Supra-bubble regime for laser acceleration of cold electron beams in tenuous plasma

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(Received 17 November 2009; accepted 14 January 2010; published online 11 February 2010)

Relativistic electrons can be accelerated by an ultraintense laser pulse in the "supra-bubble" regime, that is, in the blow-out regime *ahead* of the plasma bubble (as opposed to the conventional method, when particles remain *inside* the bubble). The acceleration is caused by the ponderomotive force of the pulse, via the so-called snow-plow mechanism. The maximum energy gain, $\Delta\gamma \sim \gamma_g a$, is attained when the particle Lorentz factor γ is initially about γ_g/a, where γ_g is the pulse group speed Lorentz factor, and a is the laser parameter, proportional to the laser field amplitude. The scheme operates at $a \lesssim \gamma_g$, yielding $\Delta\gamma$ of up to that via wakefield acceleration for the same plasma and laser parameters, $\Delta\gamma \sim \gamma_g^2$. The interaction length is shorter than that for the wakefield mechanism but grows with the particle energy, hindering acceleration in multiple stages. *© 2010 American Institute of Physics*. [doi:10.1063/1.3309488]

I. INTRODUCTION

The problem of charged particle acceleration by intense laser waves in plasma[1–4] has been attracting increased attention since the invention of ultrapowerful lasers able to produce electromagnetic pulses with relativistic intensities.[5] Recently, it has become possible to generate quasi-monoenergetic bunches of electrons with energies in the GeV range,[6–9] particularly, via the laser wakefield acceleration (LWFA) in the "bubble," or "blow-out" regime.[10–12] However, this method suffers from dephasing between particles and accelerating electrostatic field and pulse diffraction on large interaction scales,[4] which suggests also searching for alternative mechanisms of particle acceleration.

For conditions similar to those of the bubble regime, it was recently proposed that electrons can be accelerated directly by the laser field, as opposed to the induced electrostatic field in LWFA, and experience what is known as snow-plow acceleration,[13–20] or the relativistic photon mirror effect.[21,22] Particularly, a preaccelerated particle beam can be caught up with by a copropagating laser pulse, which hence pushes the beam forward. Since the group speed of the pulse in plasma is less than the speed of light, the electrons can eventually outrun the pulse (in vacuum, this would not be possible) and gain energy by sliding off its traveling ponderomotive potential (Fig. 1). As the particles remain *ahead* of the background electron density depletion (the bubble), we henceforth call this the "supra-bubble" regime of electron acceleration.

In Refs. 15 and 18, the conceptual scheme outlined here was shown to be viable in numerical simulations, and some analytical results were presented in Refs. 13–19, 21, and 22; also, the monoenergetic electron beam formation due to snow-plowing was recently demonstrated in the experiment.[23] However, the optimum parameter regime and the resulting scalings have not yet been identified; neither has the influence of the pulse transverse structure been assessed analytically, nor has the effect of the wake electrostatic potential been addressed for snow-plow acceleration.[24,25]

The purpose of the present paper is to solve these problems and summarize the main features of the snow-plow scheme in comparison with LWFA. (For brevity, we will assume LWFA to approximately follow the most common scalings from the original Ref. 1 for this comparison; for more accurate models, see Refs. 2, 3, and 12.) Specifically, we show that the maximum energy gain, $\Delta\gamma \sim \gamma_g a$, is attained when the particle Lorentz factor γ is initially about γ_g/a, where γ_g is the pulse group speed Lorentz factor, and a is the laser parameter, proportional to the laser field amplitude. We also point out that, since the scheme operates at $a \lesssim \gamma_g$, it yields $\Delta\gamma$ of up to that via wakefield acceleration for the same plasma and laser parameters, $\Delta\gamma \sim \gamma_g^2$. Simultaneously, the interaction length is shorter than that for the wakefield mechanism but grows with the particle energy, hindering acceleration in multiple stages.

