On the anomalous fast ion energy diffusion in toroidal plasmas due to cavity modes

2010 Plasma Phys. Control. Fusion 52 055014

(http://iopscience.iop.org/0741-3335/52/5/055014)

IOP Publishing Plasma Physics and Controlled Fusion

Plasma Phys. Control. Fusion **52** (2010) 055014 (13pp) doi:10.1088/0741-3335/52/5/055014

On the anomalous fast ion energy diffusion in toroidal plasmas due to cavity modes

N N Gorelenkov, N J Fisch and E Fredrickson

Princeton Plasma Physics Laboratory, PO Box 451, Princeton, NJ 08543-0451, USA

E-mail: ngorelen@pppl.gov

Received 18 September 2009, in final form 10 March 2010
Published 9 April 2010

Abstract
An enormous wave–particle diffusion coefficient along paths suitable for alpha channeling had been deduced in mode-converted ion Bernstein wave experiments on the Tokamak Fusion Test Reactor (TFTR). The only plausible explanation advanced for such a large diffusion coefficient was the excitation of internal cavity modes which induce particle diffusion along identical diffusion paths, but at much higher rates. Although such a mode was conjectured, it was never observed. However, recent detailed observations of high frequency compressional Alfvén eigenmodes (CAEs) on the National Spherical Torus Experiment (NSTX) indirectly support the existence of the related conjectured modes on TFTR. The eigenmodes responsible for the high frequency magnetic activity can be identified as CAEs through the polarization of the observed magnetic field oscillations in NSTX and through a comparison with the theoretically derived frequency dispersion relation. Here, we show how these recent observations of high frequency CAEs lend support to this explanation of the long-standing puzzle of anomalous fast ion energy diffusion on TFTR. The support of the conjecture that these internal modes could have caused the remarkable ion energy diffusion on TFTR carries significant and favorable implications for the possibilities in achieving the alpha channeling effect with small injected power in a tokamak reactor.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the final days of experimentation on the Tokamak Fusion Test Reactor (TFTR) [1], a series of experiments was carried out to test certain aspects of the predicted but undemonstrated alpha channeling effect [2]. The alpha channeling effect was predicated on mechanisms whereby waves could cool and extract energy from alpha particles, by-products of the deuterium–tritium (DT) fusion reaction, on a time scale faster than the collisional slowing down of these particles on electrons. The same waves might then be damped by the fuel ions, thereby allowing tokamaks to operate with ions hotter than electrons, the so-called 'hot-ion mode' [3]. This effect exploits population inversions via careful phase space engineering of selected particle diffusion paths that connect energetic alpha particles in the center with cold particles on the periphery, thus causing the alpha particles in the center to diffuse to the periphery while at the same time losing energy to the wave. The effect, if realized, could result in a much relaxed fusion ignition criterion and in significant savings in the cost of electricity produced by fusion energy.

Important to the viability of the alpha channeling effect was not only that suitable waves might be excited to produce the diffusion paths that accomplish the cooling but also that the power required in exciting these waves not be too large. If the wave energy is too small, then the wave–particle diffusion coefficient might be too small to accomplish the effect on the collisionless time scale. The mode-converted ion Bernstein wave, which could be excited on the low-field side in TFTR in a D–He3 plasma, had many of the necessary wave characteristics to extract energy from energetic ions [4]. Moreover, this wave, after extracting energy from alpha particles, could then damp on tritium ions [5]. In a reactor, this wave would be used together with low-frequency waves to accomplish significant diversion of alpha particle energy into the fuel ions [6, 7].

The TFTR experiment could not test the cooling concept directly because there were too few alpha particles to slow down by waves and be detected. However, the key characteristics of the IBW wave were tested and validated. Enhanced losses of fast ions, determined to be deuterium ions, were observed during ion heating experiments [8, 9]. In the so-called 'alpha channeling' experiments, the wave phase velocity was actually chosen to connect cold ions (deuterium 100 keV beam ions) in the tokamak center with hot ions (2.2 MeV) at the periphery [10]. This is the opposite phasing to what would be required for the channeling effect in a reactor, but was what could be achieved in a beam-driven experimental device. Thus, in the presence of the ion Bernstein wave, the ions were expelled and heated. The expelled ions were detected at the predicted particle energy, particle velocity pitch angle with respect to the magnetic field and the poloidal angle of the detector position. The fact that the data could corroborate the theoretical prediction in such detail meant that the diffusion paths were indeed established precisely as predicted.

The unequivocal result that was completely unexpected, however, was that the diffusion along the predicted diffusion paths was 25 $\mathrm{MeV}^2\,\mathrm{s}^{-1}$, which was about a factor of 50 greater than what might be expected with the 3 MW of injected power near the ion cyclotron frequency. Such an anomaly resisted all conventional explanations. Because the theories of wave propagation and quasilinear diffusion were well established, the largest possible diffusion coefficient could be predicted, and the kinds of effects that would affect this prediction, such as unintended reflection or dissipation effects, would only *lower* the predicted effect, making the established result even more quizzical [11].

