinitely thin layer near the walls.

In the MC simulations, charged particle trajectories are integrated in three-dimensional (3-D) $3\nu$ (three dimensions in configuration space, three dimensions in velocity space). Newton’s equations of motion are resolved using a modification of the explicit leap-frog scheme by Boris [33]. The time step of integration $\Delta t = 0.1/\omega_e$ was used in simulations, where $\omega_e$ is the particle gyrofrequency (for electrons, $\Delta t \sim 3 \times 10^{-12}$ s).

We apply the MC technique [34] to simulate electron collisions, which include collisions with neutral Xe atoms (elastic scattering, excitation, and single ionization), with channel walls [attachment, backscattering, and secondary electron emission (SEE)], and with electric field fluctuations (anomalous or “Bohm” diffusion). For simplicity, the neutral gas density $N_a$ is assumed to be uniform in the entire channel volume, and the near-wall sheath potential drop $\phi_{sh}$ is assumed to be constant along all the channel walls. To treat MC collisions, the numerically efficient null-collision method [34] is implemented in the code.

Scattering of electrons on the channel walls involves three different processes, namely, true secondary electron emission (SEE), elastic backscattering, and inelastic backscattering. For low primary electron energies, which are typical of Hall thrusters, the energy spectra of true secondary electrons and backscattered electrons merge. Thus, only the total SEE yield $\gamma(\varepsilon)$ is available for the traditional Hall thruster channel materials, such as boron-nitride. Using the approach developed in [10], we split the total SEE yield $\gamma(\varepsilon)$ into true secondary emission yield $\delta$ and total backscattering yield $\eta$. When an electron that can penetrate the sheath collides with the wall, either electron attachment, or backscattering, or true SEE can occur. The probabilities of these processes can be determined from the values of $\gamma$, $\delta$, and $\eta$ [28].

We imposed the anomalous Bohm conductivity inside the channel in order to account for fluctuation-enhanced electron transport. It was assumed that electrons scatter primarily in the azimuthal fluctuations of the electric field. When an electron undergoes a collision with the electric field fluctuation, the perpendicular, with respect to $\mathbf{B}$, electron velocity component is assumed to scatter isotropically. The parallel velocity component does not change. Thus, the guiding center of the electron orbit gets a random shift in the plane perpendicular to $\mathbf{B}$ on the order of the electron gyroradius. The frequency of Bohm diffusion collisions is assumed to be $\nu_B = \kappa_B \omega_e/16$, where $\kappa_B$ is a fitting parameter that does not depend on the electron energy. Using the customary approach, we introduce one fitting parameter, $\kappa_B$, which is applied to calculation of the anomalous collision frequency in the entire discharge volume. However, in reality, the effective $\kappa_B$ may vary across the magnetic field [21].

It is worth mentioning that for kilowatt and subkilowatt Hall thrusters most of the models that impose Bohm conductivity in the channel show that the best agreement between the experimental and simulated data is achieved when $\kappa_B$ is less than one, on the order of 0.1–0.4 [8], [9], [11]–[16].

![Fig. 4. (a) Distributions of the electron density and (b) the effective electron temperature in the channel of the 2.6-cm CHT. Solid dark rectangle in the lower left-hand side corner of the pictures ($0 < z < 6, 0 < R < 7$) represents the cross section of the central ceramic piece.](image)

In the electron transport simulations, primary electrons injected from the cathode are assumed to have monoenergetic distribution with $\varepsilon = 20$ eV. Similar energy of electrons injected from the cathode was observed in a low-power conventional Hall thruster [35]. The primary electrons are launched at the thruster exit, with a uniform distribution of the electron flux across the channel cross section. The electron current fraction is taken to be equal to 0.5 (see Section II). The electrons are followed successively one after another until both primary electrons and secondary ones (the latter being generated due to ionization and secondary electron emission from the walls), either reach the anode or get attached to the walls. EDF is determined in $z - R - \varepsilon$ phase space using the approach developed by Boeuf and Marode [36]. Electron density and effective electron temperature are determined as the corresponding moments of the EDF.