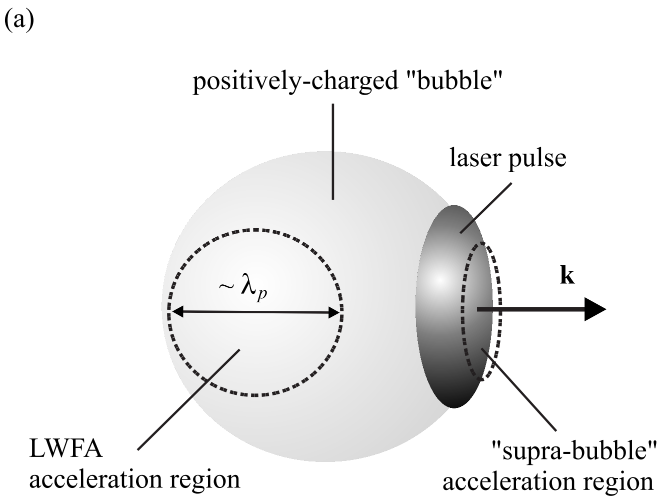
The paper is organized as follows. In Sec. II, we reintroduce, following our Ref. 20, the oscillation-center model of the laser-driven charged particle motion in plasma. In Sec. III, we formulate the optimum conditions for electron beam acceleration via the snow-plow mechanism and derive the scaling for the particle final energy. In Sec. IV, we compare the supra-bubble acceleration with LWFA, also with respect to staging. In Sec. VI, we discuss the applicability conditions of our model. In Sec. VII, we summarize our main results.

II. BASIC EQUATIONS

Consider the motion of a trial particle in a laser wave, which we assume, for clarity, to propagate along x axis; hence, the laser vector potential has the form

$$\mathbf{A} = \mathbf{A}_0\left(\frac{x - v_g t}{L_\parallel}\right)\cos[k(x - v_p t)], \tag{1}$$

where v_g and v_p are the group velocity and the phase



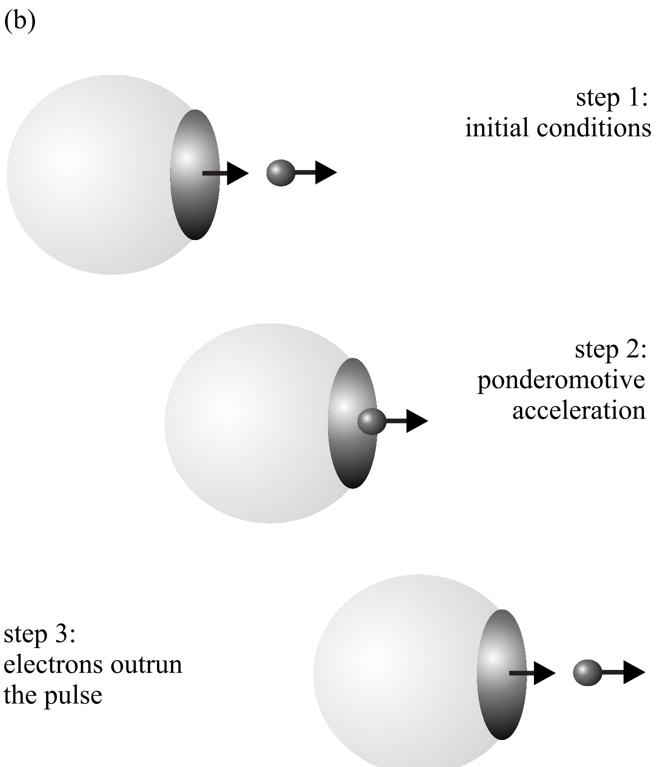


FIG. 1. Schematic of the "supra-bubble" acceleration scheme. (a) Field structure (**k** is the wavevector; λ_p is the plasma wavelength, see below). (b) Acceleration process: a preaccelerated electron beam is caught up with by a copropagating laser pulse, which hence pushes the beam forward, so the particles gain energy by sliding off the traveling ponderomotive potential, eventually outrunning the pulse.

velocity, correspondingly; $L_\parallel$ is the spatial scale of the envelope $\mathbf{A}_0$, k is the wavenumber, and $\epsilon \equiv (kL_\parallel)^{-1} \ll 1$.