What was left, when all conventional explanations were ruled out, was the remarkable explanation that the tokamak might be ringing at an internal contained mode. Such a mode would have the same frequency and toroidal mode number as the excited ion Bernstein wave, and therefore, upon the mode being excited resonantly to high amplitude, any interaction with particles would retain the same diffusion paths. But if the mode were a high-Q cavity mode, the wave amplitudes could be far higher than for the mode-converted ion Bernstein wave. Such a mechanism was in fact shown to be plausible [12]. However, if this in fact were the operative mechanism, then the implications for the alpha channeling effect in a reactor would be significant, since the effect could be accomplished with far less injected power (on the order of the anomalous enhancement or a factor of 50) than previously imagined. A more detailed description of these implications is given elsewhere [13, 14]. However, by the time this conundrum was noticed, and the theory of a ringing mode advanced as plausible, TFTR had already been shut down and thus no direct observation of the postulated internal mode could be had. Thus the explanation of the ringing mode remained a remarkable, plausible, but in the end unsubstantiated conjecture, albeit with no alternative explanation even remotely plausible.

This paper provides arguments that support the conjecture that the remarkable, anomalously large (factor 50) diffusion coefficient seen in the TFTR alpha channeling experiments was in fact due to coupling to an internal mode. The support arises from recent observations of CAE modes on the National Spherical Torus experiment (NSTX) [15]. These observations link the cavity modes with the internal modes that were conjectured to have been on TFTR and causing the anomalous diffusion. Thus, although the high toroidal mode number high-Q cavity modes conjectured to explain the very high diffusion coefficient in TFTR were never studied directly, their likely presence can now be inferred by the data on related modes in NSTX.

We emphasize that the experimental evidence offered here is only that the modes related to the conjectured high-Q cavity modes on TFTR have in fact been observed on NSTX. There is no evidence yet that the observed modes on NSTX are responsible for huge anomalous diffusion of fast ions. The high-Q cavity modes in TFTR were driven to high amplitude by the 3 MW of mode-converted ion Bernstein waves at the same frequency and at the same toroidal mode number. There is no such wave excitation in NSTX. What we do observe is that at a low amplitude level these modes do exist in NSTX—and that in itself is very interesting and revealing for it indicates that under proper excitation such lightly modes could reach high amplitude, as speculated on TFTR. RF in NSTX is applied at high harmonics of the thermal ion fundamental frequency, known as high harmonic fast wave (HHFW) heating. It is interesting, though, that the HHFW application did show power absorption by beam ions [16]. However, because of the lack of fast diagnostic the time resolution of the beam ion distribution was only 10 ms, so that the low limit for the energy diffusion emerges at its value $>0.1\,\mathrm{MeV}^2\,\mathrm{s}^{-1}$. At present HHFW frequency one cannot expect that the CAE eigenmodes are excited because of the presence of multiple cyclotron resonances. Instead, short wavevector IBW waves are expected in which more complicated wave-particle interaction mechanism should be invoked.

Both above and below the ion cyclotron frequency, instabilities of a certain class of cavity modes were observed in NSTX and were shown to be driven by fast beam ions [17–21]. An initial study of these instabilities was based primarily on the analysis of the spectra of the magnetic signal of these instabilities. It was shown experimentally that the instability frequencies correlate with the plasma Alfvén velocity, which was used to identify this magnetic activity as instabilities of compressional branch of Alfvén eigenmodes (CAEs, also called fast Alfvén or magnetosonic eigenmodes). The instabilities of these modes were suggested to be responsible for the ion cyclotron emission (ICE) in tokamaks [22–26], commonly acknowledged to be driven by fast ions such as beam ions ICRH minority ions and fusion products.

The ICE was observed in tokamaks with spectrum peaks at the harmonics of the edge background ion cyclotron frequency. It had been theoretically predicted and now observed that many CAE instabilities with narrow spectrum peaks will overlap and form a broader peak near each ion cyclotron frequency harmonic. The main difference between the more recent observations from STs and those of earlier studies is that the frequency spectrum of the observed CAE instabilities in high toroidicity plasma is discrete, so that properties of each mode can be measured separately. Each CAE mode corresponds to a line in the spectrum of the magnetic field oscillations measured at the edge of the plasma and should have three ‘quantum’ mode numbers: toroidal, n, poloidal, m, and radial, s [18, 27]. The eigenfrequency of the observed

CAEs is primarily determined by the product of the Alfvén velocity and the wavevector, k_{mns}, at the mode location as $\omega_{\rm CAE} = k_{mns} v_{\rm A}$, where k_{mns} corresponds to three mode numbers. From previous studies it is known that, due to high magnetic fields in tokamaks, the ICE frequency spectrum is populated more densely by the unstable modes, which makes the ST plasma very attractive for studying the properties of individual CAE modes and for verifying the theoretical predictions. In particular, NSTX presents a unique possibility to study mode numbers, structure, polarization, amplitude, etc. We note that, based on the CAE observations on NSTX, one would expect that ICE spectrum contains more complicated, fine structures than previously reported [28, 29]. This can be investigated in plasmas with the intermediate values of the magnetic fields (and aspect ratios), for example, with the equilibrium magnetic field $B_0 = 1$–2 T, in DIII-D plasmas.

Note that there are several mechanisms for slowing down energetic particles by waves. The concept of alpha channeling as originally envisioned supposed a large fraction of energy transfer from alpha particles to thermal ions [2]. This required arranging diffusion paths that could connect from birth energy in the center to small energy at the periphery. Other interesting mechanisms for releasing alpha particle energy [30, 31] or forcing them to be lost have also been proposed. The full beneficial effect, however, is only realized when the particles are lost to the boundary concomitant with substantial loss of energy, which requires a very special diffusion path with one wave [2] or a very special set of waves [6]. The substantial loss of energy must occur prior to energy being lost through collisions, which means that the wave diffusion coefficients must be large enough to drive the particles to the periphery prior to a collisional slowing down time. The remarkable possibility that we check here is, of course, if coupling to an internal mode could facilitate such a large diffusion coefficient.