### B. Numerical Results

The main objective of the performed numerical simulations was to determine what rate of electron cross-field diffusion could explain the observed discharge current. We performed the parametric study of the dependency of plasma parameters distribution on the electron cross-field conductivity. Numerical simulations were carried out for four different values of $\kappa_B$, with $N_a$ and $\phi_{sh}$ chosen according to the experimental constraints [28]. The main results obtained in the simulations can be summarized as follows.
The distributions of the electron density $N_e$ and effective electron temperature $T_{\text{eff}}$ obtained in simulations are shown in Fig. 4. Note that the maximum electron density is achieved in the annular part of the channel. Although there is a slight elevation of $N_e$ at the thruster axis, its value, as opposed to the results of the experiments, is lower than the density in the annular part of the channel. The plasma density spike observed at the thruster axis [see Fig. 3(c)] might be due to the convergent ion flux [37]. When $\kappa_B$ is varied, the distribution of the electron density in the channel remains similar to that shown in Fig. 4(a), with the characteristic magnitude of $N_e$ decreasing when $\kappa_B$ is increased. For different values of $\kappa_B$, the distributions of $T_{\text{eff}}$ remain very similar to each other.

When the parameter $\kappa_B$ is increased, the electron density required to conduct the observed discharge current becomes smaller. This fact is illustrated in Fig. 5, where the axial profiles of $N_e$ near the outer channel wall are plotted for different values of $\kappa_B$. As the rate of cross-field electron diffusion approaches the Bohm value $\kappa_B = 1$ [38], the electron density at $z = 5$ mm and 13.5 mm gets almost equal to the measured plasma density. As mentioned herein, the real values of the plasma density are believed to be closer to the upper bounds of the corresponding uncertainty bars in Fig. 5. Even though the match between the measured and simulated values of $N_e$ is not perfect, the trend of $N_e$ dependency on $\kappa_B$ is evident.

The present model is not expected to give a correct quantitative description of the EDF variation along the thruster channel. However, the general shape of the EDF obtained in our simulations appears to be in a good qualitative agreement with the results of work [20], where the EDF in the Hall thruster channel was determined by solving the electron Boltzmann equation.

A typical EDF in the annular part of the channel is shown in Fig. 6. The EDF averaging is performed in order to get statistically more ample phase space data. As can be concluded from Fig. 6, electron–wall collisions deplete the tail of the EDF. The resultant shape of the EDF appears to be bi-Maxwellian. Recent kinetic simulations [39] that focused on the problem of plasma–wall interaction in Hall thrusters showed that the EDF is not only depleted at high energy, but also strongly anisotropic and nonmonotonic. In the given distribution of electric field, Bohm parameter $\kappa_B$ governs the rate of electron thermal energy pumping. As $\kappa_B$ (and, consequently, $\nu_B$) decreases, the tail of the distribution function gradually weakens. For $\kappa_B = 1$ (as in Fig. 6), the ratio of the bulk and the tail electron temperatures is approximately equal to 2.1. For $\kappa_B = 0.16$, this ratio increases to 3.3, while the effective electron temperature remains approximately the same as in $\kappa_B = 1$ case. In the cylindrical part of the channel, where the electron–wall collision frequency is smaller, the influence of the walls on the EDF shape is less pronounced.

IV. DISCUSSION

In view of Fig. 5, in order to explain the observed plasma density, the electron anomalous collision frequency $\nu_B$ should be high, on the order of the Bohm value $\nu_B \sim \omega_c/16 (\kappa_B = 1)$. This conclusion can be supported also by the following argument concerning electron current conduction in the annular part of the channel. The magnetic field in the annular part of the 2.6-cm CHT is mainly radial. The average value of the magnetic field at the median is about 650 G. At $z = 5$, where the closest to the anode probe was located in the experiments, the axial electric field $E$ is about 110 V/cm. We can estimate the average electron velocity in the axial direction $U_e = \kappa_B Ee/(16 \pi m \omega_c) \sim 1.00 \kappa_B \times 10^8$ cm/s. Now, we note that in the 2.6-cm CHT the fraction of the discharge current carried by the ions varies from essentially zero at the anode ($I_I \ll I_e$) to about 0.5 at the thruster exit ($I_I \approx I_e \approx I_d/2$). Taking in to account that the overall potential drop in the annular part of the channel is not large, we conclude that the electron current in the annular part of the channel should be at least a few times larger than the ion current. Thus, in the annular part of the channel $I_e \approx I_d \approx eN_e S \xi U_e$, where $\xi = 3.77$ cm$^2$ is the anode area. Therefore, we can relate the plasma density required to conduct the observed current ($I_d = 0.6$ A) to the rate of electron cross-field transport