Start off with the particle Hamiltonian[26]

$$H = c\sqrt{m^2c^2 + p_x^2 + \left(\mathbf{P}_\perp - \frac{e}{c}\mathbf{A}\right)^2} + e\varphi, \tag{2}$$

where m and $e<0$ are the electron mass and charge, correspondingly, and φ is the electrostatic potential. Assuming that A_0 is large enough, φ turns out to be negligible (Sec. VI A). Assuming also that $L_\parallel$ is sufficiently small compared to the plasma wavelength λ_p, the linear plasma dispersion relation effectively applies for arbitrary A_0;[27,28] thus, for the field frequency $\omega = kv_p$, we use

$$\omega^2 = \omega_p^2 + c^2k^2, \tag{3}$$

so $v_g = c\beta_g$ and $v_p = c\beta_p$, where

$$\beta_g = \sqrt{1-\alpha}, \quad \beta_p = 1/\sqrt{1-\alpha}, \quad \alpha = \omega_p^2/\omega^2, \tag{4}$$

$\omega_p^2 = 4\pi ne^2/m$ is the plasma frequency and n is the density of background plasma electrons. The corresponding oscillation-center dynamics, from where the fast oscillations at the laser period are excluded, satisfies the conservation law[20]

$$\dot{\Psi}^2 + \alpha\left[1 + Q^2 + \frac{a^2(\epsilon\Psi)}{2}(1-\delta^2)\right] = \Pi^2. \tag{5}$$

Here $\Psi = k(x - v_g t)$; the dot denotes the derivative with respect to $\bar{\tau} \equiv kc\tau$; τ is the particle proper time, $d\tau = dt/\gamma$; $\gamma = H/(mc^2)$ is the electron Lorentz factor. Also, $\mathbf{Q} = \mathbf{P}_\perp/(mc)$ is the normalized canonical momentum transverse to the direction of the pulse propagation; $a = eA/(mc^2)$ is the laser parameter (the term proportional to a^2 is due to the laser ponderomotive potential[20,29–34]); Π is a constant, which is determined by initial conditions and, for a particle *outside* the pulse, equals

$$\Pi = \gamma - \beta_g\rho, \tag{6}$$

with $\rho \equiv p_x/(mc)$ being the normalized kinetic momentum in the direction of pulse propagation.

The function $\delta \lesssim 1$ depends on the laser polarization[20,35] and can significantly affect the dynamics of hot electrons.[20] However, for the case of cold ($Q=0$) electrons considered below, one has $\delta=0$ at circular polarization, and $\delta^2 \approx \alpha a^2/(16\Pi^2)$ at linear polarization, in which case $\delta^2 \lesssim 0.168$.[20] Therefore, δ is henceforth neglected, yielding that our further results are approximate, but, as such, hold for arbitrary polarization.

III. SNOW-PLOW ACCELERATION

We now use the formalism from Sec. II to describe the electron acceleration. From the definition of Ψ, one has $\dot{\Psi} = \rho - \gamma\beta_g$; combining this with Eq. (6) yields

$$\gamma = \frac{\beta_p\Pi + \dot{\Psi}}{\beta_p - \beta_g} \tag{7}$$

for a particle outside the pulse (i.e., before and after the interaction), where we used $\beta_g\beta_p = 1$. From Eq. (5), one has $\dot{\Psi} = \pm\sqrt{\Pi^2 - \alpha}$ in this case, as $a=0$; thus, assuming $Q=0$, Eq. (7) rewrites as

$$\gamma = \frac{1}{\alpha}\left[\Pi \pm \sqrt{(1-\alpha)(\Pi^2-\alpha)}\right], \tag{8}$$

where the plus and the minus correspond to the particle outrunning the pulse ($\dot{\Psi}>0$) and falling behind it ($\dot{\Psi}<0$), respectively.[36]

If no reflection occurs, then the sign of $\dot{\Psi}$ does not change; hence, the expression for γ_∞ and γ_0 coincide (the indexes ∞ and 0 denote $t \to \pm\infty$, correspondingly), i.e., there is no overall acceleration. Otherwise, from Eq. (7), the energy change $\Delta\gamma \equiv \gamma_\infty - \gamma_0$ reads as

$$\Delta\gamma = \pm \frac{2}{\alpha}\sqrt{(1-\alpha)(\Pi^2-\alpha)}. \tag{9}$$

Therefore, reflection from the pulse front leads to particle acceleration, whereas reflection from the pulse tail would lead to particle deceleration, in agreement with the relativistic mirror concept.[21,22]