Thus, the goal of the paper is to show that ICE in tokamaks and, in particular for a certain class of high, sub-ion cyclotron, frequency instabilities observed in NSTX, can in fact be described by the same theory of CAE instabilities driven by fast ions via the cyclotron resonance. This lends important support to the existence of the cavity CAE modes conjectured to play a critical role in TFTR by driving the apparently enormous anomalous diffusion [12]. It should be emphasized that by linking the conjectured modes on TFTR to the observed modes on NSTX, we have also for the first time a concrete description in detail of what dispersion relation must govern these conjectured modes, and hence the conjecture now has substantial predictive value.

The paper is organized as follows: in section 2, we review the observations of CAE activity on NSTX. In section 3, we review the theory, showing that related modes would be predicted to exist on TFTR. In section 4, we further discuss alpha channeling, review the uses of multiple waves, including the use of low-frequency waves. In section 5, we suggest what further experiments might be made to validate the theories discussed here. Section 6 summarizes our main conclusions.

2. High frequency mode observations in NSTX

Coherent high frequency modes have been observed in NSTX both with high bandwidth magnetic pick-up coils and with a reflectometer [17, 18]. Similar observations were reported from MAST [21, 32]. Observations by edge Mirnov coils show a broad and complicated frequency spectrum of such modes between 400 kHz and up to 2.5 MHz in a discharge with the fundamental cyclotron frequency of background deuterium ions (and beam deuterium ions) to be $f_{\rm cD} = \omega_{\rm cD}/2\pi = 2.3$ MHz, calculated at the vacuum magnetic field at the geometrical axis of NSTX for $B_{\rm g0} = 3$ kG. A very rich spectrum of modes is typically observed during the neutral beam injection (NBI). The mode frequencies correlate with the Alfvén velocity as the

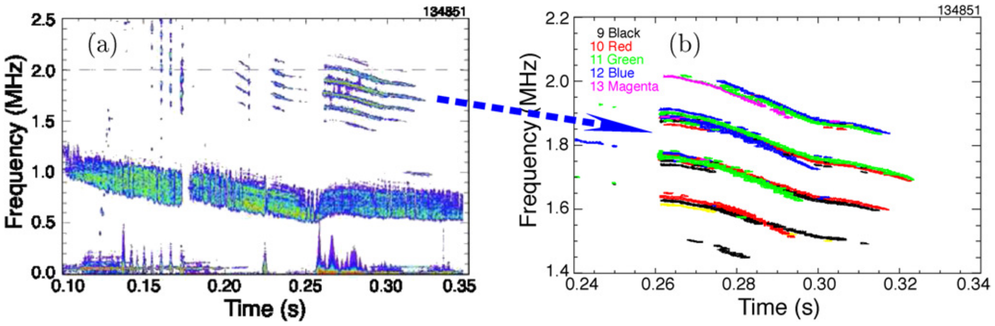

Figure 1. Spectrum of the magnetic field perturbation measured by the edge pick-up coils in NSTX discharge #134851. Shown in (*a*) is the oscillation spectrum and shown in (*b*) are the toroidal mode numbers for one cluster of instabilities, which are identified as CAEs.

magnetic field and plasma density are varied. Typically the range of operational parameters for the experiments are toroidal current $I_{\mathrm{p}} = 0.7$–1 MA, toroidal field $B_{\mathrm{g0}} = 3$–5 kG, central electron density $n_{\mathrm{e0}} = (1\text{–}5) \times 10^{13}\,\mathrm{cm}^{-3}$, central electron temperature of up to $T_{\mathrm{e0}} = 1$ keV. The plasma is heated with deuterium beams whose total power is typically $P_{\mathrm{b}} = 1.5$–3 MW, but can be as high as 6 MW. We use subscript b for beam ions parameters.

The instability is very sensitive to the distribution function of NBI ions. It was also observed that the frequency of high frequency unstable modes does not change due to the change in the NBI injection angle. It was concluded that CAEs are the plasma cavity modes, i.e. CAEs are normal modes of the plasma and may exist without fast particles. The typical magnitude of the perturbed magnetic field at the plasma edge is small $\delta B_{\parallel}/B \sim 10^{-6}$ [17].

In the identification of CAE instabilities, two main characteristic properties are typically used: frequency dispersion and a polarization. The latter is predicted to be predominantly linear and to have a magnetic field component along the equilibrium field line $\delta B_{\parallel} \gg \delta B_{\perp}$. We show an example of the high frequency spectrum in figure 1 in plasma with strong NBI heating. Figure 1(*b*) shows a zoomed region of the spectrum with the toroidal mode numbers for each line, ranging from 9 to 13. The perturbed magnetic field of these modes is mostly in the direction of the equilibrium magnetic field, which is consistent with the identification of these modes to be of the compressional branch. One example of the polarization measurements is shown in figure 2. The angle of the perturbed magnetic field, which includes independently measured poloidal and toroidal components, is close to the angle of the equilibrium magnetic field taken slightly inside the plasma from the edge. A more accurate comparison requires additional detailed analysis and, most importantly, internal mode structure measurements, which has not been published yet.