$$N_e \sim \frac{9.4}{\kappa_B} \times 10^{11} \text{cm}^{-3}. \tag{2}$$

For $\kappa_B = 1$, the value of $N_e$ acquired from this rather crude estimate, $N_e = 9.4 \times 10^{11}$ cm$^{-3}$, is in a good agreement with the
result of simulations, \( N_e = 8.2 \times 10^{11} \text{ cm}^{-3} \). More importantly, the values of \( N_e \) obtained in simulations for different values of \( \kappa_B \) follow \( 1/\kappa_B \) scaling quite well, as illustrated in Fig. 7.

It is important to mention that the value of parameter \( \kappa_B \), which, for the low-power CHT, gives the best agreement between the simulations and experiments (\( \kappa_B \sim 1 \)), is a few times larger than those obtained typically in the modeling of conventional Hall thrusters (\( \kappa_B \sim 0.1 - 0.4 \)) [8], [9], [11]–[16]. Thus, the rate of electron fluctuation-enhanced diffusion, which is required to explain the discharge current observed in the CHT, should be higher than that in conventional Hall thrusters. The anomalous electron transport in the CHT is believed to be induced by high-frequency plasma instabilities [40], [41]. Interestingly, in the frequency range below \( \sim 100 \text{ kHz} \), the 2.6-cm CHT operates quieter than the annular Hall thruster of the same size [25].

The electron–wall collisions make an insignificant contribution to the electron current conduction, as compared with the fluctuation-induced electron scattering. For the parameters of Fig. 6, the average electron–wall collision frequency \( \nu_{ew} \) is approximately equal to \( 1 \times 10^7 \text{ s}^{-1} \), while the anomalous collision frequency averaged along the corresponding field line is about \( 7.2 \times 10^8 \text{ s}^{-1} \) [28]. At the same time, the total electron–atom collision frequency \( \nu_{ea} \) is on the order of \( 2.4 \times 10^7 \text{ s}^{-1} \). Thus, \( \nu_{ew} \ll \nu_{ea} \ll \nu_B \). Both the electron–wall and the electron–atom collision frequencies decrease toward the thruster exit. The inequalities \( \nu_{ew}, \nu_{ea} \ll \nu_B \) are satisfied throughout the channel. In the real thruster, the neutral gas density decreases toward the channel exit due to ionization and the effective channel widening upon the transition from the annular to the cylindrical channel part. If a realistic neutral gas density profile was used in the simulations, \( \nu_{ew} \) would become larger than \( \nu_{ea} \) in the cylindrical part of the channel. However, in order to explain the observed discharge current, the anomalous collision frequency \( \nu_B \) would have to remain much larger than both \( \nu_{ew} \) and \( \nu_{ea} \).

Even though the electron–wall collisions appear to have little effect on the electron cross-field transport, the electron–wall interaction is very important in terms of electron energy balance [42], [43]. For the parameters of Fig. 6, for example, the electron energy loss at the walls \( q_{ew} \) is equal to about \( 9.7 \text{ J/cm}^3 \), while the energy loss due to inelastic electron–atom collisions \( q_{ea} \) is about \( 4.9 \text{ J/cm}^3 \). \( q_{ew} \) and \( q_{ea} \) both decrease toward the channel exit and have values comparable with each other. The conclusions concerning the electron cross-field transport seem, however, to be insensitive to the details of the electron energy balance.

