PHYSICS OF PLASMAS **17**, 073109 (2010)

Quasitransient backward Raman amplification of powerful laser pulses in dense plasmas with multicharged ions

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(Received 22 March 2010; accepted 14 June 2010; published online 27 July 2010)

The range of plasma parameters, where the efficient quasitransient backward Raman amplification (QBRA) of powerful laser pulses is possible, is determined for dense plasmas with multicharged ions. Approximate scalings that portray in a simple way the efficient QBRA range in multidimensional parameter space are found. The calculation, applicable to infrared, ultraviolet, soft x-ray, and x-ray laser pulses, takes into account plasma heating by the lasers. It is shown that efficient QBRA can survive even the nonsaturated linear Landau damping of the Langmuir wave mediating the energy transfer from the pump to the seed laser pulse; moreover, this survival does not require very intense seed laser pulses. © *2010 American Institute of Physics*.
[doi:10.1063/1.3460347]

I. INTRODUCTION

Through the backward Raman amplification (BRA) of laser pulses in plasmas,[1] exawatt and zetawatt optical laser pulses[2,3] might be produced in significantly smaller and cheaper devices than through chirped pulse amplification (CPA) techniques.[4] The BRA technique is also applicable to shorter wavelength laser pulses including x-ray pulses for which CPA cannot possibly work.

The first BRA experiments[5] were done for optical laser pulses in gas-jet plasmas to verify the linear and the moderately nonlinear stages of BRA in transient regimes (which are the regimes where the damping of Langmuir wave mediating energy transfer from the pump to the seed laser pulse can be neglected within the amplified pulse duration).

However, the purely transient BRA of optical laser pulses in gas-jet plasmas may exist only in a relatively narrow parameter window for low ion charge Z plasmas. This window is limited at lower electron plasma temperatures T_e by the collisional damping of Langmuir waves, and by Landau damping at the higher T_e. The typical electron concentration n_e of gas-jet plasmas is a hundred times smaller than the critical concentration for optical pulses. The purely transient BRA window is narrowed further by plasma heating through the inverse bremsstrahlung of laser pulses which can increase the plasma temperature up to the range of strong Landau damping of the Langmuir wave.[6]

On the other hand, the nonlinear saturation of the Landau damping can broaden the transient BRA window. Although a fully rigorous description of the saturation effect is not yet developed and the precision of saturation is still uncertain, useful simplified models of this effect were suggested recently.[7,8] There are also other limitations on efficient transient BRA range imposed by effects of the Langmuir wavebreaking, premature backscattering of laser pulses by noise, self-phase modulation and Raman near-forward scattering of the amplified pulse, plasma inhomogeneity, pump chirp, and seed front shape.[1,2,9–14] Some of these effects were modeled in Ref. 15 to define an efficient BRA range in density-temperature plane for optical laser pulses.

However, the next step of experiments would be designed to reach the advanced nonlinear stage of efficient BRA. In this stage, there are no longer concerns on whether the BRA is transient or not during the earlier stages of amplification. What matters is that the amplified pulse, which contracts during nonlinear BRA, ultimately enters efficient transient regimes. The class of such quasitransient (QBRA) regimes is much broader than the class of purely transient BRA regimes.

According to Ref. 16, efficient QBRA is capable of tolerating the mediating Langmuir wave damping exceeding the linear Raman growth rate up to 20 times for strong seed pulses, and up to 10 times for moderate seed pulses. This can be sufficient for tolerating even nonsaturated linear Landau damping of the Langmuir wave, thus making irrelevant the uncertainty in the extent of nonlinear suppression of the Landau damping. Furthermore, strong damping of Langmuir wave naturally suppresses deleterious instabilities, thus making QBRA more robust and technologically simple. Additionally, the broader parameter range of QBRA gives more flexibility in the selection of the most favorable technologically conditions, in particular, by allowing higher Z plasmas.