It is difficult to measure the poloidal wavelength, but the data show that the typical numbers are in the right range with the predictions by theory $m = 2$–10 (for different modes), so that the simple CAE dispersion relation

$$\omega = k_{mns} v_{\mathrm{A}}, \tag{1}$$

where

$$k_{mns} \simeq \sqrt{(m/\kappa r)^2 + (n/R)^2 + (sC/a)^2} \tag{2}$$

gives a similar estimate for m, where $\kappa \simeq 2$ is the ellipticity, and $C \simeq 2$–3 accounts for the density profile effects on the radial mode structure. The comparison with the theory shows that in the frequency range below f_{cD} only the two lowest bands of radial solutions are

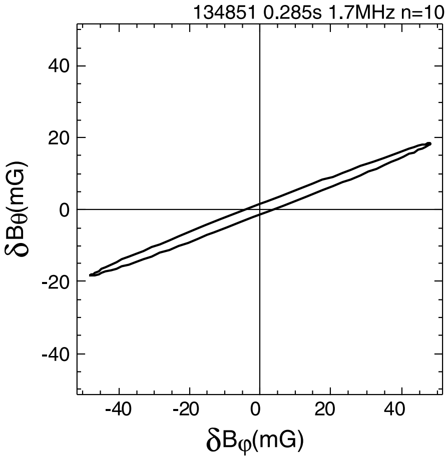

Figure 2. Edge perturbed magnetic field polarization of the observed instability in NSTX discharge #134851 at $f = 1.7$ MHz, $t = 0.285$ s, and having $n = 10$. It is among the peaks shown in figure 1.

typically observed, $s = 1, 2$, for the measured instabilities below the thermal ion cyclotron frequency [27].

The dispersion relation, equation (1), differentiates CAEs from another high frequency branch known as GAE (global AE), which has the dispersion relation

$$\omega = k_{\|} v_{\rm A}. \tag{3}$$

Because $k_{\|} \simeq m/qR - n/R$, the frequencies of GAEs evolve differently in time (for different m, n pairs) than do the frequencies for CAEs, for which k_{mns} remains approximately fixed over the plasma discharge evolution. The difference in frequency evolution of CAE and GAE instabilities is an important property which can be used experimentally to identify the mode based on the frequency spectrum evolution. In the spectrum of figure 1(*b*), CAE lines have similar evolution in time, which is another important property used for their identification.

3. CAE theory

One of the main advantages of the low aspect ratio plasma experiments for CAE studies is that the equilibrium magnetic field is typically low at the expected mode location on the low-field side, which means that the Alfvén velocity will be small compared with the velocities of the super-thermal fast ions, such as beam ions in NSTX. Such conditions facilitate a detailed study of individual CAE mode properties, which are present in larger aspect ratio tokamaks as well, but where they typically much more densely populate the magnetic frequency spectrum. The distinction should be made here that in this section we analyze the naturally driven modes by the energetic ions, so the most likely driven modes are the ones with smallest damping rates.

Recent more detailed numerical studies of the CAE spectrum support the theoretical picture of these modes as standing, cavity modes in the poloidal and radial directions characterized by the toroidal, poloidal and radial quantum numbers [27, 33]. Because of different scales associated with those dimensions the radial mode separation in frequency is predicted to be the largest ~1 MHz, followed by the poloidal mode number separation ~100–200 kHz and toroidal number separation up to 50–100 kHz. This frequency separation is in qualitative agreement with the measurements [18]. In the presented example, figure 1, the radial number is expected to separate two clusters of modes at 1 and 2 MHz, the poloidal mode number separates modes by 150 kHz (could be clearly seen in figure 1(*b*)) and the toroidal mode number separation is expected to be the finest with $\Delta f \simeq 10$ kHz.

Important for the spectrum and the radial structure of CAEs is the Hall effect, which makes the CAE structure more complicated [33]. One of the effects due to the Hall term is that CAE frequency is split for the opposite signs of m values. It is degenerate with the m sign in the ideal MHD theory.

The allowed frequencies of the observable spectrum of CAE instabilities depend to a considerable extent on the damping mechanisms, which shape the spectrum due to typically exponentially strong dependences on the plasma parameters [24, 34]. Of particular importance is the thermal ion cyclotron damping, which prevents CAE excitation when the mode frequency matches the harmonics of background ion cyclotron frequencies, so that at $f < f_{\rm cD}$ CAEs avoid such damping in toroidal plasmas in general. The CAE drive sensitively depends on $k_\perp \rho_{\rm b}$, which is sufficiently large in STs in general due to the low equilibrium magnetic field. Here $\rho_{\rm b}$ is the beam ion Larmor radius. A careful comparison of identified CAE activities in NSTX shows that the theoretically predicted excitation condition, $k_\perp \rho_{\rm b} > 1$, correlates with the range of the observed frequencies [20] over the range of plasma parameters.

Because the CAE spectrum is typically rich, i.e. various modes with close frequencies can be unstable, we offer a qualitative diagram for the expected most unstable CAE frequency in two devices, TFTR and NSTX (in the case of TFTR we refer to fusion product driven ICE instabilities). One of the most striking differences between these two machines is the ratio of the fast ion velocity to the Alfvén velocity, which in general is higher in STs due to the low magnetic field (low aspect ratio). To reflect this in the diagram, we chose the aspect ratio for the abscissa axis. It turns out that the ratio of fast ion to Alfvén velocity can be connected for both experiments as $v_\parallel / v_{\rm A} \simeq 4a/R$ (this will recover both NSTX beam ions and TFTR fusion alphas), so that the x-axis is labeled with both aspect and velocities ratio.

First, we consider the modes naturally driven by fast ions in tokamaks, that is the cavity modes with low damping. Based on the theoretical understanding, it seems plausible to expect that the CAE spectra will be different in plasmas with different aspect ratios.

