

Principles of Rotating Plasma in Plasma Propulsion Systems

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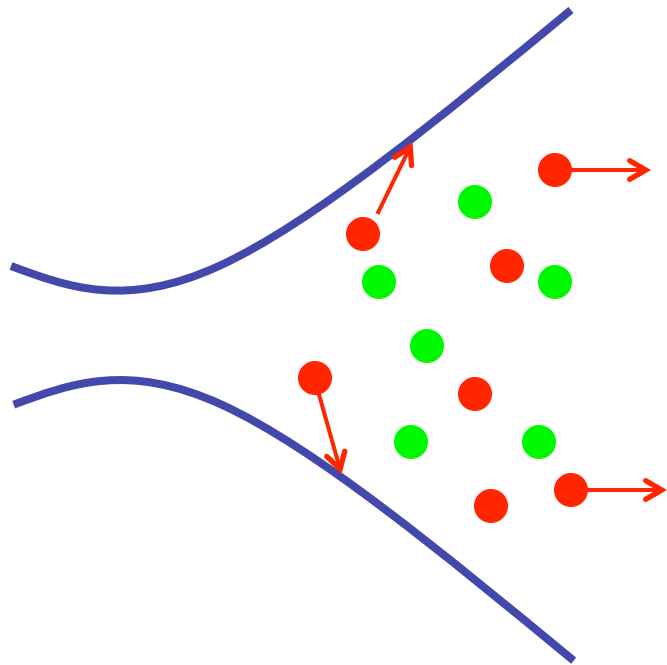
October 8, 2013

One of the main ways to propel plasma exploits rotating electron plasmas in crossed electric and magnetic fields. This talk reviews at a tutorial level some of the interesting physical effects associated with rotating clouds of electrons.

Some Fundamental Questions

- How do rotating plasmas self organize to create propulsion?
- What are the structures that propel and absorb momentum?
- What are the limiting thrust/current densities?
- Compare electric propulsion to other plasma momentum mechanisms:
 - Current drive
 - Isotope separation
 - Plasma-based accelerators

