Wave–particle interactions in rotating mirrors[a)]

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Wave–particle interactions in $E \times B$ rotating plasmas feature an unusual effect: particles are diffused by waves in both potential energy and kinetic energy. This wave–particle interaction generalizes the alpha channeling effect, in which radio frequency waves are used to remove alpha particles collisionlessly at low energy. In rotating plasmas, the alpha particles may be removed at low energy through the loss cone, and the energy lost may be transferred to the radial electric field. This eliminates the need for electrodes in the mirror throat, which have presented serious technical issues in past rotating plasma devices. A particularly simple way to achieve this effect is to use a high azimuthal mode number perturbation on the magnetic field. Rotation can also be sustained by waves in plasmas without a kinetic energy source. This type of wave has been considered for plasma centrifuges used for isotope separation. Energy may also be transferred from the electric field to particles or waves, which may be useful for ion heating and energy generation. © *2011 American Institute of Physics*. [doi:10.1063/1.3567417]

I. INTRODUCTION

Radio frequency waves are useful for ion heating,[1,2] current drive,[3] and alpha channeling[4] among many applications in plasmas. For radio frequency heating of ions, waves are injected into the plasma that resonate with an ion cyclotron frequency harmonic. It is clear from the hot plasma dispersion relation that cyclotron damping will take place, leading to energy transfer from the wave to the particles.[2]

In a single interaction with the wave, the particle speed is equally likely to increase and to decrease. The wave imparts an impulse that changes the particle velocity by $e^{i\varphi}\Delta v$, with Δv determined by the wave magnitude and the time within the resonance, and φ determined by the phase between the particle gyromotion and wave oscillation. If the phase is randomized between each interaction, the ion energy changes stochastically. Averaging over the phase, there is an average heating for each interaction, $\langle \Delta W \rangle = (1/2)m(\Delta v)^2$. This is in agreement with the amount of heating calculated using the hot plasma dispersion relation.[2]

By selectively removing particles that have lost energy to the wave and limiting energy gain, we can bias the stochastic process and change wave damping into wave amplification. This is the underlying principle for alpha channeling.[4] A key insight is that the diffusion in energy is coupled to diffusion in gyrocenter radius by the wave momentum. That is, specific waves can cause particles to move toward the axis in a tokamak if they gain energy, and toward the edge if they lose energy. The energy particles can gain from the wave that is, therefore, limited by the radial extent of the plasma. At the same time, particles that lose sufficient energy to the wave will leave the plasma through the outer flux surface. This leads to a steady state transfer of energy from alpha particles resonant with the wave to the wave itself.[5] The wave energy may then be used for ion heating.[6]

In stationary plasmas, we are limited to these two cases: transfer of energy from wave to particle or particle to wave. If we introduce a radial electric field, we may consider this as a third energy source for wave–particle interactions. As the particle diffuses in radius, it will gain or lose potential energy. There will be four additional types of energy transfer to consider, shown in Fig. 1: from the wave or particle to the electric field and from the electric field to the wave or particle.

There are many reasons one may wish to transfer energy to the field, but one important application is the elimination of the electrodes that usually produce the radial electric field. The electrodes have been a significant obstacle to obtaining high rotation speeds in rotating plasmas due to the Alfven critical ionization velocity (CIV).[7,8] This velocity limits the rotation at the electrode surface to $v_c = \sqrt{2e\phi/m_n}$, where $e\phi$ is the ionization potential of neutrals and m_n is the neutral mass. For hydrogen, $v_c = 50$ km/s, which is too small for many rotating plasma experiments to be practical. If energy is transferred to the field by waves or particles, the electrodes could be removed, and the Alfven CIV limit could be avoided.

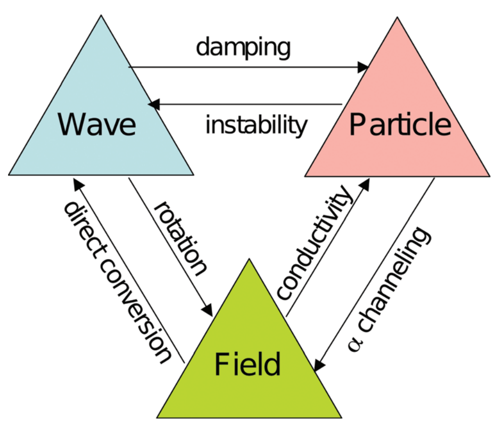

FIG. 1. (Color online) Energy transfer between the wave, particle, and field energy. Applications or descriptions of each type are written by the arrows.