It follows from the theory that fast ions exchange energy with CAEs under the cyclotron resonance condition with strong Doppler shift $\omega - \omega_{\rm c} = k_\parallel v_\parallel \simeq k_\parallel v$. The excitation condition in which fast ions transfer energy to the mode implies that $k_\perp \rho_{\rm b} = O(1)$ (it should change from 1 to 2 for CAEs to be unstable). This in turn implies that the frequency of the modes of interest, i.e. those which can be driven by fast ions, should satisfy $\omega \simeq \omega_{\rm c}(1 - |k_\parallel|/k)$. On the other hand, damping on electrons can be minimized if the phase velocity of CAEs is either too small or too large in comparison with the thermal electron velocity as in the example in [18]. In that example the damping rate was taken fixed at 1% to find the condition for $k_\parallel$. This resulted in $\zeta_{\rm e} = (k_\perp / k_\parallel)\sqrt{m_{\rm e}/m_{\rm i}\beta_{\rm e}} > 1.5$ (region above curve 1 in figure 3) or $\zeta_{\rm e} < 0.45$ (region below curve 2 in figure 3). Note that, in plotting these dependences in figure 3, we assumed that the nominal tokamak plasma beta scales with the aspect ratio, namely $\beta_{\rm e}(\%) \sim 10/(R/a)$, which is at least empirically justified based on the results from TFTR [1] and NSTX [15]. Because of the weak dependence on electron beta, a more accurate expression is not required.

A second important condition restricting the mode frequency follows from the coupling to the kinetic shear Alfvén wave (KAW) at the edge. Let us rewrite the resonance condition in the form

$$\omega = \omega_{\rm c} - \omega \frac{|k_\parallel|}{k_\perp} \frac{v_\parallel}{v_{\rm A}}. \tag{4}$$

It is not straightforward to estimate the local $k_\parallel$ for CAEs, which on average must be smaller than its value at the conversion region from CAE into KAW. The avoidance of the conversion requires $k_\parallel / k_\perp \lesssim v_{\rm A} / v_{\rm A\,edge}$ (typically less than 1), where $v_{\rm A}$ is at the CAE location and $v_{\rm A\,edge}$

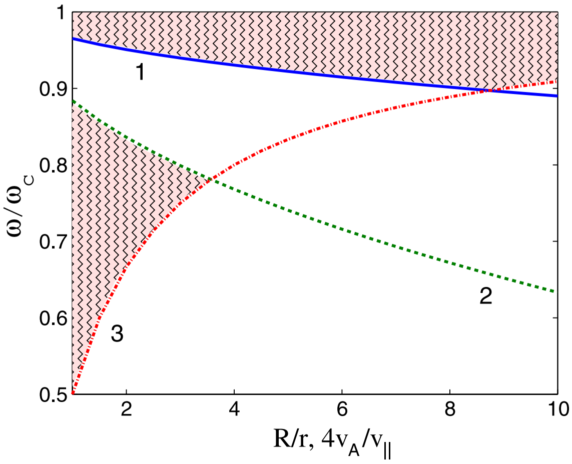

Figure 3. Regions of allowed CAE frequencies as follows from the limited mode damping. Shown are regions in the plane of aspect ratio R/a (also the ratio of fast ion parallel velocity component to the Alfvén velocity) and the mode frequency normalized to the ion cyclotron frequency.

is at the last closed surface of the plasma. Experiments in NSTX and TFTR differ in the $v_\parallel/v_A$ ratio. We obtain the inequality describing the limiting condition for the CAE mode frequency

$$\omega > \omega_c/(1 + Ca/R), \tag{5}$$

where $C = 4k_\parallel/k_\perp$. As we argued above to avoid strong continuum damping from KAW it is required that $k_\parallel < k_\perp$, which provides a high limit for C. From the low end C is limited by small $k_\parallel$. Thus, because our diagram is rather qualitative it seams reasonable to use $C = 1$ in the estimates. The parameter domain, which follows from this condition, is the region above curve 3 in figure 3 plotted for $C = 1$. If one is to consider beam ion heating, instead of curve 3 one needs to plot another curve, closer to $\omega/\omega_c = 1$ at the high $v_\parallel/v_A$ ratio limit. In tokamaks with less energetic ions in comparison with fusion alphas in TFTR constant C will be less than unity. This means that the CAE instability frequency and curve 3 will be closer to the cyclotron frequency of the fast ions. We note that figure 3 is rather schematic without any specific plasma profiles.

Figure 3 illustrates how NSTX and TFTR are related in terms of the regions of most likely unstable CAE modes, based on minimizing the damping rates due to thermal particles. The regions of allowed frequencies for CAEs are shown as shaded areas in figure 3 and are obtained by overlapping the three parameter domains. It is clear that in general in conventional tokamaks such as in TFTR CAEs are not expected to be excited far from the cyclotron frequency due to rather high Alfvén velocity, i.e. low Doppler shift in comparison with the mode frequency. Only by creating a special condition with a significantly lower magnetic field (lowered from 2.1 to 0.6 T) in DIII-D in similarity experiments a spectrum similar to that in NSTX was observed [35].

Figure 3 shows the regions in which modes can be made unstable in the presence of energetic ions, or, put differently, modes which are high-Q cavity modes. In the case of the TFTR mode conversion experiments in which the ion Bernstein wave was excited, the driven frequencies corresponded to the frequencies of high-Q cavity CAEs, which would be susceptible to reaching large amplitude either by being driven by resonant energetic ions or by a resonant wave such as excited by the IBW antenna. CAEs with frequencies above the edge cyclotron frequency have the same properties as CAEs excited by beam ions in NSTX.