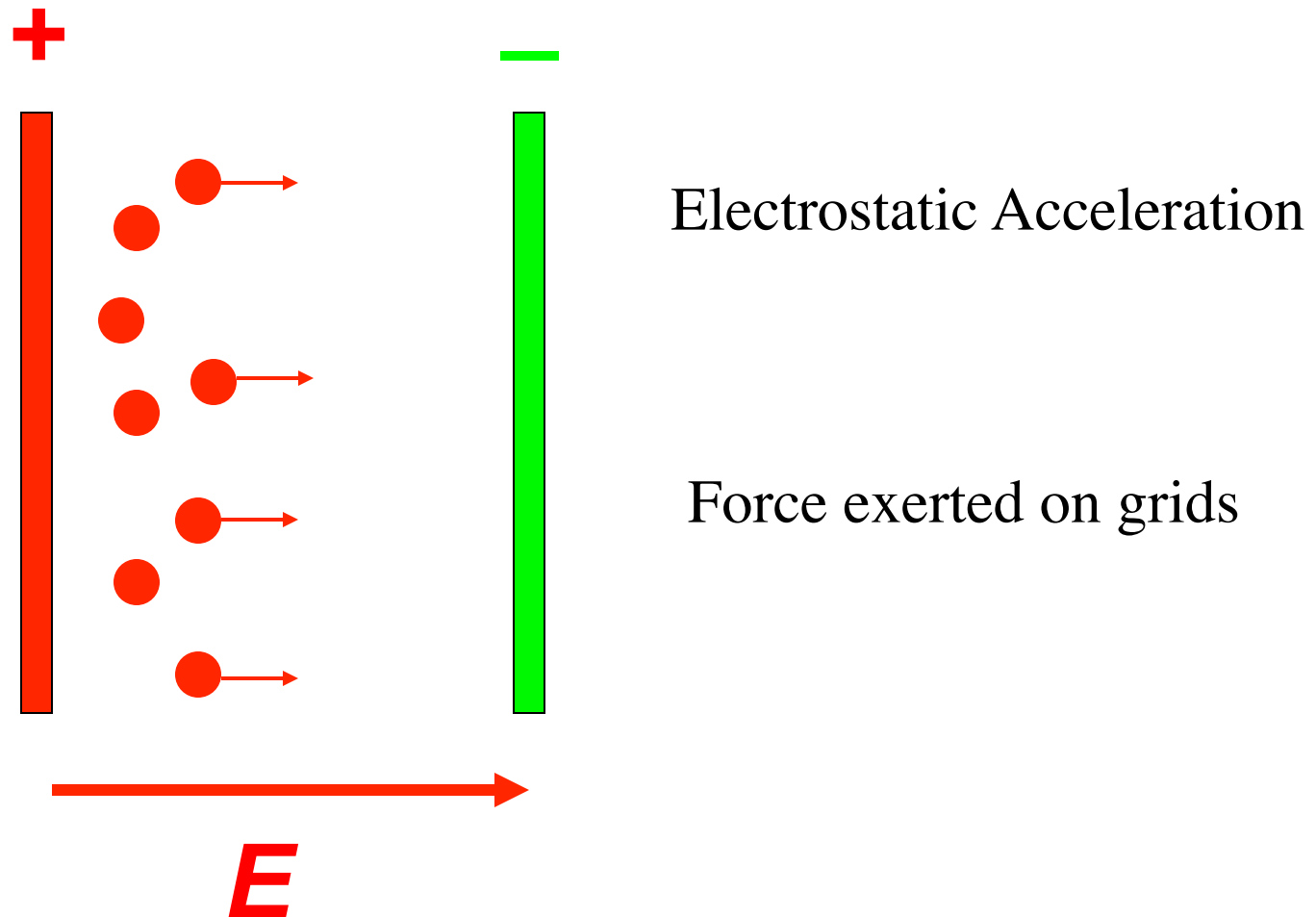
Mechanisms of Propulsion



Electro-thermal
Acceleration

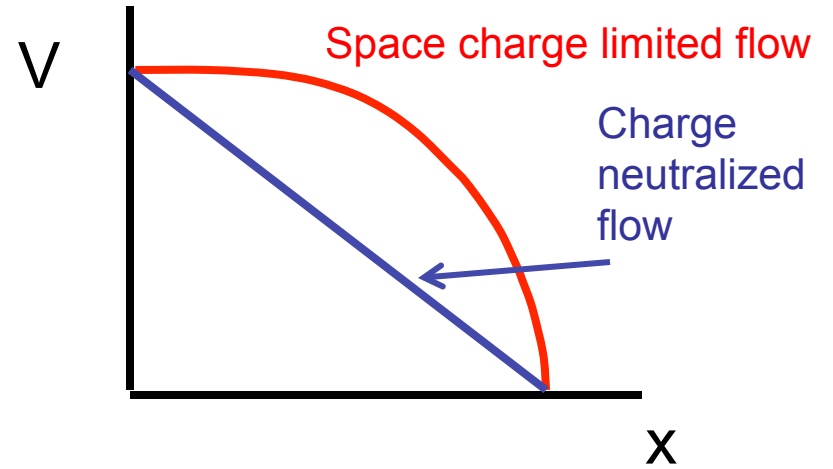
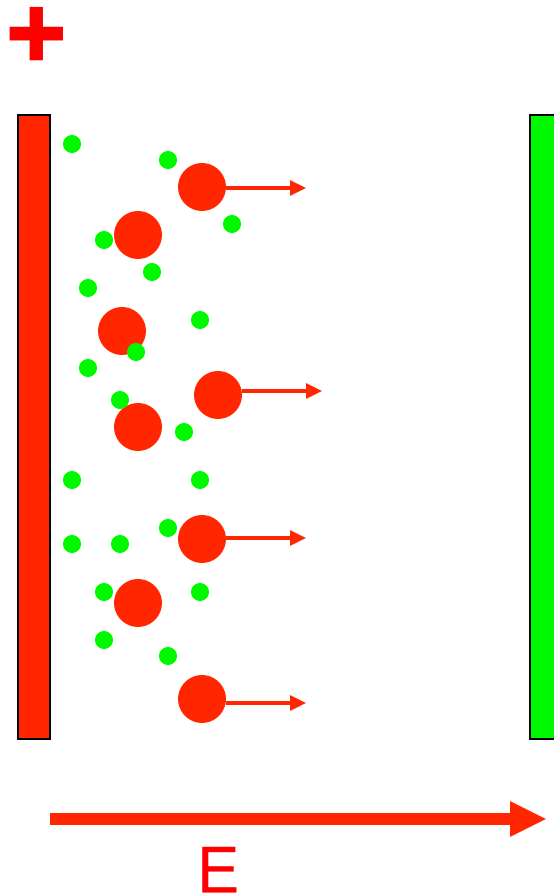
Momentum transfer to walls

Mechanisms of Propulsion -- Ion Thruster



Space charge limited flow

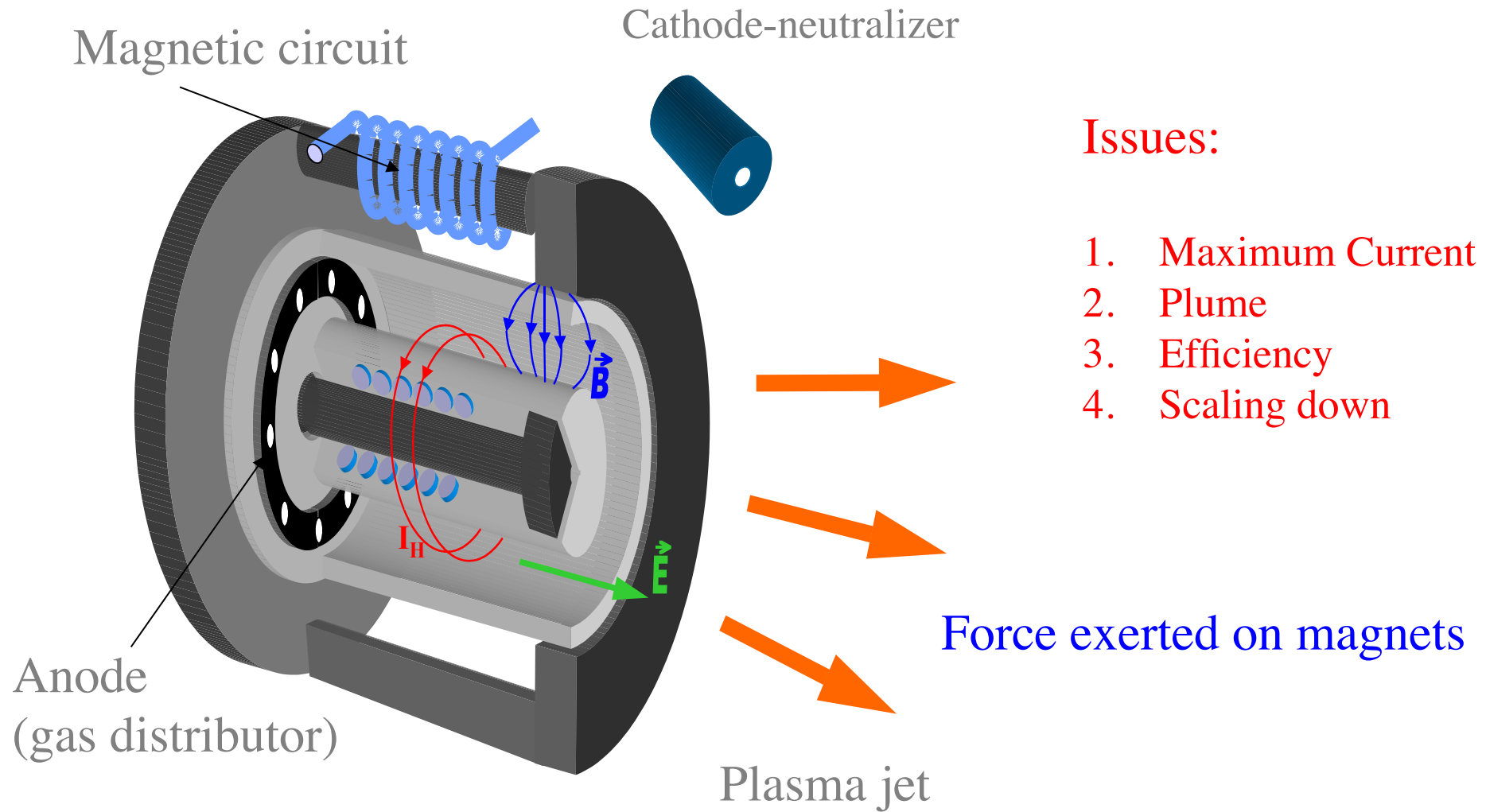
$$J_{i\max} = J_i^{CL} = \frac{\sqrt{2q\phi_0^{3/2}}}{9\pi d^2 \sqrt{M_i}}$$



How do you maintain charge neutralization, without drawing a lot of power?

Need electrons not to move axially!

Hall Thruster (Schematic)



Issues:

1. Maximum Current
2. Plume
3. Efficiency
4. Scaling down

What is the Maximum Current Density?

Thrusters

Electro-thermal:

Wall Temperature

Ion Thruster:

Space Charge Limit

Hall Thruster:

Magnetic Field limit

Ion current limit in Hall thrusters

Consider slab geometry

Electron momentum equation:

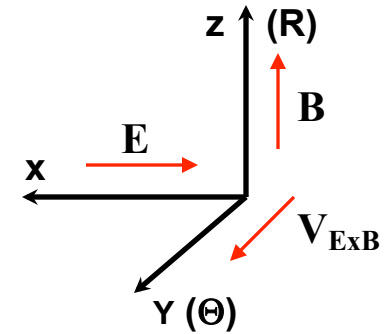
$$n_e m_e \frac{d\vec{v}_e}{dt} = -e\vec{E}n_e - \frac{en_e \vec{v}_e \times \vec{B}}{c} - \nabla p_e - \frac{m_e n_e (\vec{v}_e - \vec{v}_i)}{\tau_{coll}}$$



$$en_e \frac{d\varphi}{dx} = + \frac{eBj_y}{c}$$

$$j_y = n_e v_{ey} = \frac{c}{4\pi e} \frac{dB}{dx}$$

Ampere's Law



Ion acceleration in axial electric field: $m v_i n_i \frac{dv_i}{dx} = -q n_i \frac{d\varphi}{dx} = -\frac{q n_i}{en_e} \frac{d}{dx} \left(\frac{B^2}{8\pi} \right)$ $\beta = \frac{p_e}{(B^2/8\pi)} \ll 1$