The goal of this paper is to outline the boundaries of efficient QBRA range in the multidimensional parameter space for ultraintense infrared, ultraviolet, soft x-ray, and x-ray laser pulses in dense plasmas. The short-wavelength boundary of this range was evaluated earlier in Ref. 17. The parameters of interest here include, in particular, the pump laser wavelength λ, intensity I, fluence w, the plasma electron concentration n_e, electron temperature T_e, and ion charge Z. The temperature range is to be selected self-consistently, taking into account the plasma heating through the inverse bremsstrahlung of laser pulses. The laser seed is taken intense enough to meet the requirements of efficient QBRA tolerating the assumed rate of Langmuir wave damping.[16] The plasma is assumed to be completely ionized due to high laser intensities and high plasma density and temperature, so that effects of partial ionization, like those modeled in Ref. 18, are negligible.

II. BASIC EQUATIONS

To proceed, it is convenient to introduce dimensionless parameters

$$q_L = \frac{\omega_e}{\omega} \ll 1, \tag{1}$$

where

$$\omega_e = \left(\frac{4\pi n_e e^2}{m}\right)^{1/2}, \quad \omega = \frac{2\pi c}{\lambda} \tag{2}$$

are the plasma and pump laser frequencies, respectively, and

$$q_T = \frac{v_{Te}^2}{v_{\text{ph}}^2} \equiv \frac{T_e}{T_M} \ll 1, \tag{3}$$

where $v_{Te}=\sqrt{T_e/m}$ is the electron thermal velocity,

$$v_{\text{ph}} \approx cq_L/2 \tag{4}$$

is the phase velocity of resonant Langmuir wave in cold plasma, and $T_M=mv_{\text{ph}}^2$. In these formulas, c is the speed of light in vacuum, m is the electron mass, and e is the positron charge.

It is also convenient to introduce the ratio of the pump pulse intensity I to the intensity value corresponding to the threshold for breaking of the mediating Langmuir wave in the pump depletion regime,

$$q_I = I/I_{\text{thr}} \lesssim 1, \tag{5}$$

$$I_{\text{thr}} = n_e mc^3 q_L/16. \tag{6}$$

In efficient BRA regimes, q_I should not be much larger than 1, since both the laser coupling and BRA efficiency decrease for intensities much exceeding the threshold.

The pump fluence w will be normalized to $w_{\max}=m^2c^4/e^2\lambda$, the largest tolerable fluence of the amplified pulse. This largest possible fluence arises from the avoidance of near-forward Raman scattering and self-phase modulation instabilities, as discussed in Refs. 1 and 2. Thus, the dimensionless fluence, $q_w \equiv w/w_{\max}$, is defined through

$$w = \frac{q_w}{\lambda}\left(\frac{mc^2}{e}\right)^2. \tag{7}$$

For a uniform pump intensity I, the fluence w is proportional to the pump duration

$$t_{\text{pmp}} = w/I. \tag{8}$$

For highly efficient amplification, resulting in the largest tolerable fluence, $q_w \sim 1$.

The plasma length will be taken equal to half of the pump length

$$L = ct_{\text{pmp}}/2, \tag{9}$$

which is the optimal value.

As was shown in Ref. 17, important restrictions on the efficient BRA parameter range are imposed by the inverse bremsstrahlung effect. The inverse bremsstrahlung rate can be evaluated[19] as

$$\nu_{ib} = \nu_{ei} q_L^2, \tag{10}$$

where ν_{ei} is the rate of electron-ion collisions in the plasma.

Upon traversing the plasma length L, laser pulses lose to inverse bremsstrahlung the energy fraction

$$q_{ib} = \nu_{ib} L/c \ll 1, \tag{11}$$

where the smallness of q_{ib} is indicated as a necessary condition for efficient BRA.

There is yet another requirement associated with the plasma heating via the inverse bremsstrahlung of laser energy. Under conditions when the electron cooling by thermoconductivity and ions is negligible, the inverse bremsstrahlung increases the electron plasma temperature by $\delta T_e = \nu_{ib} t_h I/(C_e n_e c)$, where t_h is the time of heating, and $C_e=3/2$ is the specific heat per electron. Assuming no focusing, the largest heating occurs at the edge where the pump enters the plasma. There, $t_h=2L/c$ and

$$\delta T_e = \frac{4 q_{ib} I}{3 n_e c}. \tag{12}$$

Since the temperature increase must be less than the final temperature, it follows that

$$q_{ibT} \equiv \frac{\delta T_e}{T_e} = \frac{q_{ib} q_I}{3 q_L q_T} \leq 1. \tag{13}$$

For intense enough pump pulses,

$$q_I \gg 3 q_L q_T, \tag{14}$$

the requirement $q_{ibT} \leq 1$ is stricter than $q_{ib} \ll 1$.

