Channeling of Fusion Alpha-Particle Power Using Minority Ion Catalysis

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Maintaining fuel ions hotter than electrons would greatly facilitate controlled nuclear fusion. The parameter range for achieving this temperature disparity is shown here to be enhanced by catalyzing the α-channeling effect (wave-induced simultaneous expulsion and cooling of α particles) through minority-ion heating. Specifically, a wave can extract energy from hot α particles and transfer it to colder minority ions, which act as a catalyst, eventually forwarding the energy to still colder fuel ions through collisions. In comparison with the traditional α-channeling mechanism, the requirements are thereby relaxed on the waves that accomplish the α channeling, which no longer have to interact simultaneously with α particles and fuel ions. Numerical simulations illustrate how the new scheme may increase, for example, the effective fusion reactivity of mirror-confined plasmas.

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Introduction.—Alpha particles born in deuterium-tritium fusion reactions carry almost 20% of the released fusion energy. Before this energy is lost to electrons through collisions or to plasma instabilities [1–8], it would be advantageous to transfer it rapidly to fuel ions, with electrons kept cold, so that the effective fusion reactivity can be increased [9–11]. The means to accomplish such a transfer is the so-called α-channeling technique [12], where the α particle energy is extracted by waves which heat fuel ions simultaneously via resonant interactions. The α-channeling wave can be marginally stable, interacting with both species simultaneously [13] (possibly with external wave feedback control), or convectively stable, first excited by α particles and then damped on fuel ions [14,15]. However, finding appropriate modes that would be able to interact vigorously enough with both α particles and fuel ions in a practical device can be challenging [14–17]. (For example, in mirror machines, no such weakly damped modes have been identified yet that would allow transferring most of the energy [18].) Thus, expanding the parameter range within which the α channeling can be practiced would be highly advantageous.

Here, a mechanism is proposed that relaxes the requirements on suitable waves by combining the traditional α-channeling effect with minority-ion heating [19–22]. Specifically, we show that there exists a wave which can extract energy from hot α particles and transfer it to colder minority ions [23]. The minority ions act as a catalyst, eventually forwarding the energy to still colder fuel ions through collisions. We call the wave extracting energy from the α particles the "extracting wave." With the minority catalyst, the extracting waves no longer have to interact with α particles and fuel ions simultaneously. To illustrate how this concept can broaden considerably the parameter space of useful waves, we focus on α channeling in mirror geometry [26], where roughly suitable waves have already been identified [18].

Wave interaction with minority species.—Using minority species as a mediator means that the operating wave is not required to interact resonantly with the fuel ions. However, the minority ions must be in resonance with the wave, meaning that $\omega \approx n\Omega_i$, where ω is the wave frequency, Ω_i is the gyrofrequency of the minority ions, and n is an integer number. Similarly, the wave frequency ω must obey a resonance condition with α particles satisfying $\omega \approx m\Omega_\alpha$ for some integer m, where Ω_α is the α particle gyrofrequency [27]. The choice of the low-energy minority ions is thus limited by the combination of these two conditions:

$$n\Omega_i \approx m\Omega_\alpha. \tag{1}$$

Yet another limitation is that satisfying Eq. (1) should not imply a simultaneous resonance with fuel ions; otherwise, a wave would be strongly damped, hindering extraction of the α-particle energy. In particular, deuterium (D) and tritium (T) resonant parallel velocities, which can be written as $v_{D\text{res}}^{(\ell)} = (\omega - \ell\Omega_D)/k_\parallel$ and $v_{T\text{res}}^{(\ell)} = (\omega - \ell\Omega_T)/k_\parallel$, should be much larger than the thermal ion velocities for all resonance numbers ℓ. Here $k_\parallel$ is the projection of the wave vector $\mathbf{k}$ on the direction of the background magnetic field $\mathbf{B}$.

To assess the choice of the minority ions quantitatively, we focus, for simplicity, on mirror geometry, where two candidate waves suitable for α channeling were identified: the fast Alfvén wave and the ion Bernstein wave [18]. Two distinct regimes are then possible, depending on the parameter $k_\perp\rho_i$, where ρ_i is the minority-ion gyroradius and $k_\perp$ is the perpendicular projection of $\mathbf{k}$.