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It is possible to transfer either particle kinetic energy or wave energy to the potential.[9] In fusion plasmas, alpha particles provide an ideal energy source for the rotation. By recycling energy from alpha particles to rotation, it is possible for the plasma to operate with no recirculating power. The rotation energy heats neutrals when they are ionized, leading naturally to a hot ion mode. In addition, by removing alpha particles quickly the fusion reactivity might be approximately doubled.[6] For plasma centrifuges, the rotation energy must be provided by the wave energy. Waves can also be used to drive countercurrent flow patterns to increase the separative power of the centrifuges.[10]

The process of transferring potential energy to particles and waves from the field can also be useful. In rotating plasma experiments, one may wish to do heating to separately control the rotation speed and ion temperature. If the energy is transferred to particles from the potential rather than the wave, there is no antenna coupling inefficiency because the wave does not carry any energy. Another application would be to slow down a flowing plasma and extract energy, as in magnetohydrodynamic (MHD) energy conversion.[11] This could be done by converting potential energy to wave energy in a linear, rather than cylindrical, device.

This paper will be organized as follows. In Sec. II, we will describe the wave–particle interaction in a moving plasma. The transfer of energy to the field will be covered in Sec. III, and the reverse process of removing energy from the field will be described in Sec. IV. We will then describe stationary waves and contained modes in rotating plasmas in Sec. V.

II. WAVE–PARTICLE INTERACTION

The particular type of wave–particle interaction we consider is at the cyclotron resonance. In this case, the direction of acceleration varies on the timescale of the cyclotron motion, so the ion can gain or lose energy depending on the phase. Consider a wave with frequency ω in the plasma frame and a parallel wave number $k_{||}$. The condition for resonant interaction of an ion moving along the field with $v_{||}$ is $\omega - k_{||}v_{||} = n\Omega_{ci}$, where n is the cyclotron harmonic number and Ω_{ci} is the cyclotron frequency.

If the phase between the wave and particle is destroyed between interactions by turbulence or nonlinear effects, the wave can cause stochastic diffusion of the particle in phase space.[1] For a given wave, the particle can only absorb momentum and energy in the proportions they exist in the wave (in linear theory, the wave is either damped or amplified). Therefore, the particle moves along a one dimensional path in terms of its constants of motion: energy W, magnetic moment μ, and angular momentum P_θ.[4]

A change in the particle energy and momentum will also lead to a change in the gyrocenter position. In the frame moving with the plasma, the change in velocity Δv_x will lead to a change in position $\Delta y_{gc} = -\Delta v_x/\Omega_{ci}$. We can relate this to the change in energy $\Delta W = m v_x \Delta v_x$ knowing the interaction takes place where the particle velocity matches the wave phase velocity, $v_x = v_{ph} = \omega/k_x$.[1] Therefore, the change in position $\Delta y_{gc} = -\Delta W k_x/(m\omega\Omega_{ci})$.

In the lab frame, this leads to a change in the potential energy, $e\Delta\Phi = -eE_y\Delta y_{gc}$. Adjusting to the non-Doppler shifted frequency, $\omega \to \omega - k_x v_E$, so $e\Delta\Phi = \Delta W v_E/(\omega/\ k_x - v_E)$, with ω now defined as the lab frame frequency. We use this to define the branching ratio or the increase in potential energy divided by the decrease in kinetic energy, $f_E = v_E/(v_E - \omega/k_x)$. This may be used to characterize the type of energy transfer: for $f_E \approx 0$ energy is transferred between wave energy and kinetic energy, for $f_E \approx 1$ it is transferred between potential and kinetic energy, and for $f_E \approx 1$ it is transferred between wave and potential energy.[9]

III. TRANSFERRING ENERGY TO THE FIELD

In past rotating plasma experiments, energy has been transferred to the radial electric field by means of electrodes connected to an external power source.[8] Contact with these electrodes can lead to velocity limitation by the Alfven CIV phenomenon.[7,12] Because of the electrodes, there must be sufficient conductivity along the field lines to carry power and maintain the rotation. If instead the rotation is produced by waves, the plasma density could be reduced where the field line intersects the containment vessel. The Alfven CIV limitation might then be avoided.

In deuterium–tritium fusion, alpha particles provide power to maintain the plasma at fusion temperatures. Without alpha channeling, the alpha particle energy is transferred first to electrons and then to ions through collisions. Alpha channeling allows the alpha particles to directly heat ions by using waves as an intermediary. The ion temperature could then exceed the electron temperature, improving fusion reactivity.[4] In rotating plasmas, a natural intermediary is the radial electric field, which produces hot ions when cold neutrals are ionized and enter the rotating frame.[13]

Whether the plasma is rotating or not, for alpha channeling in a mirror we can create a diffusion path that connects the fusion alpha particles to the loss cone in phase space[14,15] (Fig. 2). Particles that reduce their perpendicular energy may exit, but particles that increase their perpendicular energy are better trapped in the mirror. The result is that in steady state, alpha particles on average transfer energy to the wave.