Using above formulas, the requirement $q_{ibT} \leq 1$ can be presented in the form

$$q_{ibT} = \frac{16}{3}\frac{\nu_{ei}}{\omega_e}\frac{q_w}{q_T q_L} \leq 1. \tag{15}$$

It also can be expressed in the terms of ratio of the Langmuir wave collisional damping

$$\nu_{Lei} = \nu_{ei}/4 \tag{16}$$

to the linear Raman growth rate

$$\Gamma_R = \omega_e \frac{q_L \sqrt{q_I}}{4\sqrt{2}}. \tag{17}$$

This gives

$$q_{ibT} = \frac{\nu_{Lei}}{\Gamma_R}\frac{8 q_w \sqrt{2 q_I}}{3 q_T} \leq 1. \tag{18}$$

For intense enough pump pulses, such that

$$8 q_w \sqrt{2 q_I} > 3 q_T, \tag{19}$$

the requirement $q_{ibT} \leq 1$ automatically implies that $\nu_{Lei} < \Gamma_R$, i.e., collisional damping of the Langmuir wave does not additionally restrict the parameter range of efficient BRA.

The total damping of the Langmuir wave ν,

$$\nu = \nu_{Lei} + \nu_{\mathrm{Lnd}}, \tag{20}$$

includes, however, Landau damping ν_{Lnd} that may significantly exceed the collisional damping. The Landau damping could then additionally restrict the parameter regime of efficient BRA. The linear Landau damping rate for Maxwellian electron distribution is given by the formula

$$\nu_{\mathrm{Lnd0}} = \frac{\omega_e\sqrt{\pi}}{(2q_T)^{3/2}}\exp\left(-\frac{1}{2q_T}-\frac{3}{2}\right) \tag{21}$$

and is sensitive to the exact value of the parameter $q_T \ll 1$ (defined above as the square of the ratio of the electron thermal velocity to the Langmuir wave phase velocity in cold plasma). Because of this sensitivity, one should not use rough estimates of this paper to express q_T in Eq. (21) through other parameters. This could be clearly seen, for instance, from modification of Eq. (21) produced by substituting there q_T definition (3) with the rough evaluation $v_{\mathrm{ph}} \approx cq_L/2$ (4) instead of the exact formula for the Langmuir wave phase velocity in cold plasma

$$v_{\mathrm{ph}} = \frac{cq_L}{\sqrt{1-q_L^2}+\sqrt{1-2q_L}}.$$

The rough estimates are justified, however, when q_T is treated as an independent parameter (so that it is known exactly) and other parameters are expressed as functions of q_T. This is done throughout this paper.

If the condition $\nu_{\mathrm{Lnd0}} < \Gamma_R$ were really required, it would severely restrict the parameter range of efficient BRA. In fact, however, this condition is not required, and much larger damping of the Langmuir wave can be tolerated, so that QBRA can be efficient for[16]

$$\nu < 20\Gamma_R. \tag{22}$$

Therefore, the much milder condition,

$$\nu_{\mathrm{Lnd0}} < 20\Gamma_R,$$

is sufficient to enable efficient QBRA, even without positing nonlinear saturation of the Landau damping.