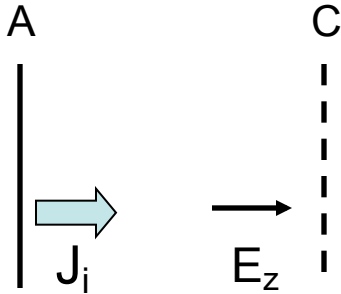
Plasma is quasineutral: ($qn_i=en_e$):

$$m J_i v_{if} = \frac{B_0^2}{8\pi} - \frac{B_1^2}{8\pi}$$

$$J_i = n_i v_i = \text{const}$$

Limitation of ion current in Ion and Hall thrusters

Ion thruster



$$J_{i\max} = J_i^{CL} = \frac{\sqrt{2q\phi_0^{3/2}}}{9\pi d^2 \sqrt{m_i}}$$

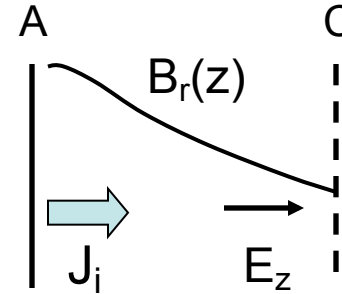
For $\phi_0 = 1100$ V, $d = 1$ mm

$$J_i^{CL} \sim 17 \text{ mA/cm}^2,$$

while real $J_i \sim 2 - 6 \text{ mA/cm}^2$

(NSTAR, XIPS-25)

Hall thruster



$$m_i J_{i\max} v_{if} = q \frac{B_0^2 - B_1^2}{8\pi}$$

For $\phi_0 = 300$ V, $B = 200$ G

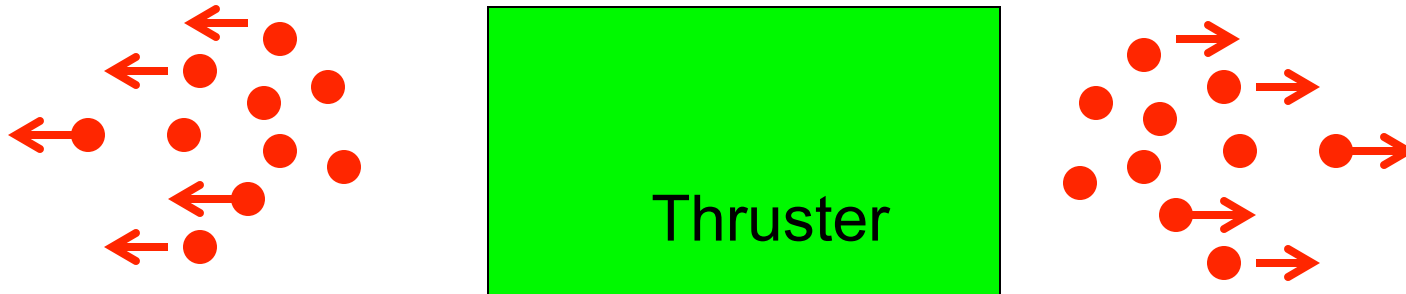
$$J_{i\max} \sim 560 \text{ mA/cm}^2,$$

while real $J_i \sim 200 \text{ mA/cm}^2$

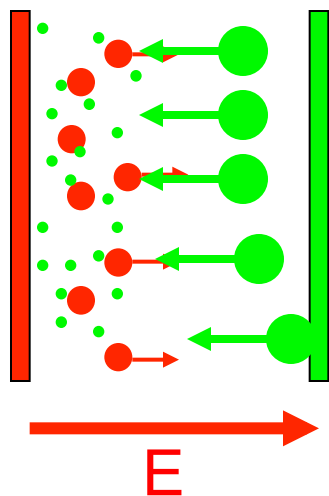
(SPT-140)

One possibility to overcome the limitations – injection of negatively charged ions (dust) at the cathode or between the electrodes.

What about maximum current density?



Can one get a larger current density in one direction, if a counter-steaming beam is introduced in the opposite direction?



Try large negatively charged ions or dust?

Space charge limit -- double layer

Not a thruster, but perhaps an ion injector.

Ion Thruster (Diode) With Negative Ion Injection

1. Injection from the cathode ($z_0=d$)

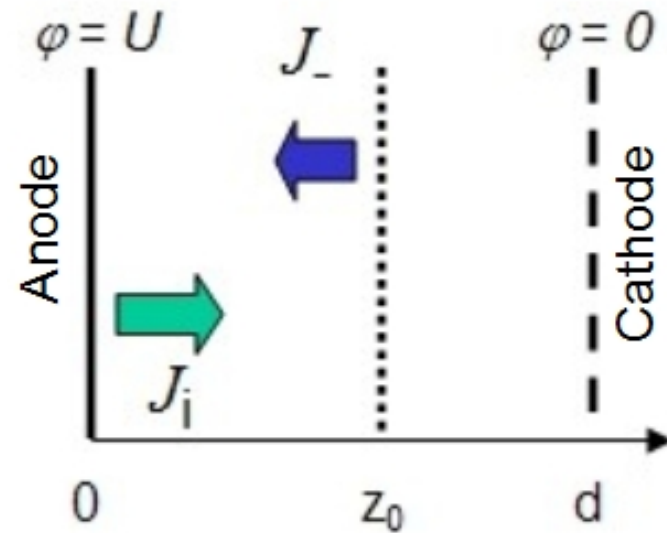
$$\delta J_i = 0.378 \sqrt{\frac{qM_-}{QM_i}} \delta J_- \gg \delta J_-$$

BUT:

For infinite supply of negative ions

$$\frac{J_i}{J_i^{CL}} = \frac{J_-}{J_-^{CL}} \approx 1.865$$

I. Langmuir, *Phys. Rev.* **33**, 954 (1929).

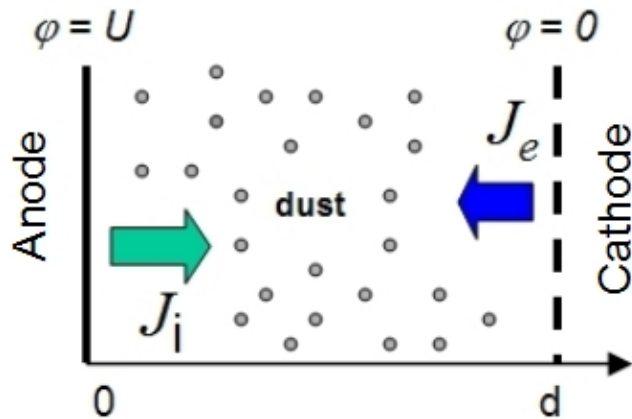


2. Injection between the electrodes

$$J_{i \max} \approx 4.59 J_i^{CL}$$

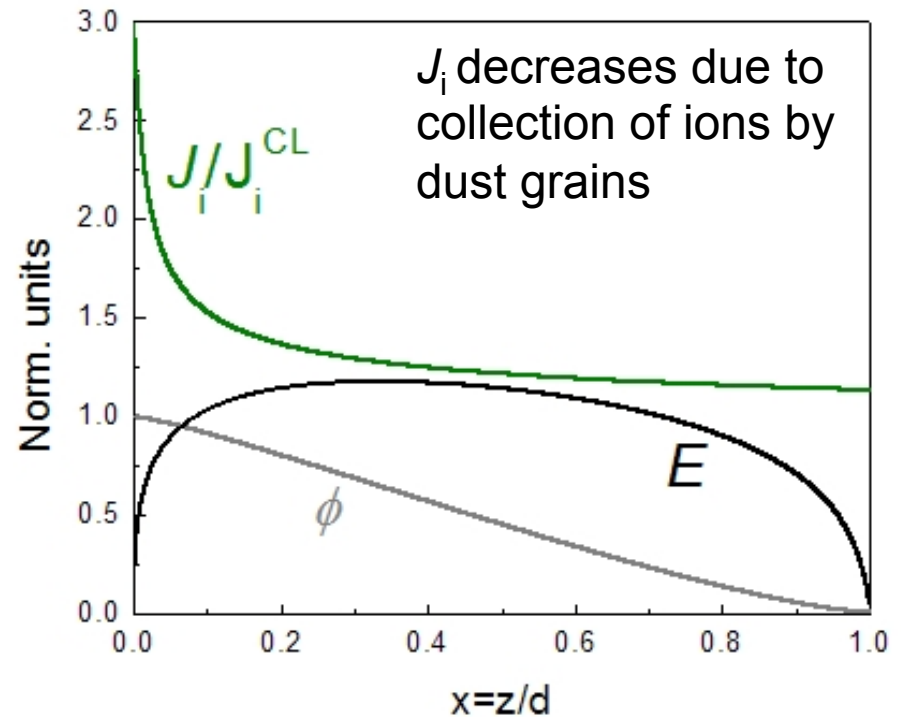
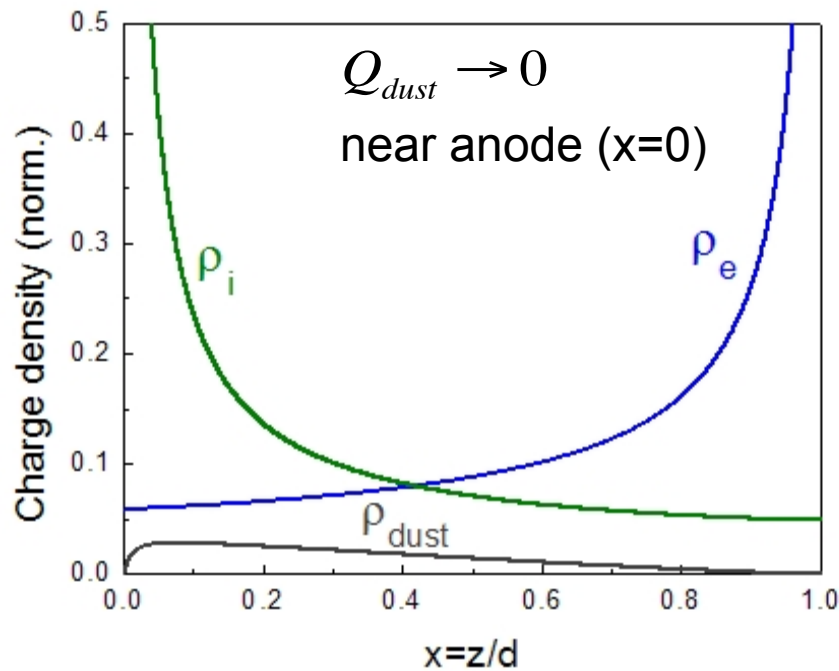
$$(z_0)_{opt} \approx 0.44d$$

Ion Thruster (Diode) With Negatively Charged Dust



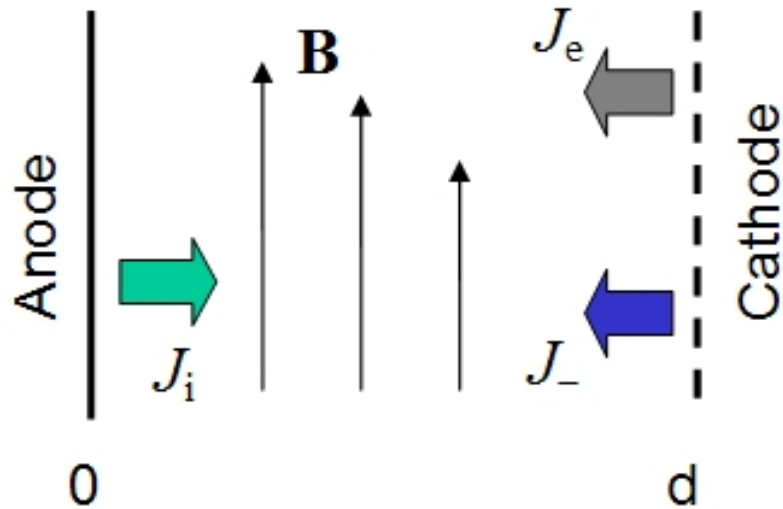
$Q_{dust} \sim 10^4 - 10^6 e$ electron impact charging

$$(M/Q)_{dust} \gg (M/q)_{ion}$$



Smirnov, Raitses, and Fisch (2005)

Summary: Hall Thruster With Negative Ion Injection



Without negative ions:

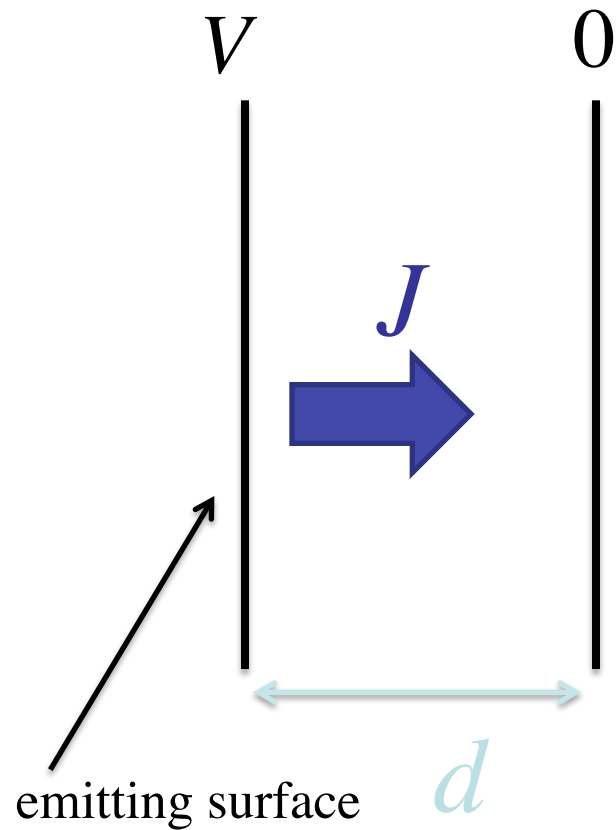
$$mJ_i v_{if} = \frac{B_a^2 - B_c^2}{8\pi}$$

Zharinov and Popov (1967)