First, consider the fast Alfvén wave with $k_\perp\rho_i \ll 1$. In this case, the minority-ion cyclotron heating power decreases with n and thus, $n = 1$ is advantageous. Choosing $n = 1$ automatically requires that $m = 1$ also [Eq. (1)], because hydrogen ions cannot be used as minority species as their gyrofrequency is exactly twice of that of the fuel deuterium. This means that the minority ions should have $\Omega_i \approx \Omega_\alpha$. In particular, ions with $Z = N - \Delta Z \gg 1$ are suitable, where Z and N are the number of protons and neutrons in the nucleus and $\Delta Z \ll N$. On the other hand, one must also have

$$\omega/k_\parallel = \xi_e v_e, \qquad (\omega - \Omega_\alpha)/k_\parallel = \xi_\alpha v_\alpha, \tag{2}$$

where the coefficients satisfy $\xi_\alpha \lesssim 1/2$ in order to capture a large fraction of α particles and $\xi_e \gtrsim 3$ to avoid strong wave damping on electrons. Here v_α is the birth velocity of 3.5 MeV α particles and v_e is the electron thermal velocity. Combining these two equations, one obtains

$$\frac{\omega - \Omega_\alpha}{\omega} = \frac{\xi_\alpha}{\xi_e}\frac{v_\alpha}{v_e}. \tag{3}$$

For 10 keV plasmas, the frequency difference $\omega - \Omega_\alpha$ should be smaller or comparable to $0.05\Omega_\alpha$, which suggests that light ions (lighter than oxygen) would be unsuitable for the minority-ion heating technique.

Now consider waves with $k_\perp \rho_i \gtrsim 1$. In this case, higher cyclotron resonances can be employed, both for α-particle energy extraction and minority-ion heating. However, the extracting wave may need to have much larger intensity for α channeling to remain efficient, because the quasilinear diffusion coefficient decays with $k_\perp \rho$ [20,28,29]. It is for this reason that in the following we focus our attention on fast Alfvén waves.

Regimes for the catalytic effect.—The requirement that minority ions must cool down preferentially on fuel ions imposes an upper bound on the extracting wave intensity. Specifically, the heating must be slow enough such that the minority species remain at energies much lower than 1 MeV, for otherwise they would heat electrons instead. To avoid overheating, minority ions can be injected in regions where the wave energy density is smaller than its peak value (say, where the wave is evanescent). Another way would be to detune the extracting wave from the exact resonant with the minority ions. The heating would then be determined by the tail of the wave spectrum.

On the other hand, a lower bound on the extracting wave intensity appears if the minority-ion injection energy is lower than the majority ion thermal energy. In this case, if the wave intensity is not sufficiently high, the dominant energy transfer mechanism will be minority-ion heating by the background plasmas. To avoid this, one should either use higher-intensity waves or inject minority ions with the energy close to or even above the energy of the background species.

Finally, assuming that most minority ions are lost through the device ends rather than at the walls, another limitation is connected with collision-driven loss of minority species through the loss cone. Specifically, since minority ions are energetic, the energy outflow associated with the particle escape must be kept small compared to the total energy flow mediated by the minority population.

Numerical model.—To address these effects quantitatively, we conducted numerical simulations of the minority-ion and α particle dynamics by solving the Fokker-Planck equation [30]

$$\frac{\partial f}{\partial t} + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}(v_\perp J_\perp) + \frac{\partial J_\parallel}{\partial v_\parallel} = S. \tag{4}$$

Here $f(v_\parallel, v_\perp, t)$ is the particle distribution function, $v_\parallel$ and $v_\perp$ are the parallel and the perpendicular particle velocities, S is the particle source, and

$$J_\perp = -D_{\perp\perp}\frac{\partial f}{\partial v_\perp} - D_{\perp\parallel}\frac{\partial f}{\partial v_\parallel} + F_\perp f, \tag{5}$$

$$J_\parallel = -D_{\parallel\perp}\frac{\partial f}{\partial v_\perp} - D_{\parallel\parallel}\frac{\partial f}{\partial v_\parallel} + F_\parallel f, \tag{6}$$

where the diffusion tensor $\hat{D}$ and the particle drag $\mathbf{F}$ contain terms due to both collisions with the background plasma and quasilinear particle diffusion [28,30]. The particles are assumed trapped in a mirror cell with a homogeneous magnetic field. The mirror loss cone was simulated by using a boundary condition $f(v_\parallel, v_\parallel/\sqrt{R-1}, t) = 0$, where R is the mirror ratio.