V. RECENT EXPERIMENTAL RESULTS AND PLANS FOR FUTURE WORK

The effect of the magnetic field on the discharge characteristics and efficiency of the low-power CHTs with channel outer diameters of 2.6 and 3 cm was investigated recently [25], [44]. In this section, we briefly describe a few interesting results obtained in these experiments. The observed effects (even though the underlying physics remains largely unexplored) have important implications for the problem of electron cross-field transport and suggest the directions for further studies.

The variation of the current in the back magnetic coil of the CHT mainly changes the magnetic field magnitude without altering the shape of magnetic field surfaces. It is generally observed that the increase of the back coil current leads to the monotonic decrease of the discharge current. The variation of
the front coil current changes the shape of the magnetic field surfaces, with the most pronounced changes occurring in the cylindrical part of the channel. When the current in the front coil is counter-directed to that in the back coil ($I_{\text{front}} < 0$), the “cusp” magnetic field with an enhanced radial component is created (see Fig. 2). Swapping the polarity of the front coil current ($I_{\text{front}} > 0$) leads to the enhancement of the axial component of the magnetic field (see Fig. 8) and generation of a stronger magnetic mirror near the thruster axis. The goal of the performed experiments was to investigate the dependence of the discharge current and generated thrust on the current in the front magnetic coil.

The experiments were performed in the Electric Propulsion and Plasma Dynamics Laboratory (EPPDyL), Princeton University, Princeton, NJ [45]. The operating background pressure of xenon in the EPPDyL vacuum facility was about one order of magnitude smaller than that in the Small Hall Thruster facility at PPPL. Importantly, it was observed that the magnetic field configuration that minimizes the discharge current depends on the background gas pressure in the tank. In Fig. 9, the variation of the discharge current $I_d$ with the current in the front coil $I_{\text{front}}$ is shown for the EPPDyL and PPPL facilities. All discharge parameters are the same (anode flow rate $\mu = 0.4$ mg/s, $U_a = 250$ V, $I_{\text{back}} = +3$ A), except for the background xenon pressure, which is equal to $6 \times 10^{-6}$ torr for the EPPDyL tank and $6.7 \times 10^{-5}$ torr for the PPPL tank.

![Fig. 9. Dependencies of the discharge current on the current in the front magnetic coil in the 2.6-cm CHT for the EPPDyL and PPPL facilities. All discharge parameters are the same (anode flow rate $\mu = 0.4$ mg/s, $U_a = 250$ V, $I_{\text{back}} = +3$ A), except for the background xenon pressure, which is equal to $6 \times 10^{-6}$ torr for the EPPDyL tank and $6.7 \times 10^{-5}$ torr for the PPPL tank.](image)

Now, at low background pressure, the increase of $I_{\text{front}}$ above $\sim 1$ A leads to the negligible variation of the discharge current. The decrease of $I_{\text{front}}$, on the contrary, brings about rather sharp increase of $I_d$. Along with it, as the magnetic field configuration is changed from direct to cusp, the generated thrust slightly decreases (see Fig. 10). Consequently, in the voltage range from 200 to 300 V, the anode efficiency in the direct configuration is approximately factor of 1.5–1.7 larger than that in the cusp configuration.

The fact that the discharge current decreases with the increase in $I_{\text{front}}$ (see Fig. 10) implies that the electron transport to the anode is suppressed more strongly in the direct magnetic field configuration than in the cusp configuration. Indeed, from the data shown in Fig. 10 it follows that

$$I_C^e \sim 0.95 I_d \sqrt{\frac{\varepsilon d}{\varepsilon c}}$$

(3a)

$$I_C^d \sim 1.37 I_d.$$  

(3b)

Here, $I_I$ is the ion current, $\varepsilon$ is the mean ion energy, and superscripts $d$ and $c$ refer to the direct and cusp polarities, respectively. From (3), we obtain the ratio of the electron currents in the cusp and direct configurations

$$\frac{I_C^e}{I_C^d} \approx 1.37 + \frac{I_d}{I_C^d} \left(1.37 - 0.95 \sqrt{\frac{\varepsilon d}{\varepsilon c}}\right).$$

(4)

When the thruster magnetic field configuration is changed, it is very unlikely that the average ion energy varies by more than about factor of 2. Thus, the ratio $I_C^e/I_C^d$ is about 1.3–1.5.