With negative ions:

$$m_i J_i v_{if} = m_- J_- v_{-f} + \frac{B_a^2 - B_c^2}{8\pi}$$

Child-Langmuir Law: Space-Charge Limited Flow

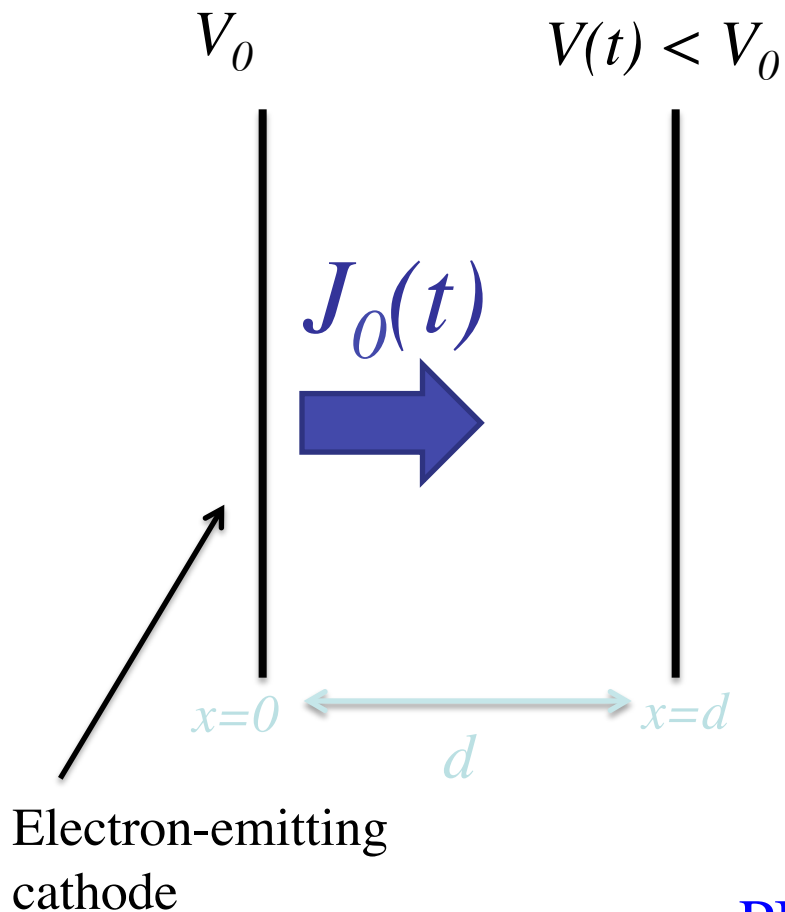


$$J_{CL} = \frac{4}{9} \epsilon_0 \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2}$$

Generalizations

- nonzero injection velocity (Langmuir, 1923)
- Relativistic (Chetvertkov, 1985)
- Time-Varying (Kadish, Peter, Jones 1985)
- Quantum (Y.Y. Lau, 1991)
- Multi-Dimensional (Luginsland, Lau and Umstattd, 2002)
- Short Pulses (Y.Y. Lau, Valfells, 2002)
- Nonlinear and Unsteady (Caffisch and Rosin, 2012)
- Coulomb Blockade (Zhu and Ang, 2012)
- Magnetic Mirror (Son and Moon, 2013)

Time-Dependent Boundary Conditions



Instantaneous current leaving the diode can exceed the steady-state limit.

But what about the average current?

(Unremarkable) Upper-Bound Proof for Time-Averaged Current Density

Griswold, Fisch and Wurtele (2010)

PIC simulations suggest time-dependent limit cannot exceed the steady-state limit.

Statement of the (unsolved) Problem:

Find the *rigorous least upper bound* for the time-averaged current

or

Is there a function $J_0(t)$ that maximizes the average current?

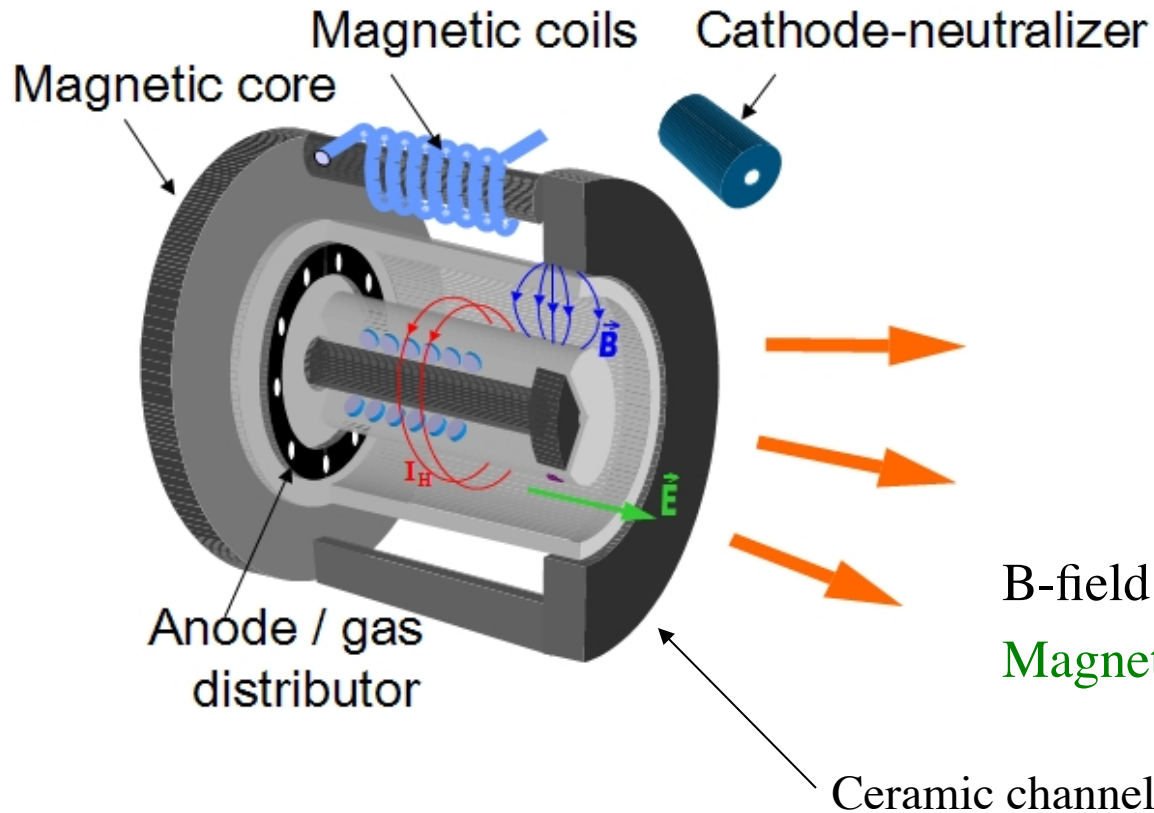
$$\frac{\partial^2}{\partial t^2} v(t, t_0) = \frac{q}{m} \left(\frac{\partial}{\partial t} E_0(t) + \frac{1}{\epsilon_0} J_0(t) \right)$$

$$E_0(t) = \frac{V}{d} - \frac{1}{\epsilon_0} \int_0^t J_0(t') dt' + \frac{1}{\epsilon_0 d} \int_0^t x(t, t'_0) J_0(t'_0) dt'_0$$

subject to the boundary condition: $E_0(t) \geq 0$, for all t .

And, similarly unsolved, for $J=J(t)$, $B=B(t)$,
prove rigorous upper bound for *unsteady* Hall thruster current.