The α-particle extraction time, the steady-state minority-ion energy, and the energy loss due to escaping ions were calculated in our simulations by finding the stationary solutions of Eq. (4). These solutions were obtained using the Monte Carlo method: starting with no particles in the system ($f = 0$), new particles sampling the distribution S were injected and traced until the particle distribution reached the steady state.

While accurate solutions of the minority-ion distribution function can be found by solving Eq. (4), the steady-state minority-ion energy and collisional energy flows can be estimated using a simpler approach exploiting the fact that for sufficiently large wave amplitudes, the minority-ion distribution function is highly anisotropic ($v_\perp \gg v_\parallel$ for most particles). By integrating over $v_\parallel$, in steady state Eq. (4) can then be put in the form of a one-dimensional (1D) equation [31]:

$$F_\perp g - D_{\perp\perp}\frac{\partial g}{\partial v_\perp} = \frac{1}{v_\perp}\int_0^{v_\perp} v'_\perp \mathcal{S} dv'_\perp, \tag{7}$$

where $\mathcal{S} = \int_{-\infty}^{\infty} S dv_\parallel$ and $g(v_\perp, t) = \int_{-\infty}^{\infty} f dv_\parallel$. To simplify the boundary conditions, we take $S \sim \delta(v_\perp - v_{\perp 0})\delta(v_\parallel - v_{\parallel 0})$ with $v_{\perp 0} \gtrsim v_{\parallel 0}$ and assume that the particle longitudinal motion is approximately unaffected. In this case, most particles escape the device with $v_\parallel \approx v_{\parallel 0}$, corresponding to the boundary condition $g(v_{\parallel 0}/\sqrt{R-1}, t) = 0$. Notice that the exact location of the loss boundary does not affect the profile of the particle distribution function above $v_{\perp 0}$.

Minority-ion heating and heating of fuel ions.—Numerical simulations, employing the model described above, demonstrate the feasibility of catalyzing α channeling through minority-ion heating for plasma parameters of practical interest. Specifically, we used $R = 5$, densities $n_e = 3 \times 10^{13}$ cm^{-3}, $n_D = 0.9 n_e$, $n_T = 0.1 n_e$, and the temperatures $T_e = T_D = T_T = 10$ keV, where the indices e, D, and T denote electrons, deuterium, and tritium, correspondingly. The background magnetic field was varied to optimize the α-channeling efficiency, i.e., to minimize the α-particle extraction time for the fixed wave intensity. Recalling that $|\omega - \Omega_\alpha|$ should be comparable to $0.05\Omega_\alpha$, we chose ^{22}Ne with $\Omega_\alpha - \Omega_i \approx 0.1\Omega_\alpha$ and a more rare isotope ^{21}Ne with $\Omega_\alpha - \Omega_i \approx 0.05\Omega_\alpha$ as minority ions in all our simulations.

Let $\mathcal{E}_{\mathrm{fuel}}$ and $\mathcal{E}_e$ be the collisional energy flows from minority to majority ions and from minority ions to electrons correspondingly. The catalyzing technique is practical if the wave intensity I is larger than $I_{\min}$, below which $\mathcal{E}_{\mathrm{fuel}} < 0$, but smaller than $I_{\max}$ above which minority ions are overheated and $\chi = \mathcal{E}_{\mathrm{fuel}}/\mathcal{E}_e$ becomes comparable or less than one. Specifically, we define $I_{\max}$ as a wave intensity above which $\chi < 5$ and less than approximately 80% of energy dissipated by minorities on background plasma particles goes to the fuel ions. The wave intensity must also be sufficient to extract α-particle energy before the α particle slows down. Introducing I_α as a wave intensity sufficient for extracting α particles on a time $\tau_\alpha/2$, where τ_α is the characteristic collisional slowing-down time, we require that $I > I_\alpha$. Calculating numerically $I_{\min}$, $I_{\max}$, and I_α, one can check if $I_{\min} < I_\alpha < I_{\max}$. If this inequality does not hold, then the conditions must be modified in one of two ways: (a) if $I_\alpha < I_{\min}$, the wave intensity or the minority-ion injection energy should be increased for the technique to work and (b) if $I_{\max} < I_\alpha$, the α-channeling wave overheats minorities, which can be avoided by injecting ions in the wave evanescent regions.