The fact that the electron current in the direct configuration is smaller does not necessarily imply that the rate of electron cross-field transport is smaller. Plasma measurements, similar to those described in Section II, are required to understand how the magnetic field configuration and background gas pressure influence the electron anomalous transport. Studying the dependence of the plasma parameters on the magnetic field and gas pressure is a subject of ongoing research.
VI. CONCLUSION

Scaling to low-power Hall thrusters requires the magnetic field to be increased inversely with length, as the thruster channel size is decreased. In a strong magnetic field of a low-power Hall thruster, the rate of electron cross-field diffusion, required to sustain the discharge, can differ from that in a Hall thruster operating in the conventional kilowatt or sub-kilowatt power range. Thus, understanding of the mechanisms of electron transport is essential for the development of higher efficiency low-power thrusters and for scaling to small sizes.

The conventional (annular) Hall thrusters become inefficient when scaled to small sizes because of the large surface-to-volume ratio and the difficulty in miniaturizing the magnetic circuit. Also, the erosion of the walls of a small annular channel can severely limit the thruster lifetime. An alternative approach, which may be more suitable for scaling to low power, is a CHT. The 9-cm CHT, operated in the sub-kilowatt power range, and the miniature 2.6- and 3-cm CHT, operated in the power range 50–300 W, exhibit performance comparable with the conventional state-of-the-art annular Hall thrusters of the same size. Ion acceleration in the CHTs occurs mainly in the cylindrical part of the channel and beyond the thruster exit. Thus, CHTs, having lower surface-to-volume ratio as compared with conventional annular design Hall thrusters, should suffer lower erosion of the channel walls and, therefore, have a longer lifetime.

Plasma potential, ion density, and electron temperature profiles were measured inside the 2.6-cm cylindrical Hall thruster, operated in the vacuum facility with a relatively high background gas pressure ($<10^{-1}$ torr). The electron cross-field transport was studied for the typical operating regime. To analyze electron dynamics in the channel region of the 2.6-cm CHT, a Monte Carlo code was developed. The numerical model takes into account elastic and inelastic electron collisions with atoms, electron–wall collisions (backscattering, attachment, and secondary electron emission), and Bohm diffusion. The comparison of numerical and experimental results shows that in order to explain the discharge current, observed in the 2.6-cm CHT, the electron anomalous collision frequency $\nu_B$ has to be high. As opposed to most of the conventional Hall thruster models, which predict the ratio $\nu_B/\omega_e$ to be on the order of $10^{-2}$, we find that in the 2.6 cm CHT $\nu_B$ has to be on the order of the Bohm value, $\nu_B \sim \omega_e/16$. The anomalous cross-field electron transport in the CHT is believed to be induced by high-frequency plasma instabilities. The EDF in a Hall thruster is depleted at high energy due to electron loss at the walls, thus indicating that the contribution of secondary electrons to cross-field transport is likely insignificant.

The effect of the magnetic field on the discharge current and generated thrust in the 2.6- and 3-cm CHTs was studied in the experiments performed at low background gas pressure ($<10^{-5}$ torr). These experiments demonstrated that the optimal regimes of thruster operation at low background pressure are, in fact, different from those at higher pressure. For instance, for both the 2.6- and 3-cm CHTs the discharge current decreases and the generated thrust slightly increases as the magnetic field configuration is changed from cusp to direct. This, most likely, implies that the electron transport to the anode is suppressed more strongly and the directionality of ion acceleration is better in the direct magnetic field configuration than in the cusp configuration. The thruster efficiency is accordingly larger in the direct configuration. Future experiments will address the question of how the rate of electron cross-field transport depends on the magnetic field configuration, channel geometric parameters, and the background gas pressure in the tank.

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