Let us now find the conditions under which the reflection does occur. At the reflection point (further denoted by index r), one has $\dot{\Psi}=0$, in which case Eq. (5) yields

$$\alpha a_r^2 \approx 2\Pi^2 \tag{10}$$

for negligible δ. Background plasma electrons have $\Pi=1$, meaning that the whole plasma is snow-plowed when the maximum amplitude of the pulse satisfies $\alpha a_{\max}^2 \gtrsim 2$. However, in this case, a significant electrostatic potential will build up, which is not included into our model; thus, we henceforth assume

$$\alpha a^2 \lesssim 1. \tag{11}$$

Because of this condition, reflection is possible only for $\Pi \ll 1$, which requires [see Eq. (6)] that electrons copropagate with the pulse and have $\gamma_0 \gg 1$, assuming that $\alpha \ll 1$ here and further. However, particles should not be too fast either, as they have to be caught up with by the pulse, whose group-velocity Lorentz factor $\gamma_g \approx \alpha^{-1/2}$ is finite. Thus, for snow-plow acceleration to take place, one must satisfy

$$\gamma_g > \gamma_0 \gg 1. \tag{12}$$

From Eq. (10), $\Pi^2 \gg \alpha$ for ultraintense $(a \gg 1)$ pulses of interest; thus, Eq. (8) rewrites as $\Delta\gamma \approx 2\gamma_g^2\Pi$, where we took the plus sign, assuming reflection on the front of the pulse. In the regime (12), Eq. (6) is Taylor-expanded as $\Pi \approx (2\gamma_0)^{-1}$ (see Ref. 20). Thus, $\Delta\gamma \approx \gamma_g^2/\gamma_0$, meaning that the maximum $\Delta\gamma$ corresponds to the least γ_0 at which reflection is possible. From Eq. (10), such optimum γ_0 is

$$\gamma_0^{(\mathrm{opt})} = \frac{\gamma_g}{a_{\max}\sqrt{2}}, \tag{13}$$

yielding that the maximum energy attainable via snow-plow acceleration of a cold electron beam equals

$$\Delta\gamma_{\max} \approx \gamma_g a_{\max}\sqrt{2}. \tag{14}$$

(A similar limit, $\Delta\gamma_{\max} \sim \gamma_g a_{\max}$, was derived in Ref. 20 also for acceleration of *hot* electrons produced by electron-ion collisions inside laser field.) Therefore, acceleration becomes stronger at smaller densities $n \propto \gamma_g^{-2}$, which result in a higher speed of the pulse-formed "relativistic mirror," and larger laser amplitudes a, which result in a larger height of the ponderomotive potential for the electrons to slide off from.

Now one can also estimate the acceleration length L, particularly as follows. During the interaction time t_{int}, a particle attains $\gamma \gg \gamma_g$ and therefore, in comparison with the pulse propagating at the group velocity v_g, can be considered traveling with the speed of light c. By definition of t_{int}, the difference in the particle displacement ct_{int} and the pulse displacement $v_g t_{\mathrm{int}}=L$ is about $L_\parallel$. Thus, $t_{\mathrm{int}} \sim L_\parallel/(c-v_g)$, yielding

$$L \sim L_\parallel c/(c-v_g). \tag{15}$$

Using that $\beta_g \approx 1-1/(2\gamma_g^2)$, one finally gets

$$L \sim L_\parallel \gamma_g^2, \tag{16}$$

as also derivable from Eq. (5) or as discussed in Ref. 14.

IV. STAGED SNOW-PLOW ACCELERATION

For the optimum initial energy (13) and the acceleration length (16) to remain finite, the supra-bubble acceleration scheme can operate only at finite γ_g, or nonzero plasma densities, and cannot operate in vacuum. (For more precise limitations see Sec. VI.) On the other hand, to increase the limit on the electron final energy [Eq. (14)], staged acceleration could be used, such as in the case of LWFA,[4,37–39] meaning that the same pulse could interact with electrons multiple times, transferring its energy to particles in stages. This will require that the laser group speed be increased from one stage to another, in order to let the pulse catch with the particles which outran it before. Modulating the group speed is possible via introducing additional vacuum sections *between* those with equal plasma density; alternatively, the plasma density can be varied monotonically from one section to another. Both approaches result in the same scalings for the particle energy and the acceleration length.