So far, the rate of electron-ion collisions ν_{ei} has not been specified; it can be written for a nearly ideal quasiclassical plasma as

$$\nu_{ei} \approx \frac{4}{3}\sqrt{\frac{2\pi}{m}}\frac{Z\Lambda n_e e^4}{T_e^{3/2}}, \tag{23}$$

where Λ is the Coulomb logarithm,

$$\Lambda = \begin{cases} 24-\ln(n_e^{1/2}T_e^{-1}), & T_e > 10Z^2\ \text{eV} \\ 23-\ln(n_e^{1/2}T_e^{-3/2}Z), & T_e < 10Z^2\ \text{eV}. \end{cases} \tag{24}$$

Taking into account Eq. (23), condition (15) can be presented in the form

$$q_T^{5/2}q_L^3 \geq q_w\Lambda Z r_1/\lambda \equiv Q, \tag{25}$$

where

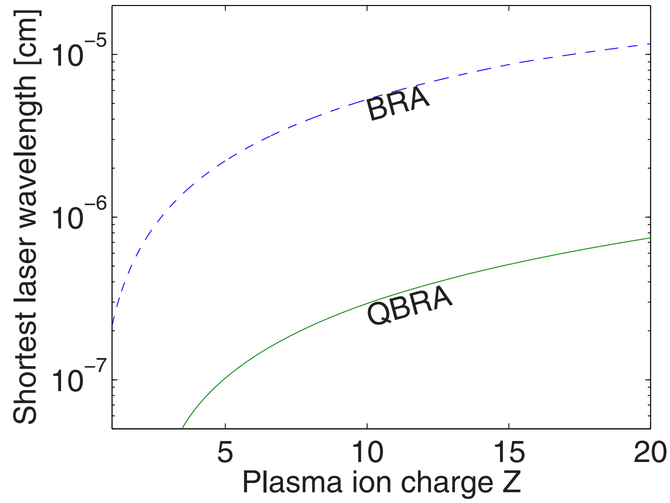

FIG. 1. (Color online) The shortest laser wavelength for QBRA (solid line) and transient BRA (dashed line) regimes in plasmas with ion charge Z.

$$r_1 = \frac{256\sqrt{2\pi}}{9}\frac{e^2}{mc^2} = 2\times 10^{-11}\ \text{cm}. \tag{26}$$

III. EFFICIENT QBRA RANGE

To outline the efficient QBRA range based on the above rough estimates, it is convenient to define formally the range boundary. Considering the inaccuracy of the estimates, the definition is to some extent arbitrary and can be chosen to simplify everything as much as possible. Consider then the implications of the above for efficient QBRA range, assuming the top fluence and reasonable ranges of parameters q_T and q_L,

$$q_w = 1, \quad q_T \leq 1/3, \quad q_L \leq 1/3. \tag{27}$$

Physically, the restriction $q_L \leq 1/3$ means that laser pump frequency is at least three times larger than the plasma frequency, so that the laser seed frequency is at least twice larger than the plasma frequency, and the approximation of highly undercritical plasma density is at least roughly applicable. This approximation conveniently simplifies formulas; yet $q_L \leq 1/3$ is reasonably close to the indispensable limitation $q_L \leq 1/2$.

The restriction $q_T \leq 1/3$ is at least roughly justifying the approximation of the electron thermal velocity significantly smaller than the Langmuir wave phase velocity. This approximation conveniently simplifies formulas; yet $q_T \leq 1/3$ is pretty close to the very condition of the linear Langmuir wave existence in Maxwellian plasma. Note that the condition of the nonlinear electron plasma wave existence could be somewhat milder, see for instance Ref. 20, but the possibility of accessing such nonlinear kinetic waves within the ultrafast QBRA considered here still has to be more carefully studied.

The shortest pump laser wavelength λ allowed by Eq. (25), as a function of plasma ion charge Z, is presented in Fig. 1. This QBRA short-wavelength limit corresponds to