Single Particle Confinement in Conventional Hall Thruster



$$\rho_e \ll L \ll \rho_i$$

Electrons are magnetized
Ions are not

$$\mathbf{E} = -V_e \times \mathbf{B}$$

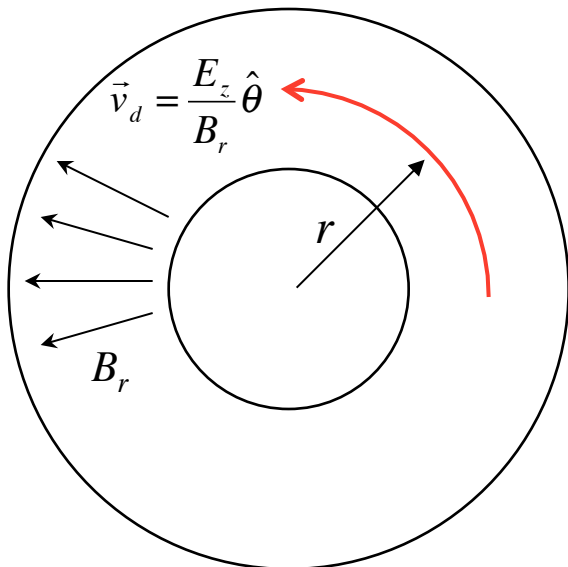
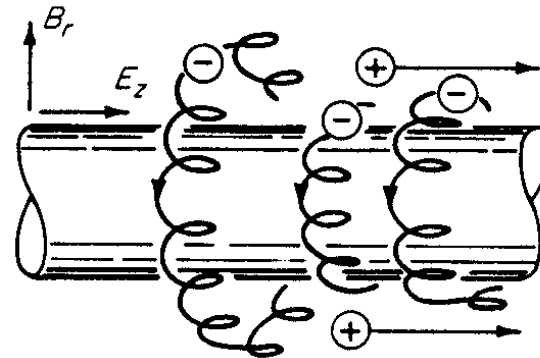
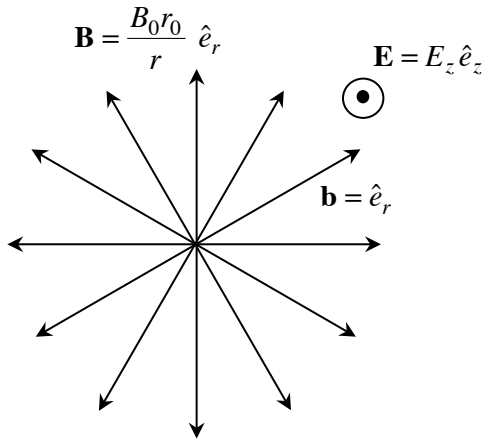
B-field is radial; electric field is axial
Magnetic Surfaces are Equipotential
(if $T_e=0$)*

Space charge limit on ion current density replaced by B-limit

*This assertion will be challenged

Electron Motion in crossed radial magnetic and axial electric fields (Hall Thruster)

Courtesy: Lyon B. King, IEPC 2005



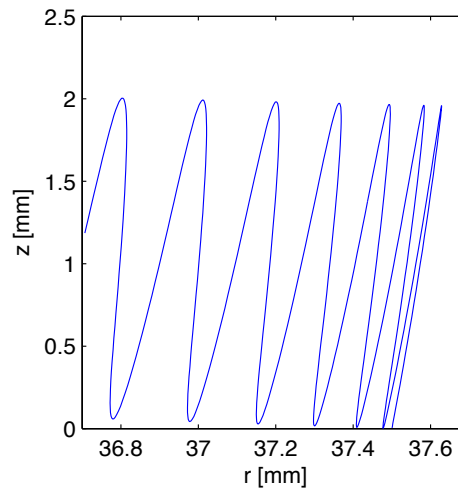
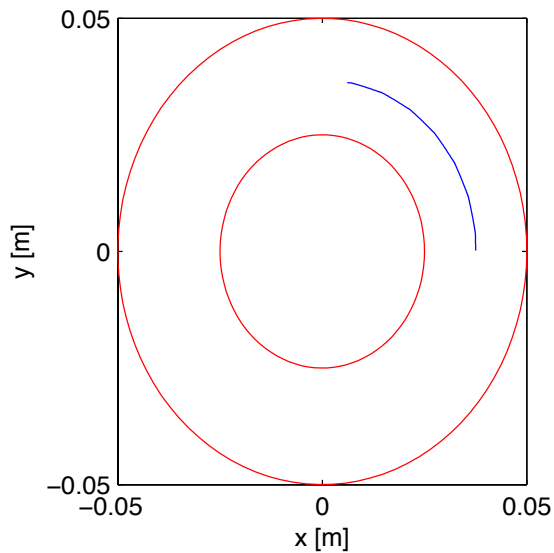
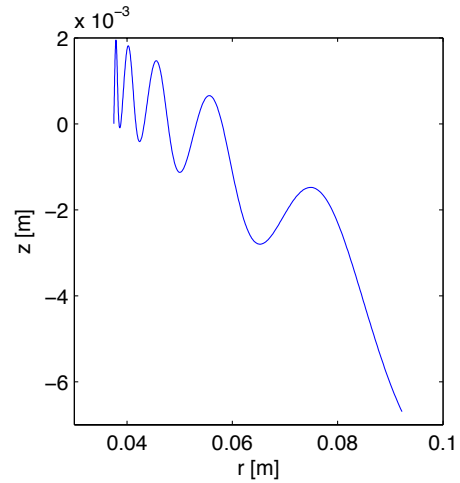
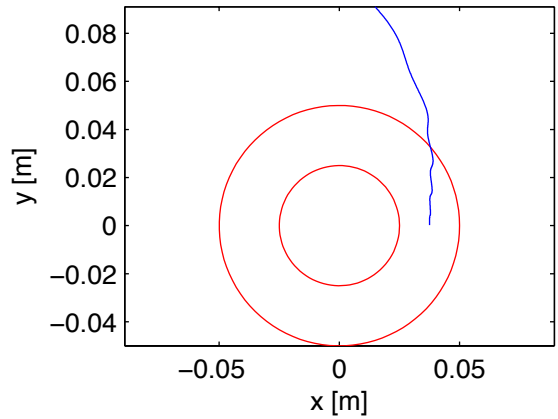
$$\dot{v}_{\parallel} = \frac{u_{\perp}^2}{2r} + \frac{E_z^2 r}{B_0^2 r_0^2}$$