In our simulations, we studied dependencies of $I_{\min}$, $I_{\max}$, and I_α on the wave intensity, wave polarization, and $k_\perp/B$ entering the quasilinear diffusion tensor. The parameter $k_\perp/B$, which has a large effect on the α-particle extraction time, was chosen to minimize this time for a fixed wave intensity. Following Ref. [18], we used the fast Alfvén branch for the operating mode. In that case, the ratio f_+/f_- of the amplitudes of the left- and right-hand polarized components equals $|\Omega_i - \Omega_\alpha|/\Omega_\alpha$, thus yielding $f_+/f_- \approx 0.1$ for ^{22}Ne ions and $f_+/f_- \approx 0.05$ for ^{21}Ne.

Our calculations of $I_{\max}$ were carried out by finding the stationary minority distribution function solving Eq. (7) for different parameter values. Fully ionized ^{22}Ne and ^{21}Ne atoms were injected at 1.1 keV and the loss boundary was at 300 eV. As expected, χ was shown to be monotonically decreasing with I [Fig. 1(a)] since hotter minority ions tend to collide more frequently with electrons, but not ions. The dependence of χ on $k_\perp/B$, on the other hand, turned out to be nonmonotonic with a local minimum at $k_\perp \sim \rho_i^{-1}$ [Fig. 1(b)], where ρ_i is the characteristic gyroradius of the hot minority ions. This suggests that low-$k_\perp$ waves might be more advantageous for limiting minority-ion heating by a strong α-channeling wave. Finally, comparing two polarizations $f_+/f_- \approx 0.05$ and $f_+/f_- \approx 0.1$, the former was shown to be characterized by a twice larger $I_{\max}$ [Fig. 1(a)].

To calculate $I_{\min}$ and I_α, we performed numerical simulations of minority-ion and α particle diffusion by solving Eq. (4) using the Monte Carlo method. First, we found I_α and the optimal values of $k_\perp/B$. Since the extracting wave does not interact with deeply trapped α particles having $v_\parallel < \xi_\alpha v_\alpha$, we focused our attention on extraction of α particles with $v_\parallel > \xi_\alpha v_\alpha$. The source of α particles was thus chosen to be monoenergetic with $v_\parallel > \xi_\alpha v_\alpha$ and

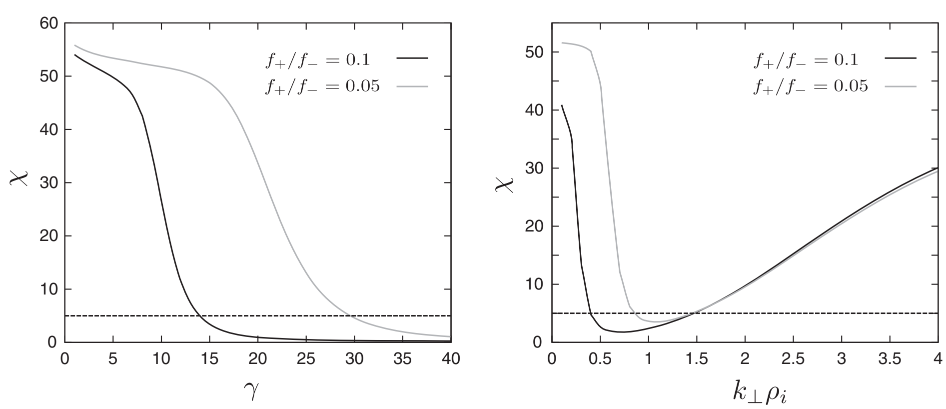

FIG. 1. The ratio χ of the energy absorbed by the fuel ions and that absorbed by electrons, obtained via numerical simulation of Eq. (7) for ^{22}Ne$(f_+/f_- = 0.1)$ and ^{21}Ne$(f_+/f_- = 0.05)$ ions with injection energy 1.1 keV and loss energy 300 eV: (a) $\chi(\gamma)$ at $k_\perp \rho_i = 0.45$; (b) $\chi(k_\perp \rho_i)$ at $\gamma = 15$. Here ρ_i is the gyroradius of 100 keV ions and $\gamma = I/I_0$, where I is the wave intensity and I_0 is the wave intensity, for which the α particle quasilinear diffusion coefficient $D^\alpha_{\perp\perp}$ can be approximated as $(v_\alpha^2/\tau_\alpha)(J_0 f_+ + J_2 f_-)^2$. In our simulations, the quasilinear diffusion coefficient for minority ions was then approximated by $D^i_{\perp\perp} \approx \gamma(\Omega_i/\Omega_\alpha)^4(v_\alpha^2/\tau_\alpha) \times (J_0 f_+ + J_2 f_-)^2$, where J_n is the Bessel function of order n with an argument $k_\perp v_\perp/\Omega_i$.