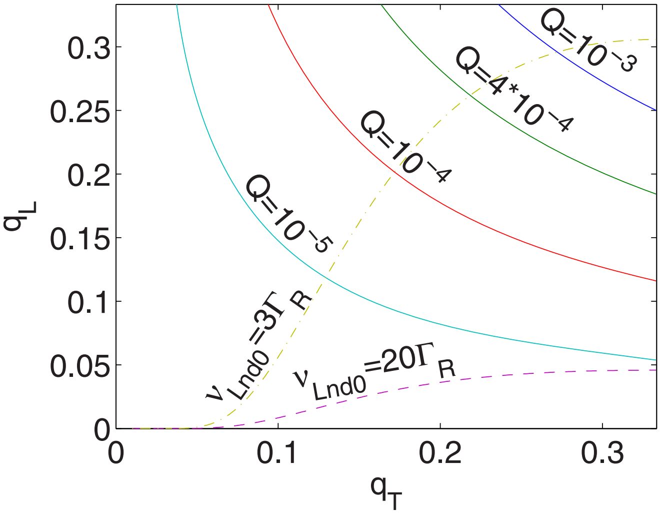

FIG. 2. (Color online) Lines $q_T^{5/2}q_L^3=Q=$const, for several values of Q (solid lines), and lines where the linear Landau damping $\nu_{\rm Lnd0}$ exceeds the linear Raman rate Γ_R by 3 (dash-dotted line), or 20 (dashed line) times. The efficient QBRA is not affected by even the unsuppressed linear Landau damping in the regions above the dashed lines, for intense seed pulses, or dash-dotted lines, for less intense seed pulses.

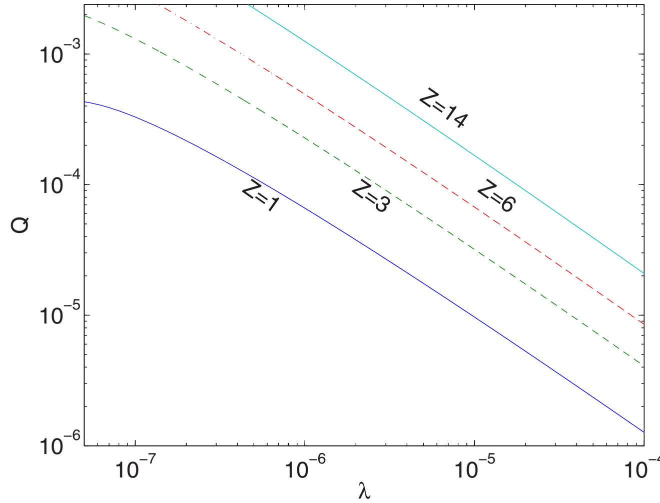

FIG. 3. (Color online) Dependence of the parameter Q of the pump wavelength λ, at the lower boundary of the region (25), $Q=q_T^{5/2}q_L^3$, for several values of plasma ion charge Z. The curves are calculated for $q_L=1/3$, but are in fact nearly the same for all q_L's within the efficient QBRA range.

$$q_T=q_L=1/3, \quad Q=0.0024, \tag{28}$$

$$\nu_{\rm Lnd0}=2.75\Gamma_R. \tag{29}$$

The transient BRA short-wavelength limit, corresponding to

$$q_T=1/8, \quad q_L=1/3, \quad Q=0.0002, \tag{30}$$

$$\nu_{\rm Lnd0}=\Gamma_R, \tag{31}$$

is shown in Fig. 1 by the dashed line.

For laser wavelengths larger than the QBRA short-wavelength limit, condition (25) defines a region in the q_T-q_L plane, where efficient QBRA may occur. The lower boundary of this region approximately coincides with a line $Q=$const, since the Coulomb logarithm variation along this line is relatively small. The line $q_T^{5/2}q_L^3=Q=$const [which is approximately the lower boundary of the region (25)] is shown in Fig. 2, for several values of Q. The lower boundary of the region (22), for the unsuppressed linear Landau damping $\nu_{\rm Lnd}=\nu_{\rm Lnd0}$, is shown by a dashed line in Fig. 2. As seen, even the unsuppressed linear Landau damping does not limit the region of efficient QBRA at not too small Q's, $Q>10^{-5}$. Thus, Landau damping does not jeopardize the possibility of efficient QBRA for intense enough laser seed pulses for which condition (22) is acceptable. Moreover, there is a broad region of very moderately damped QBRA regimes, $\nu_{\rm Lnd0}<3\Gamma_R$, which are efficient even for very moderately intense seed laser pulses[16] even for the unsuppressed linear Landau damping. The lower boundary of this region is shown by the dash-dotted line in Fig. 2.