First, suppose that the plasma density is a monotonic steplike function, so $\alpha_{n+1} < \alpha_n$, where n is the section number. Assume that each nth section yields the particle output Lorentz factor γ_n that corresponds to the optimum initial energy [Eq. (13)] for the $(n+1)$th section. Then the energy multiplication factor scales as a^2:

$$\gamma_{n+1} \sim a^2\gamma_n, \tag{17}$$

as flows from Eq. (13) with Eq. (14); hence,

$$\gamma_n \sim a^{2n}\gamma_0. \tag{18}$$

Similarly, from Eq. (13), one would need $\alpha_n \sim (a\gamma_{n-1})^{-2}$. Thus, using Eq. (17), one also gets

$$\alpha_n \sim a^{2-4n}\gamma_0^{-2}, \quad \alpha_{n+1} \sim a^{-4}\alpha_n, \tag{19}$$

and, from Eq. (16),

$$L_n \sim a^{4n-2}\gamma_0^2 L_\parallel, \quad L_{n+1} \sim a^4 L_n. \tag{20}$$

For example, for $a \sim 10$, this yields $L_2 \sim 10^4 L_1$ for the length of the second section. Assuming L_1 is a fraction of that attainable for LWFA today (see also Sec. V), as limited by the laser pulse diffraction spreading, the possibility of sectioned acceleration in the supra-bubble regime may not be technologically feasible. For, say, $n \sim 10^{17}$ cm^{-3} $(\alpha \sim 10^{-4})$ and $a \sim 10$, this limits the maximum energy gain via the supra-bubble regime to about half GeV.

An alternative scheme employs acceleration in multiple vacuum sections alternating with plasma sections (say, with the same α) needed to slow down the laser *average* group speed. The energy gain, found from the vacuum solution,[20] again satisfies Eqs. (17) and (18). Obtained similarly to Eq. (16), the interaction length $L_n \sim L_\| \gamma_n^2$ in nth vacuum section then follows the same scaling as in Eq. (20). Therefore, in terms of the energy gain and the interaction length, this scheme of acceleration staging is identical to the one discussed above.

V. COMPARISON WITH LWFA

The maximum energy that can be attained in one stage in a single-stage supra-bubble acceleration [Eq. (14)] compares to the one attainable via LWFA as

$$\frac{\Delta\gamma_{\max}}{\Delta\gamma_{\max}^{(\mathrm{LWFA})}} \sim a\sqrt{\alpha}, \tag{21}$$

where we used that $\Delta\gamma_{\max}^{(\mathrm{LWFA})} \sim \gamma_g^2$ (see Ref. 1). Considering the applicability condition (11), one gets

$$\Delta\gamma_{\max} \lesssim \Delta\gamma_{\max}^{(\mathrm{LWFA})}, \tag{22}$$

for the same plasma and laser parameters. On the other hand, the interaction length in LWFA,[1] $L^{(\mathrm{LWFA})} \sim \lambda_p \gamma_g^2$ (here $\lambda_p = c/\omega_p$), exceeds that in the supra-bubble regime [Eq. (16)]; particularly,

$$\frac{L^{(\mathrm{LWFA})}}{L} \sim \frac{\lambda_p}{L_\|} \gg 1. \tag{23}$$

Therefore, in the latter case, the energy from the laser pulse can be transferred to particles more rapidly than in LWFA. For one-stage accelerator, this could make the supra-bubble regime more favorable. However, having multiple stages would require that the plasma density $n \propto \alpha$ varies between different sections as described in Sec. IV, resulting in unrealistically large L starting from the second section. LWFA, on the contrary, allows equal densities in all plasma sections and therefore is favorable for electron acceleration to energies above half GeV or so.

VI. APPLICABILITY CONDITIONS

In this section, we explore the limitations of our model, particularly, the conditions when the electrostatic potential can be neglected, the one-dimensional (1D) approximation for the particle dynamics applies, and the pulse distortion over the interaction distance is negligible.