$\xi_\alpha = 1/2$. Fixing the same plasma parameters as we used for determining $I_{\max}$, I_α was calculated for both polarizations and different $k_\perp/B$ values by finding a wave amplitude sufficient to extract α particles on time $\tau_\alpha/2$. (The characteristic extraction time is equal to the total number of particles for the stationary solution divided by the particle injection rate.) For both wave polarizations, the minimum of I_α was achieved at $f_+ J_0(k_\perp \rho_\alpha) + f_- J_2(k_\perp \rho_\alpha) \approx 0$, where $\rho_\alpha = v_\alpha/\Omega_\alpha$. Given the optimal $k_\perp/B$, the corresponding magnetic field equals $B \approx 0.9$ T for both polarizations. The obtained minimum values of I_α were then compared to the corresponding values of $I_{\max}$. For $f_+/f_- = 0.1$, I_α was found to be equal to $0.43 I_{\max}$, while for $f_+/f_- = 0.05$, $I_\alpha \approx 0.2 I_{\max}$. This means that the α-channeling wave is not expected to cause minority-ion overheating in both scenarios.

We also calculated $I_{\min}$ for the optimal value of $k_\perp/B$. For $f_+/f_- = 0.1$, $I_{\min}$ was calculated to be equal to approximately $0.15 I_{\max}$, or $0.35 I_\alpha$, while for $f_+/f_- = 0.05$, $I_{\min} \approx 0.2 I_{\max}$ and hence $I_{\min} \approx I_\alpha$. These results suggest that the minority-ion heating with the α-channeling wave characterized by $I = I_\alpha$ is feasible for ^{22}Ne ions. For ^{21}Ne ions, however, using wave intensities higher than I_α might be necessary.

Energy loss with escaping particles.—Let us now calculate energy flows $\mathcal{E}_{\mathrm{fuel}}$ and $\mathcal{E}_e$ and compare their sum $\mathcal{E}$ to the energy loss $\mathcal{W}$ associated with minority ions. Solving numerically the 2D Fokker-Planck equation (4) with $k_\perp \rho_\alpha \approx 5.6$, we obtained the following results. For $f_+/f_- = 0.05$, one has $\mathcal{E}/\mathcal{W} = 1$ for $I \approx 2.3 I_\alpha$, and $\mathcal{E}/\mathcal{W} \approx 3.5$ at $I \approx I_i$. For $f_+/f_- = 0.1$, $\mathcal{E}/\mathcal{W} = 1$ for much smaller wave intensity $I \approx 0.7 I_\alpha$, and, finally, $\mathcal{E}/\mathcal{W} \approx 5.5$ at $I \approx 2 I_\alpha$. Thus, the energy lost with minority ions can be made several times smaller than the total energy mediated by minorities by increasing the wave amplitude to a sufficiently high level. Note that the successful implementation of the proposed technique may require external wave control to ensure that the α-channeling mode parameters remain near their optimal values.

Conclusions.—A technique for improving the efficiency of magnetic fusion reactors by catalyzing α channeling through minority-ion heating is proposed. Considering a mirror geometry, we show that instead of heating fuel ions directly, the α-channeling wave can transfer energy to injected minority ions, which act as a catalyst, forwarding the absorbed energy to yet colder fuel ions. Very hot minority ions with energies of several MeV tend to heat electrons rather than ions. However, by solving the Fokker-Planck equation numerically, we identify wave regimes for which the wave intensities are large enough for efficient α channeling, yet small enough to avoid minority-ion overheating. We also show that, under a proper choice of the wave amplitude, the energy loss through losing energetic minority ions can be much smaller than the total energy mediated by the injected particles. Note that the proposed technique may also catalyze α channeling in rotating plasma [32]. It stands to reason as well that similar catalytic techniques can facilitate α channeling in tokamaks, except that the operating modes would be different in toroidal geometry.

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