Magnetic Mirror Force

Centrifugal Force

Radial Electric Field Necessary for Confinement

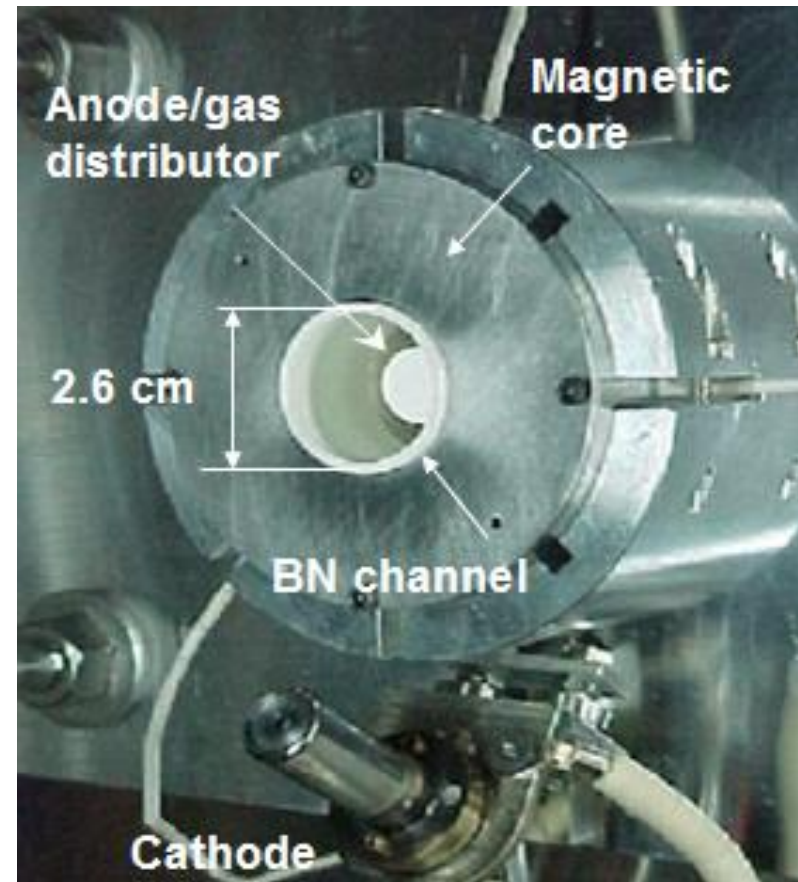
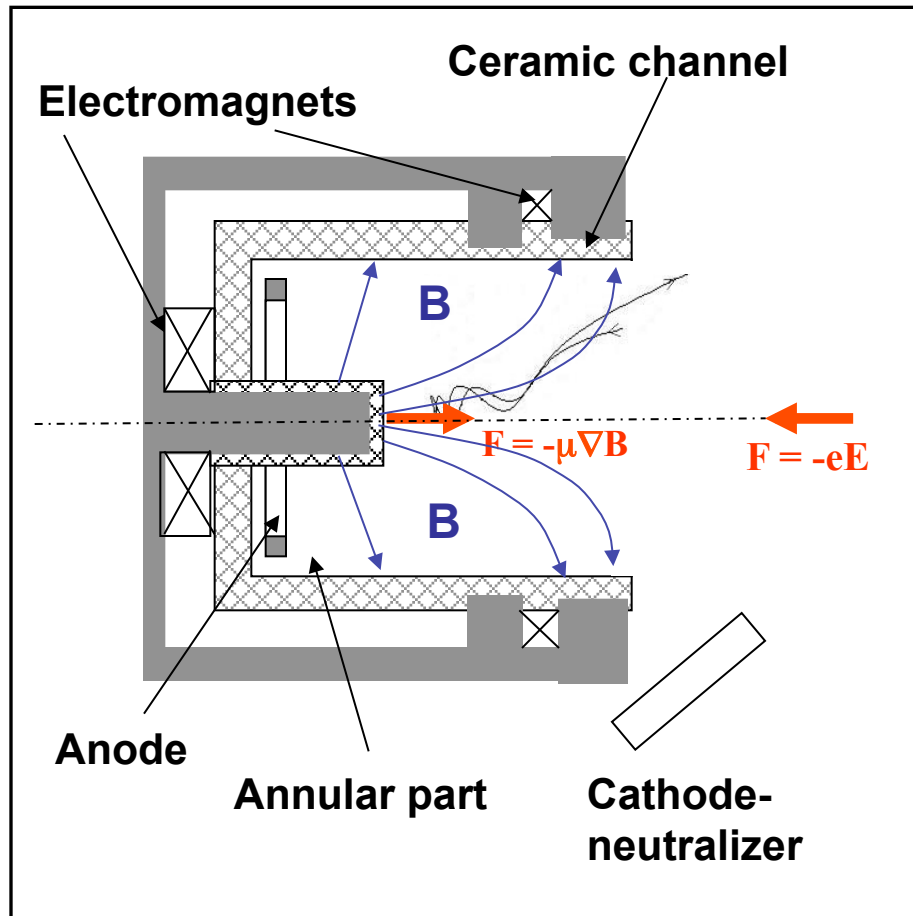
in addition to compensating space charge



Do axial oscillations matter?

Relax Constraint of Axial Rigidity → Cylindrical Hall Thruster

Fundamentally different from conventional HT:
Electrons are confined in a hybrid magneto-electrostatic trap.



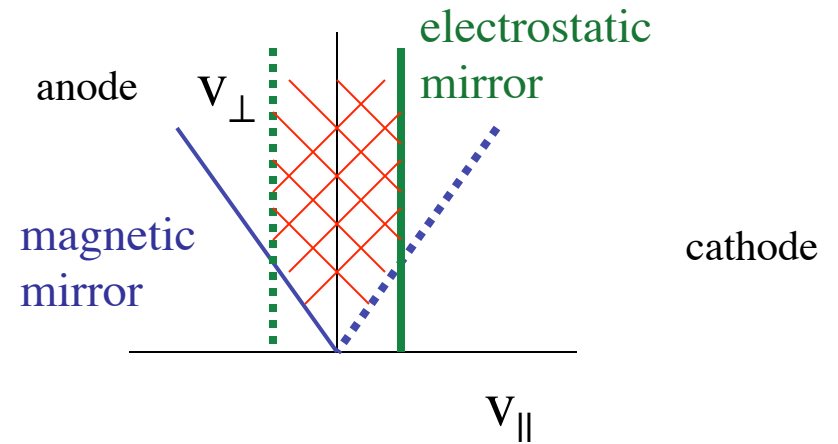
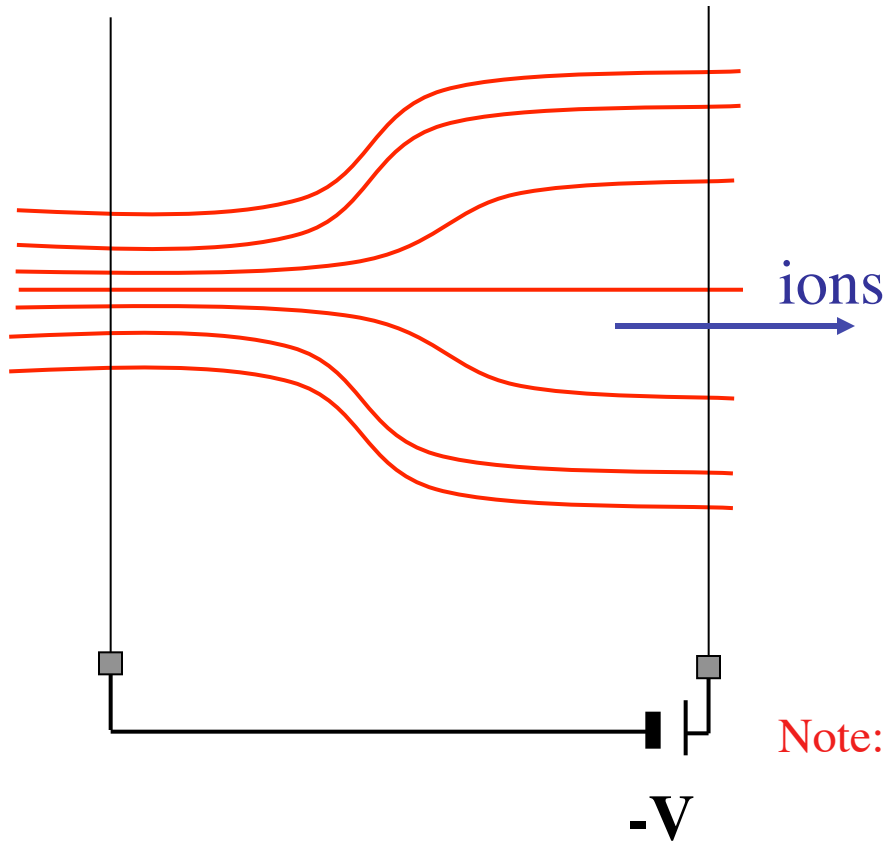
PPPL CHT: $P = 50 - 300 \text{ W}$
 $OD = 2.6 \text{ cm}$
 $T = 2 - 12 \text{ mN}$