Figure 3 shows how the lower boundary of the region (25), $Q=q_T^{5/2}q_L^3$, depends of the pump laser wavelength λ, for several values of plasma ion charge Z. These curves depend on q_L (and $q_T=Q^{2/5}q_L^{-6/5}$) just through the Coulomb logarithm, and are therefore nearly the same for all q_L's within the efficient QBRA range. Specific curves presented in the Fig. 3 are calculated for $q_L=1/3$.

Figures 4 and 5 show how the efficient QBRA region looks in the T_e-n_e plane for infrared, ultraviolet, soft x-ray, and x-ray ranges of laser wavelengths. This region (curve-triangular in shape) shrinks at shorter wavelengths and at larger ion charges. The dashed and dash-dotted lines in these figures are images of the respective lines in Fig. 2. Note that the dashed lines are nearly absent in these figures, which indicates that, for intense enough laser seeds, QBRA regimes are not affected by even the unsuppressed linear Landau damping. The regimes above the dashed-dotted line are not affected by even the unsuppressed linear Landau damping for even moderately intense laser seeds.

As seen from Fig. 4, there is an ample parameter range for efficient QBRA of infrared (like $\lambda=1$ μm wavelength) laser pulses in plasmas. The required plasmas could be produced, in volumes sufficient for ultrapowerful QBRA regimes, by ionization of dense gases, or by ionization and further expansion of the lowest density solids. Such solids are already available now at densities as low as 1 mg/cc which were used, for instance, in experiments.[21] This corresponds to the electron concentration $n_e=3\times10^{20}$ cm^{-3}, so that the expansion of such an ionized sheet by just three times might provide a plasma with density suitable for QBRA of ultrapowerful 1 μm wavelength laser pulses.

Furthermore, Fig. 4 shows that there is still an ample parameter range for efficient QBRA of ultraviolet laser pulses (like $\lambda=0.1$ μm wavelength). Here, the required plasmas could be produced by ionization of the lowest density solids directly, even without waiting for a further expansion. Ultraviolet laser pulses include, in particular, those in the range of 1/3 μm wavelength, which are utilized at the U.S. National Ignition Facility (NIF) ($\lambda=0.351$ μm, energy 2 MJ). There are proposals on compression and focusing these laser pulses to the vacuum breakdown intensities. In