The internal modes that may have been excited on TFTR would have resided above the shaded regions in figure 3.

4. Alpha channeling

The conjecture that high-Q eigenmodes were excited on TFTR was plausible only because no other explanation could be offered to explain the very high diffusion coefficient [11, 12]. However, apart from this plausibility argument, no other evidence was offered, since the contained modes were not directly observed. Were a contained mode excited, though, it would of necessity have the same frequency as the mode conversion ion Bernstein wave (MCIBW) on TFTR. Similarly, since a tokamak is axisymmetric, it would have the same toroidal mode number. Since the diffusion paths of resonant particles are determined uniquely with specification of the frequency and toroidal mode number only, the diffusion paths caused by the excited high-Q cavity mode would be identical to those caused by the mode-converted IBW wave. Only the magnitude of the diffusion coefficient would differ, in a way that might explain the anomalous diffusion coefficient, while retaining agreement with the other observations on TFTR.

What figure 3 tells us is that CAE modes likely exist at the same frequency as the IBW wave. The shaded region at the upper part of figure 3 shows that CAE modes with frequencies on the order of (or higher than) the cyclotron frequency can exist at many different minor radii. Why is this frequency needed? Since after undergoing mode conversion, the mode-converted IBW wave in TFTR would be locally near the ion–ion hybrid resonance. In traversing the hybrid resonant region, it would then approach from the low-field side the frequency of the ion species with the lower gyro-frequency—or in the case of the ‘beam-blip’ experiments in D-He3 in TFTR that would be deuterium. Thus, the mode-converted wave would need to have frequency somewhat higher than the local deuterium gyro-frequency, and through Doppler effects could interact resonantly with energetic ions. But this frequency region is exactly the upper shaded region of figure 3, which only bounds from below in frequency the CAE region. The fact that high frequency modes (conjectured here as CAEs in the TFTR case) were observed both on TFTR and on NSTX in the shaded regions lends confidence to this picture—and in turn lends confidence to the likelihood of excitation within this upper region on TFTR.

Besides the frequency, it is important that the CAE mode also exists with the requisite toroidal n. Here, the fact that the upper region in figure 3 is so dense in lightly damped modes means that both high n and high m modes might be found. The conjectured modes would have n about 40 and m about 25, which could be accommodated. This is the key result of this paper: the fact that the contained modes could exist on TFTR near the excited IBW frequency and with the required mode number n lends support to the conjecture that they might have been excited as a high-Q cavity mode.

There are very significant implications for the prospects of using the alpha channeling mechanism in a reactor: controlling the fast ion energy diffusion, thereby effecting the energy channeling, might be accomplished at ~50 times less injected power than what would have been necessary without the contained mode excitation. This conclusion is based on the assumption that there is a linear relation between the injected RF (radio frequency) power and the energy contained in the excited cavity CAE modes, which is in turn proportional to the energy diffusion coefficient [12]. The significant reduction (order of magnitude or more) in the required ICRH power can be expected, which makes the idea of alpha channeling more attractive.

Note that in a reactor, as opposed to in TFTR, a 2-wave scenario for controlling the alpha particles would likely be used. In a 2-wave scenario, one wave would be used for moving

Table 1. NSTX and C-mod parameters for the proposed alpha-channeling experiment. Note that energy of fast ions can be changed from its nominal value given in the table. f_{bc} is the cyclotron frequency of fast ions.

Device	Plasma	Heating	Fast ions	f_{bc} (MHz)	Energy (keV)	B (T)	$\frac{v_A \kappa}{2\pi a}\vert_{\mathrm{edge}}$ (MHz)	$\vert m\vert_{\vert \mathrm{at}\, f_{\mathrm{bc}}}$
NSTX	He^3/D mix	NBI/ICRH	NBI ions, D	2.1	80	0.5	0.4	5
C-mod	He^3	ICRH	H-minority	57	50	5	3.6	16
TFTR	He^3/D mix	NBI/ICRH	NBI ions, D	27.5	100	4.8	1.4	20

particles radially without much energy loss, whereas the second wave might move them radially only with significant energy loss. The first wave could be a low-frequency Alfvén mode which in any event would not draw the main power requirement. The second wave would likely be the MCIBW wave [7]. If an internal mode could be excited in the manner conjectured here, then the power requirements for exciting the MCIBW wave would be significantly less and hence the alpha channeling effect could be accomplished with relatively small power investment.

5. Implications for present day experiments

Although the observations on TFTR and NSTX lend support to the high-Q eigenmode excitation, the validity of the prediction requires more complete tests. It will be important to test the interpretation supported here of the TFTR results on the anomalous fast ion energy diffusion. We show here that definite predictions can be made and tested on existing toroidal devices such as NSTX as well as on more conventional tokamaks, such as C-mod and DIII-D.

First, we identify the cavity mode parameters, which would allow lightly damped CAE excitations by fast ions. As a guidance for the experiments, we consider two plasmas similar to NSTX and C-mod, which differ first of all by the value of the equilibrium magnetic field, 0.5 T and 5 T, respectively. We also compare them with the TFTR plasma ($B = 4.8$ T), where the energy diffusion of the fast beam ions was measured. Table 1 summarizes some relevant parameters of the envisaged experiments. When computing the poloidal mode numbers we assumed that $n \ll m$ as required by edge antenna coupling. Obviously the magnetic field parameters can be adjusted if required.