A. Electrostatic potential

Even when background plasma electrons are not snowplowed, provided that the condition (11) is satisfied, an electrostatic wake field $\mathbf{E} = -\nabla\varphi$ is built up due to these electrons being displaced by the pulse. To see how it affects the beam electrons being accelerated, calculate their energy change due to $\mathbf{E}$ on the interaction length L:

$$\gamma_\varphi \sim eEL/(mc^2). \tag{24}$$

From Poisson's equation, assuming the field scale is of the order of λ_p, one obtains $E \sim en\lambda_p$; thus,

$$\frac{\gamma_\varphi}{\Delta\gamma} \sim \frac{L_\|}{\lambda_p}\frac{\gamma_g}{a}, \tag{25}$$

where we used Eq. (16) for L and Eq. (14) for $\Delta\gamma$. Since $\lambda_p \approx \gamma_g/k$, the electrostatic potential is negligible (i.e., $\gamma_\varphi \ll \Delta\gamma$) when

$$a \gg kL_\|. \tag{26}$$

Hence, for our above calculations to hold, Eq. (26) must be satisfied, which is feasible for existing lasers.[5]

B. Transverse effects

The finite width of the pulse $L_\perp$ results in the electron average displacement δr across the direction of the pulse propagation as the particle gains $Q \sim a$. For this displacement to be negligible on the acceleration time scale, one must satisfy $L_\perp \gg \delta r$. Calculate $\delta r \sim v_\perp t_{\mathrm{int}}$ using that the transverse velocity is $v_\perp \sim ca/\gamma_\infty$, where γ_∞ is found from Eq. (14), and $t_{\mathrm{int}} \sim L/c$, with the substitution for L from Eq. (16). Hence, $\delta r \sim \gamma_g L_\|$, so the validity condition for the 1D approximation is

$$L_\perp \gg \gamma_g L_\|. \tag{27}$$

While the above condition may not be satisfied in laboratory experiments, the 1D approximation yet remains an adequate estimate for calculating the effect of supra-bubble acceleration, especially for particles traveling close to the pulse central axis. This is seen in Fig. 2, which presents the results of our numerical simulations of single-particle orbits in a given laser field with finite $L_\perp$.

Another limitation of our 1D analysis in application to finite $L_\perp$ is due to the particle transverse motion with quiver momentum, also of the order of mca, which results in the electron effectively "seeing" the laser amplitude to oscillate. The average-motion model accepted in this paper (see Ref. 20 for details) holds only when these oscillations are minor; otherwise, nonadiabatic effects take place. In Fig. 2(a), these are seen as quasiperiodic modulation of the energy gain as a function of the particle initial transverse location y_0. From an argument similar to the one we used to derive Eq. (27), one finds that the modulation disappears when $L_\perp \gg \lambda_p$, which is a less strict condition than Eq. (27). This result is confirmed by Fig. 2(b) showing that, indeed, γ_∞ does not oscillate with y_0 when the laser pulse is sufficiently wide.

C. Pulse distortion

Finally, let us estimate the conditions under which the pulse distortion, ignored above, does remain negligible. Due to the group velocity dispersion (GVD), the longitudinal spreading of the pulse becomes significant on the distance L_s found from[40]