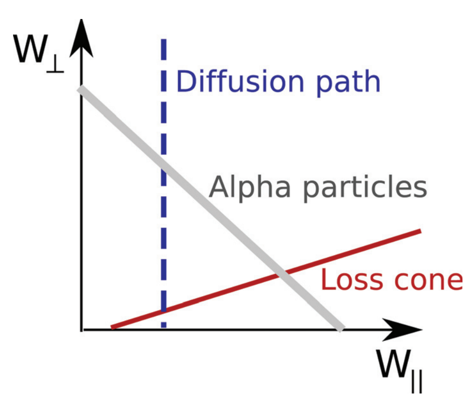

FIG. 2. (Color online) Phase space in a mirror with a perpendicular wave. The dashed line represents the diffusion path, connecting the alpha particle source with the loss cone.

However, in a rotating mirror, which happens when the plasma has a large radial electric field, there is another opportunity. By using waves with a branching ratio $f_E = 1$, we convert the kinetic energy of particles entirely into potential energy. In the rest frame, this is obvious, since $f_E = 1$ when $\omega = 0$ and so the wave contains no energy. As the particle reduces its energy, the particle also moves outward radially. The decrease in energy is in fact the movement of the particle up the potential gradient.

While the particle energy is provided by the electric potential, the azimuthal momentum is provided by the wave and ultimately the antenna structure. This momentum allows the particle to move radially. When the particle exits through the loss cone, most of the angular momentum remains in the plasma, and this allows us to drive rotation in the bulk plasma.

Simulations of alpha channeling with stationary waves demonstrate that enough energy can be recovered from alpha particles to make the rotating mirror fusion reactor self-sustaining.[16] Using four wave regions, an average of 2.5 MeV was recovered from particles subject to alpha channeling. There remains room for improvement, however, as 42% of alpha particles were not removed by the waves. One way the efficiency might be increased is the excitation of contained modes.[17] These are further discussed in Sec. V.

In the case of the plasma centrifuge, the energy and momentum must both be provided by the wave. Because collisions play a critical role in separating the isotopes, we cannot rely on creating a non-Maxwellian ion distribution as in alpha channeling. This in no way limits our ability to drive rotation. The energy may be provided by the radio frequency antenna, and the resulting waves lead to the nonambipolar diffusion of ions. With a plasma source near the device axis, the radial diffusion of ions leads to a charge separation and drives rotation.[10] This method for producing a radial electric field is similar to the centrifuge produced by a magnetic nozzle[18] or the natural rotation of a pure ion plasma.[19] Another relevant separation scheme is the autoresonant ion cyclotron isotope separation scheme, which uses fixed magnetic structures to separate ions.[20]

IV. TRANSFERRING ENERGY FROM THE FIELD

In the reverse of the processes just discussed, we consider applications for transferring energy from the radial electric field to either kinetic or wave energy. The first case might be used as an alternative to ion cyclotron radio frequency (ICRF) heating, useful for heating fusion plasmas or plasma experiments. In the rotating frame the interaction appears exactly as ICRF heating, but in the rest frame the wave may have little or no energy.[17] The advantage to this process is that there is no inefficiency introduced by wave–plasma coupling. The power is provided directly through electrodes (if they are used to drive rotation) or indirectly through waves or alpha particles. This interaction might be thought of as a decrease in the plasma resistance, which leads to increased power dissipation at fixed voltage.

We can also take advantage of the conversion of field energy to wave energy to extract power from the plasma bulk motion, like in a gyrotron. This can be useful in MHD energy conversion, the reverse of the plasma centrifuge, in which the plasma rotation speed is increased using wave energy. For MHD energy conversion, the bulk motion will be slowed and the energy may be extracted through an antenna near the plasma. This allows the MHD generator to operate without electrodes, which are subject to wear and reduce overall efficiency. The application of alpha channeling principles to MHD energy conversion is a subject of ongoing research.