We evaluate the poloidal mode number of CAEs with the frequency taken at the edge cyclotron frequency of the fast ions. It is given in the table as an absolute value, $|m|_{\mathrm{at}\, f_{\mathrm{bc}}}$, because, for CAEs in STs that are standing waves in the poloidal direction, there are two dominant poloidal harmonics with signs of poloidal mode numbers, $\pm m$. However, in tokamaks one sign of m number dominates [27, 33]. At higher ICRH frequency $|m|$ increases linearly with f. For excitation of the cavity modes in tokamaks, one should plan to have the mode frequency near the edge fast ion cyclotron frequency to avoid ion cyclotron absorption. Another way of exciting CAEs is through the mode conversion from IBW. In STs and NSTX in particular, one can also try lower frequency modes, such as $f = 700\text{–}1000\,(\mathrm{kHz}) = (1/3\text{–}1/2) f_{\mathrm{bc}}$, which are already observed and are expected to have good coupling. Thus, in tokamaks one can aim at small $k_{\|}$ or $n = m/q$, where q is to be taken at the point of CAE localization $r/a = 1/(1+\sigma)$, σ is a plasma density exponent $n_{\mathrm{e}} = n_{\mathrm{e}0}(1 + r^2/a^2)^{\sigma}$. Typically in experiments a broad spectrum of n numbers is present in the antenna signal. Strong excitation is expected of modes with small $k_{\|}$ and weak electron Landau damping.

The toroidal canonical angular momentum is related to the energy change in the case of the axisymmetric plasma (see, for example, [36])

$$\Delta P_\varphi \frac{\omega}{n} = \Delta \mathcal{E}. \tag{6}$$

We can rewrite this condition in a more simple approximate way

$$\frac{\Delta P_\varphi}{P_\varphi} \frac{\omega}{\omega_{\rm bc}} \frac{r^2}{\rho_{\rm b}^2} \frac{1}{qn} = \frac{\Delta \mathcal{E}}{\mathcal{E}}, \tag{7}$$

where q is the safety factor on a particle orbit and the assumption that orbit width is small in comparison with the plasma minor radius was made, which requires $\Delta_{\rm b} \ll a$, where $\Delta_{\rm b} = q\rho_{\rm b}$ is the passing particle drift orbit width. If the diffusion step is given by equation (7) the total number of steps required for a fast particle to leave the plasma is

$$N_{\rm steps} = \frac{P_\varphi}{\Delta P_\varphi} = \frac{\mathcal{E}}{\Delta \mathcal{E}} \frac{a^2}{\Delta_{\rm b}\rho_{\rm b}} \frac{1}{n},$$

where we used that $\omega \simeq \omega_{\rm bc}$. If a particle is accelerated strongly enough it can be lost even from the inner plasma regions in which case for trapped particles the following condition is satisfied: $q\rho_{\rm b}/a \simeq \sqrt{a/R}$.

The typical toroidal mode number spectrum is relatively wide in present day antennas. Hence, in order to demonstrate the energy diffusion we need to maximize the change in the energy at the point of particle impact on the plasma wall. If we allow for $\Delta P_\varphi \sim P_\varphi$ for lost particle, strong energy diffusion would require high (cyclotron) frequency and moderate n number ~10. The next step could be the controlling and optimizing of the fast ion diffusion paths in phase space. The antenna would need to be phased appropriately to achieve $n = 30$–40 [12].

Note that on C-mod, because of the absence of the fast ions with fixed injection energy, one would need to create an H-minority at high energy first. But the experiments on C-mode could be planned in analogy with TFTR, where the anomalous fast ion energy diffusion was observed. On C-Mod, the mode-converted IBW could be situated as it was in TFTR, somewhat on the low-field side. On NSTX, further detailed study of such diffusion is possible due to a large frequency separation between the CAE modes so that the resonant layer could be shifted throughout the plasma.

6. Summary

We demonstrated the close connection between the observed CAE modes in NSTX and ICE instabilities in TFTR. We argued that both belong to the same branch, which may exist at harmonics of the fundamental ion cyclotron frequency in TFTR and at its fraction in STs. The new evidence of the CAE polarization together with the magnetic spectrum structure unambiguously identified these modes as the compressional Alfvén branch in NSTX.

We demonstrated how the same fast ion driven CAE instability theory reconciles CAE excitation in both STs and TFTR with the above-mentioned properties. The observations of CAEs on NSTX, together with the agreement in essential details with theoretical predictions, substantiates their existence on TFTR. Moreover, these observations imply that CAEs have very low excitation thresholds with very weak damping, which makes them an ideal candidate for the mechanism suggested by Clark *et al* [12].

In addition to the important implications for interpreting the TFTR data, it also follows from this connection that one might expect and should look for more complicated fine structures in the ICE spectrum, perhaps in plasmas with intermediate values of the equilibrium magnetic fields. In addition one should look for the ICE-like high harmonics of the ion cyclotron frequency features in the magnetic activity of NSTX.

The CAE modes were suggested to be responsible for the anomalous energy diffusion of beam ions in TFTR experiments [10–12]. Now, with the positive experimental identification of the closely related CAEs, further evidence is offered that the concept of ‘alpha channeling’ may be considerably facilitated by internal modes. These observations point naturally to further experiments, even on existing devices, that might verify and further extend this very unusual, but possibly extremely useful, effect. Moreover, the demonstration of the role played by these modes, together with their theoretical description, carries significant implications for how this very unusual effect might be extrapolated with confidence to achieve significant improvements in the tokamak reactor concept. Since this mechanism if successfully deployed can mean a dramatic reduction in the cost of electricity produced by fusion, the newly inspired confidence in extrapolating these results may lead to extremely important follow up experiments.