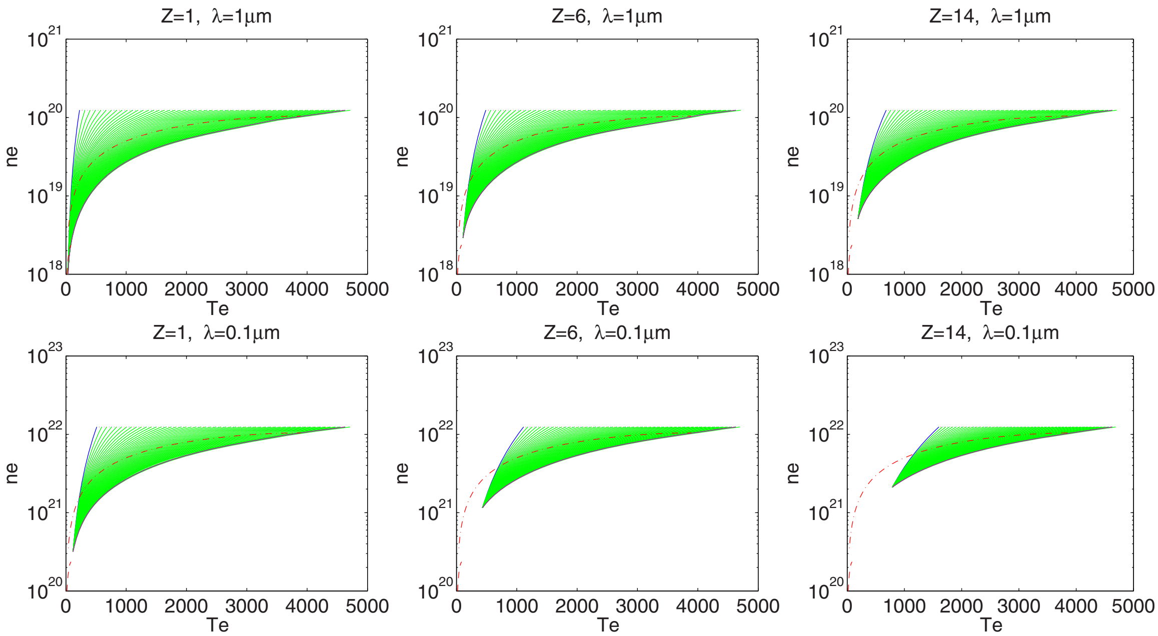

FIG. 4. (Color online) Electron temperature-concentration (T_e-n_e) regions of efficient QBRA of infrared, $\lambda=1$ μm, and ultraviolet, $\lambda=0.1$ μm wavelength, laser pulses in plasmas with ion charges $Z=1$; 6 and 14 (H, C, and Si plasmas). Lines where the linear Landau damping ν_{Lnd0} exceeds the linear Raman rate Γ_R by 3 (dash-dotted line), or 20 (dashed line) times. Note that the dashed lines are nearly absent in these regions, which indicates that, for intense enough laser seeds, the efficient QBRA regimes are not affected by even the unsuppressed linear Landau damping.

particular, the BRA scheme[3] suggests the electron concentration $n_e \sim 10^{21}$ cm^{-3} for the final stage of the pulse compression. This concentration corresponds to the plasma density about 3 mg/cc. Such plasmas could be produced by direct ionization of the lightest stiff solids already available in this density range. Figure 6 shows that efficient QBRA region does include these parameter values.

For soft x-ray pulses (like $\lambda=10$ nm wavelength), ionization of even denser solids (close to a regular condensed matter) could be used to produce the needed plasmas, as seen from Fig. 5. The smaller size of samples needed here might also simplify technologically the plasma production.

Figure 5 also shows that the parameter range for efficient QBRA shrinks considerably for x-ray pulses of $\lambda=1$ nm wavelength already. The required electron plasma concentrations are those of compressed condensed matter. In this regime, large ion charge can no longer be tolerated. However, for smaller output pulse fluence, it may be possible to oper-