V. STATIONARY WAVES

A simple and convenient way to transfer energy between particles and the field is to use waves with zero frequency in the lab frame.[16] These stationary waves have no energy in the lab frame, and so the branching ratio $f_E = 1$. They may be produced by a fixed magnetic ripple near the plasma edge. In order to satisfy the resonance condition $\omega - k_{||}v_{||} - n_\theta\Omega = \Omega_{ci}$, where n_θ is the azimuthal mode number and Ω is the rotation frequency, the mode must have $|n_\theta| = \Omega/\Omega_{ci} \gtrsim 20$ for realistic plasma parameters. Although in a vacuum this mode decays like $r^{|n_\theta|}$, it is possible to reach sufficient wave amplitudes for alpha channeling as well as ion heating.[16,17]

While the magnetic ripple is sufficient for alpha channeling, it may be possible to increase its efficiency by using contained modes. When ion Bernstein waves were used to test alpha channeling in the tokamak fusion test reactor (TFTR), the measured alpha particle diffusion rate was 50 times higher than expected.[21] The eventual explanation for this surprising result is that contained Alfven eigenmodes were excited in the tokamak.[22] Recent observations on the national spherical torus experiment support the existence of contained modes and their contribution to alpha channeling.[23]

In tokamaks, the contained mode originates from the radial variation in azimuthal wave number and Alfven velocity.[24] There exist waves that only propagate in a localized region of the device, and so energy remains in the wave rather than being dissipated. In mirrors with sheared rotation, there is a similar radial variation in azimuthal wave number and a variation in the plasma-frame wave frequency due to the

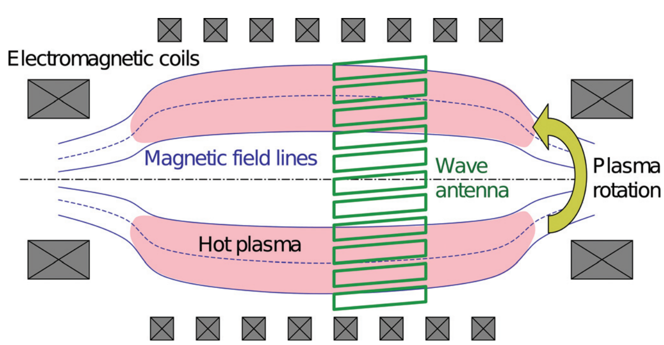

FIG. 3. (Color online) A diagram of a mirror driven with waves. The wave antenna is a stationary magnetic ripple that produces rotation by interacting with alpha particles.

Doppler shift. These can lead to a similar contained mode that might be excited by a stationary magnetic ripple.[17]

The condition for the contained mode to be stationary ($\omega = 0$), and therefore couple to a magnetic ripple, is that the rotation speed is equal to the phase velocity. For Alfven modes, this means $v_\theta \approx v_A$, where v_θ is the peak plasma rotation speed and v_A is the Alfven velocity. These velocities are practical for experimental validation, for example, on the Maryland centrifugal experiment.[25,26]

By exciting contained modes, significantly higher diffusion rates may be expected in alpha channeling or ion heating experiments. In addition, the contained mode is localized near the radius of peak rotation, which is ideal for both heating and alpha channeling. While both process are already very efficient due to the use of stationary waves, the excitation of contained modes would reduce the required ripple amplitude, limiting the plasma perturbation and further decreasing the dissipated power.

VI. CONCLUSION

Wave–particle interactions are fundamentally different in plasmas flowing across a magnetic field because of the additional energy source of the perpendicular electric field. We have shown that field energy can be transferred to particles and waves, and vice versa. We have focused our research on superthermally rotating mirrors, which are useful as fusion reactors and plasma centrifuges, among many applications.[8]

It was shown that by the transfer of energy to the radial electric field, waves can drive rotation in plasmas and replace electrodes that limit rotation speeds to the Alfven CIV. The method for driving rotation is an extension of the alpha channeling effect in tokamaks. The rotation energy may be provided directly by alpha particle kinetic energy or externally by radio frequency waves.

Alpha channeling can be accomplished in rotating plasmas using a fixed azimuthal ripple, a simple magnetic structure that may be placed in the vacuum vessel (Fig. 3). The ripple does not require a significant power source because the field has no energy, but the ripple serves to break the invariance of the alpha particle magnetic moment and angular momentum. The alpha particle interacts with the ripple to diffuse radially and must give up its energy to the radial potential in order to exit the mirror. In addition to driving rotation without electrodes, this increases the efficiency of the fusion reactor, and the prompt removal of fusion ash may approximately double the fusion reactivity.[6] These results challenge the basic assumptions about mirrors as fusion reactors and make centrifugal mirrors again attractive as a fusion concept.

The described wave–particle interactions suggest a number of other applications in which energy is transferred from the field to particles or waves. Increasing particle energy using field energy can efficiently create higher ion temperatures. The conversion of field energy to waves might be useful in turning plasma flow into a radio frequency power source. These applications are currently under investigation.

ACKNOWLEDGMENTS

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