Acknowledgment

This work supported by DoE contract No DE-AC02-09CH11466.

References

[1] Hawryluk R 1999 *Phil. Trans. R. Soc. Lond.* A **357** 443

[2] Fisch N J and Rax J-M 1992 *Phys. Rev. Lett.* **69** 612

[3] Fisch N J and Herrman M C 1994 *Nucl. Fusion* **34** 1541

[4] Fisch N J, Fruchtman A, Karney C F F, Herrmann M C and Valeo E J 1995 *Phys. Plasmas* **6** 2375

[5] Valeo E J and Fisch N J 1994 *Phys. Rev. Lett.* **73** 3536

[6] Fisch N J and Herrman M C 1995 *Nucl. Fusion* **35** 1753

[7] Herrmann M C and Fisch N J 1997 *Phys. Rev. Lett.* **79** 1495

[8] Darrow D *et al* 1996 *Phys. Plasmas* **3** 1875

[9] Darrow D, Majeski R, Fisch N J, Heeter R, Herrmann H, Herrmann M, Zarnstorff M and Zweben S 1996 *Nucl. Fusion* **36** 509

[10] Fisch N J *et al* 1997 *Proc. 16th IAEA Fusion Energy Conf. on Plasma Physics and Controlled Nuclear Fusion Research (Montreal, Canada, 1996)* vol 1, pp 271–9

[11] Herrmann M C 1998 *PhD Thesis* (Princeton, NJ: Princeton University)

[12] Clark D S and Fisch N J 2000 *Phys. Plasmas* **7** 2923

[13] Fisch N J and Herrmann M C 1999 *Plasma Phys. Control. Fusion* **41** A221

[14] Fisch N J 2000 *Nucl. Fusion* **40** 1095

[15] Menard J *et al* 2007 *Nucl. Fusion* **47** S645

[16] Rosenberg A L *et al* 2004 *Phys. Plasmas* **11** 2441

[17] Fredrickson E D *et al* 2001 *Phys. Rev. Lett.* **87** 145001

[18] Gorelenkov N N, Cheng C Z, Fredrickson E D, Belova E, Gates D, Kaye S, Kramer G J, Nazikian R and White R B 2002 *Nucl. Fusion* **42** 977

[19] Gorelenkov N N, Belova E, Berk H L, Cheng C Z, Fredrickson E D, Heidbrink W W, Kaye S and Kramer G J 2004 *Phys. Plasmas* **11** 2586

[20] Fredrickson E D, Gorelenkov N N and Menard J 2004 *Phys. Plasmas* **11** 3653

[21] Appel L C, Fülop T, Hole M J, Smith H M, Pinches S D, Vann R G L and The MAST Team 2008 *Plasma Phys. Control. Fusion* **50** 115011

[22] Mahajan S M and Ross D W 1983 *Phys. Fluids* **26** 2561

[23] Coppi B, Cowley S, Kulsrud R, Detragiache P and Pegoraro F 1986 *Phys. Fluids* **29** 4060

[24] Gorelenkov N N and Cheng C Z 1995 *Nucl. Fusion* **35** 1743

[25] Fülöp T, Kolesnichenko Y I, Lisak M and Anderson D 1997 *Nucl. Fusion* **37** 1281

[26] Kolesnichenko Y I, Fülöp T, Lisak M and Anderson D 1998 *Nucl. Fusion* **38** 1871

[27] Gorelenkov N, Fredrickson E, Heidbrink W, Crocker N, Kubota S and Peebles W 2006 *Nucl. Fusion* **46** S933

[28] Cauffman S and Majeski R 1995 *Rev. Sci. Instrum.* **66** 817

[29] Cottrell G A, Bhatnagar V P, Costa O D, Dendy R O, Jacquinot J, McClements K G, McCune D C, Nave M F F, Smeulders P and Start D F H 1993 *Nucl. Fusion* **33** 1365

[30] Gates D, Gorelenkov N N and White R B 2001 *Phys. Rev. Lett.* **87** 205003

[31] Wong K L, Heidbrink W W, Ruskov E, Petty C C, Greenfield C M, Nazikian R and Budny R V 2005 *Nucl. Fusion* **45** 30

[32] Appel L C, Akers R J, Fülöp T, Martin R and Pinfold T 2004 *Proc. 31th European Physical Society Conf. on Plasma Physics (London, England, 2004)* vol 28G, ed R M Pick and P Helfenstein (European Physical Society, CCLRC Rutherford Appleton Laboratory and Euratom/UKAEA Fusion Association Culham Laboratory) p 4.195

[33] Smith H M and Verwichte E 2009 *Plasma Phys. Control Fusion* **51** 075001

[34] Gorelenkov N N, Fredrickson E D, Belova E, Cheng C Z, Gates D, Kaye S and White R B 2003 *Nucl. Fusion* **43** 228

[35] Heidbrink W W, Fredrickson E D, Gorelenkov N N, Rhodes T and Van Zeeland M A 2006 *Nucl. Fusion* **46** 324

[36] White R B 2001 *The Theory of Toroidally Confined Plasmas* 2nd edn (London, UK: Imperial College Press)