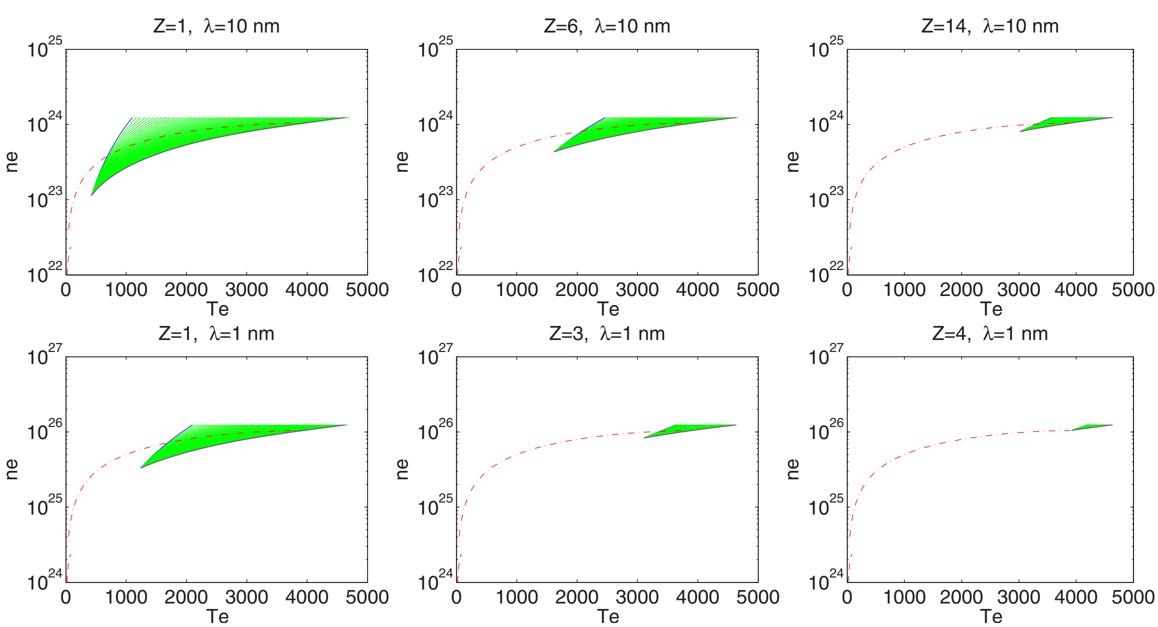

FIG. 5. (Color online) Electron temperature-concentration (T_e-n_e) regions of efficient QBRA of x-ray, $\lambda=10$ nm and $\lambda=1$ nm wavelength, laser pulses in plasmas with ion charges $Z=1$, 3, 4, 6, and 14. Lines where the linear Landau damping ν_{Lnd0} exceeds the linear Raman rate Γ_R by three (dash-dotted line), or 20 (dashed line) times. Note that the dashed lines are nearly absent in these regions, which indicates that, for intense enough laser seeds, the efficient QBRA regimes are not affected by even the unsuppressed linear Landau damping.

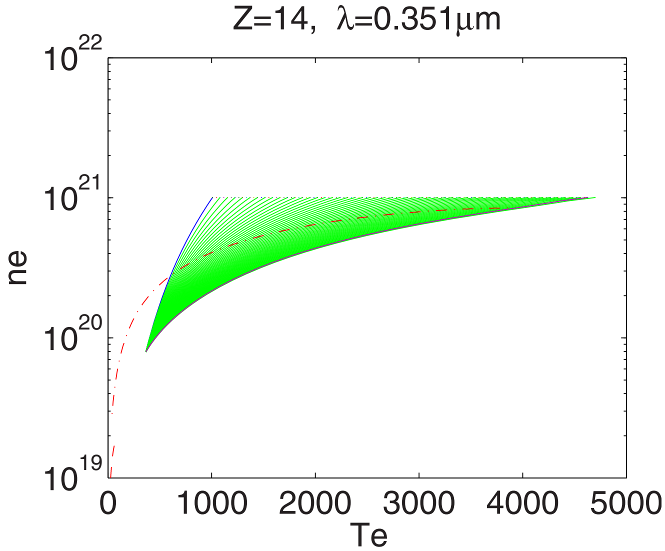

FIG. 6. (Color online) Electron temperature-concentration ($T_e - n_e$) region of efficient QBRA of NIF $\lambda = 0.351$ μm micron wavelength laser pulses in plasmas with ion charge $Z = 14$. The dash-dotted line shows where the linear Landau damping ν_{Lnd0} exceeds the linear Raman rate Γ_R by three times; above this line the damping is even smaller.

ate in less technologically challenging regimes. For example, as seen from Eq. (25), reducing q_w by half would double the allowed ion charge Z.

IV. CONCLUSION

The major results of this paper include the following.

- Approximate scalings are found and used to reduce the dimensionality of the parameter space of efficient QBRA.
- Based on this, the electron-temperature-concentration ($T_e - n_e$) region of efficient QBRA is identified for all possible laser wavelengths.
- Examples of this region are presented for infrared, ultraviolet, soft x-ray, and x-ray intense laser pulses in dense plasmas with multicharged ions.
- It is shown that even an unsuppressed linear Landau damping of the mediating Langmuir wave does not affect efficient QBRA regimes using intense enough seed laser pulses.
- Moreover, even for moderately intense seed laser pulses there is a broad parameter range where efficient QBRA is not affected by even unsuppressed Landau damping.
- It is predicted that an efficient QBRA regime exists for a vacuum breakdown experiment[3] utilizing lasers in the range of $1/3$ μm wavelength and energies in the range of 2 MJ (such as in use at NIF) and plasma densities in the range of 3 mg/cc. It is suggested to produce the required plasmas by ionization of the lightest stiff solids that are already available in this density range.

ACKNOWLEDGMENTS

This work was supported in part by the NNSA under the SSAA Program through DOE Research Grant Nos. DE-FG52-04NA00139 and DE-FG52-08NA28553.

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