$$L_s \frac{dv_g}{d\omega}\frac{\Delta\omega}{c} \sim L_\|, \tag{28}$$

where $\Delta\omega/c \sim L_\|^{-1}$ is the pulse spectrum width. Thus,

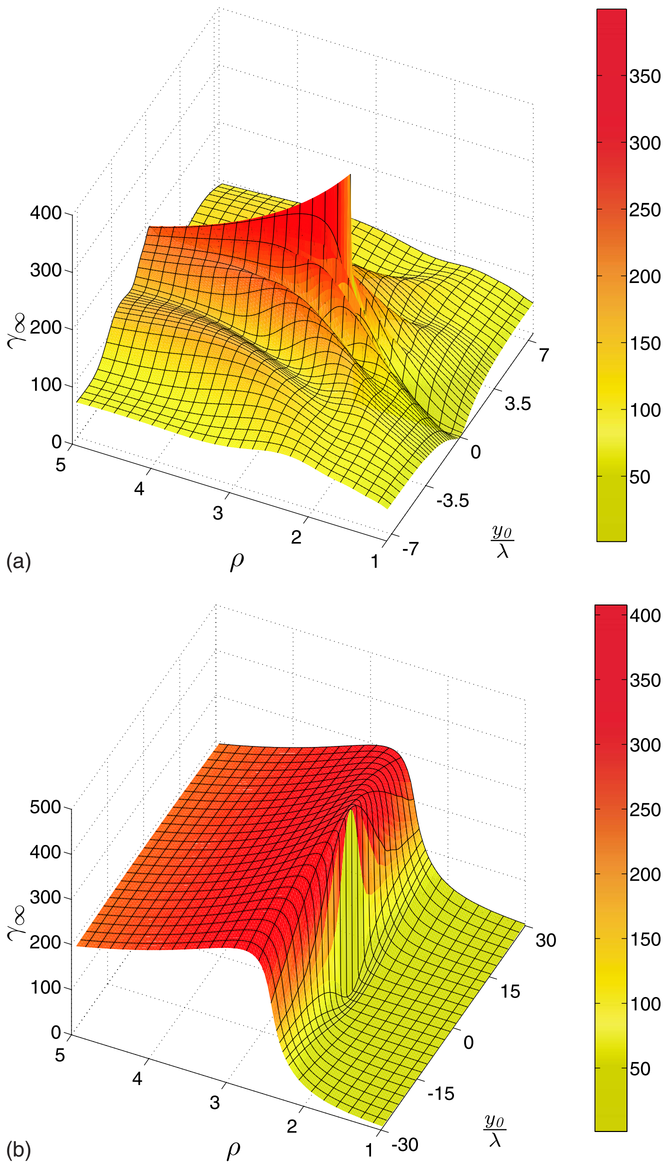


FIG. 2. (Color online) The normalized energy γ of electrons accelerated by a laser pulse $a=a_{\max}\exp(-x^2/L_\parallel^2-y^2/L_\perp^2)$ with $L_\parallel=3\lambda$ in a low-density plasma vs the initial electron longitudinal momentum $\rho\equiv p_x/(mc)$ and the initial transverse displacement y_0/λ for $a_{\max}=10$ and $\alpha=10^{-3}$ ($\lambda\equiv 2\pi/k$ is the laser wavelength): (a) $L_\perp=25\lambda$, (b) $L_\perp=100\lambda$.

$$L_s \sim L/\epsilon, \tag{29}$$

which is much bigger than the acceleration length L [Eq. (16)]. Therefore, GVD does not affect the electron acceleration. As for the pulse transverse spreading due to diffraction, it becomes significant on the Rayleigh length $L_R \sim kL_\perp^2$ and, thus, can be neglected if $L \ll L_R$. Hence, required is

$$kL_\perp \gg \gamma_g \epsilon^{-1/2}, \tag{30}$$

which is a condition weaker than Eq. (27).

VII. CONCLUSIONS

What we have shown is that relativistic electrons can be accelerated by an ultraintense laser pulse in the "supra-bubble" regime, that is, in the blow-out regime *ahead* of the plasma bubble (as opposed to the conventional method, when particles remain *inside* the bubble). The acceleration is caused by the ponderomotive force of the pulse, via the so-called snow-plow mechanism. The maximum energy gain, $\Delta\gamma \sim \gamma_g a$, is attained when the particle Lorentz factor γ is initially about γ_g/a, where γ_g is the pulse group speed Lorentz factor, and a is the laser parameter, proportional to the laser field amplitude. The scheme operates at $a \lesssim \gamma_g$, yielding $\Delta\gamma$ of up to that via wakefield acceleration for the same plasma and laser parameters, $\Delta\gamma \sim \gamma_g^2$. The interaction length L is shorter than that for the wakefield mechanism, which makes the supra-bubble regime more favorable for a single-stage accelerator. For plasma densities of, say, 10^{17} cm^{-3}, this limits the maximum energy gain to about half GeV. On the other hand, since L grows with the particle energy, staging the acceleration in the supra-bubble may not be feasible; thus, for acceleration beyond the said energies, LWFA is preferable.

ACKNOWLEDGMENTS

This work was supported by the NNSA under the SSAA Program through DOE Research Grant Nos. DE-FG52-04NA00139 and DE-FG52-08NA28553.

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