Charge neutralization by trapped electrons

Magnetic-electric mirror

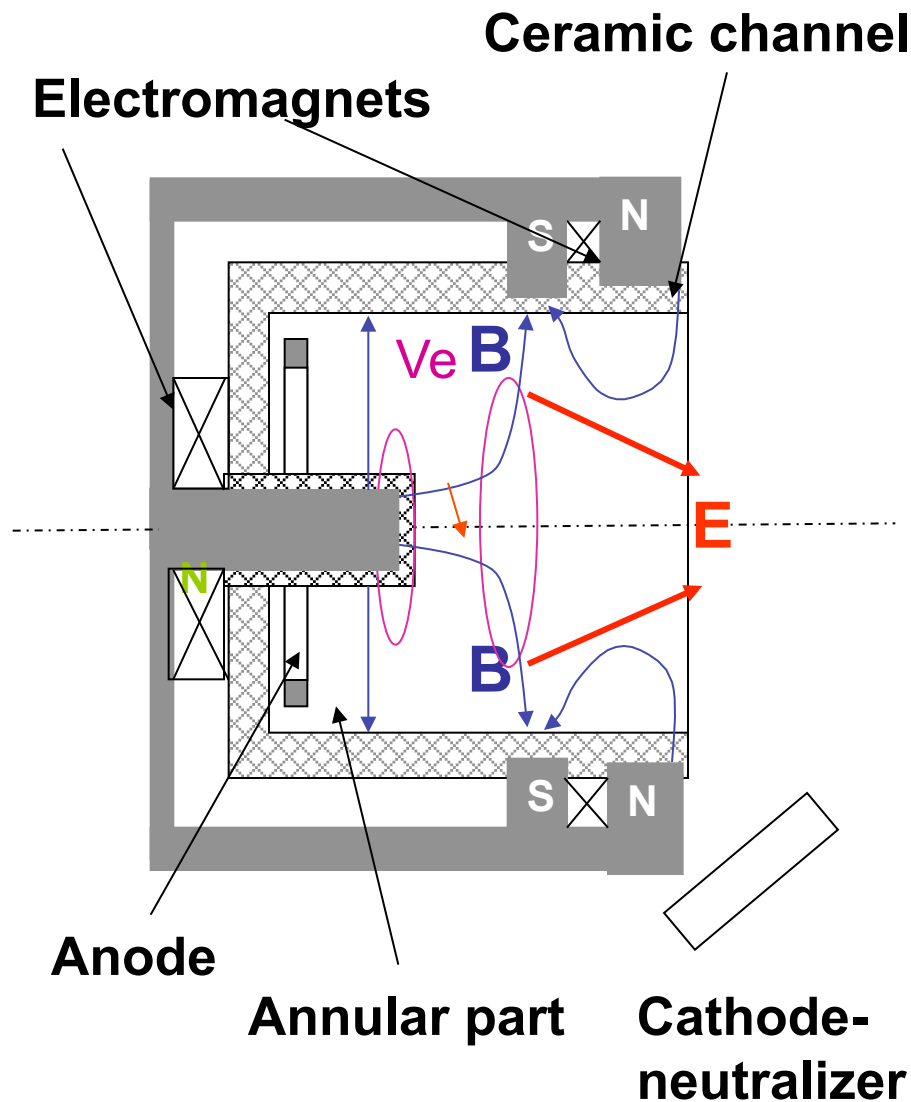
Electrons are mirror trapped on left and electrostatically on right

Note: electrons are still confined for “bent section” possibly useful to control thrust vector



Note: low-energy electrons leave towards anode
high-energy electrons leave towards cathode

Cylindrical Hall Thruster



larger volume to surface ratio

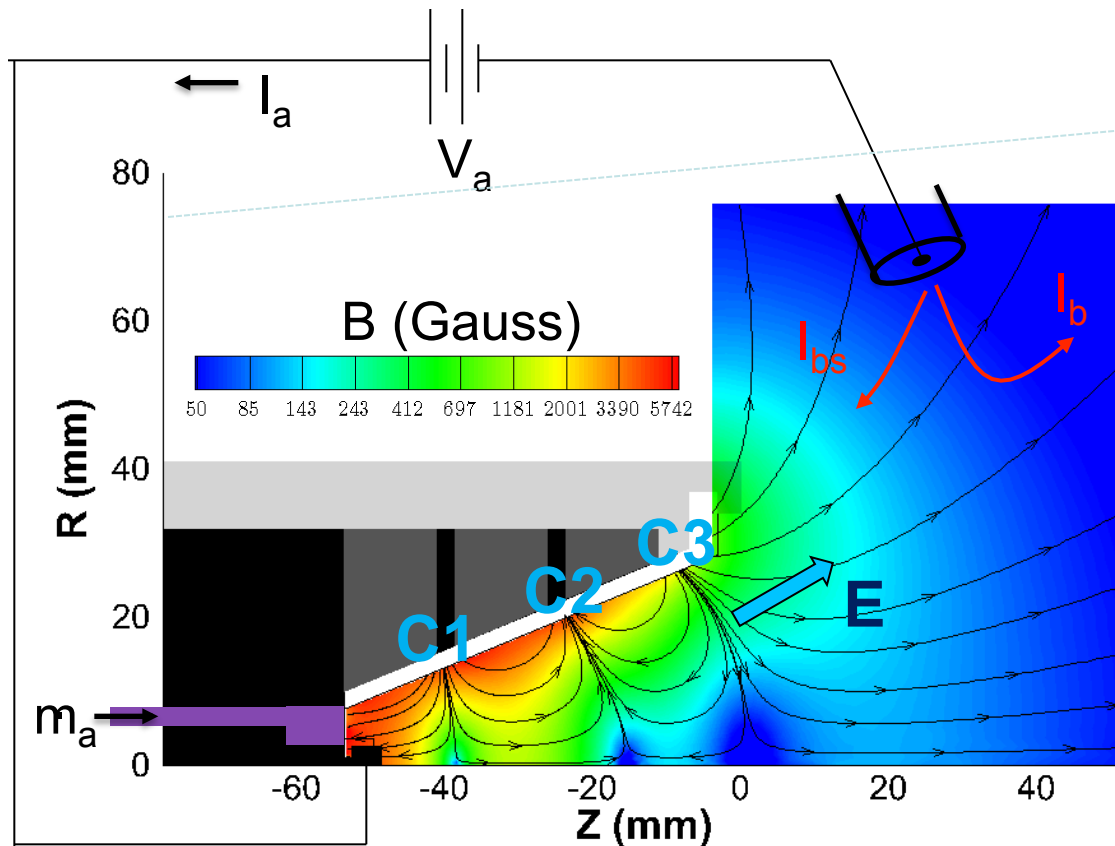
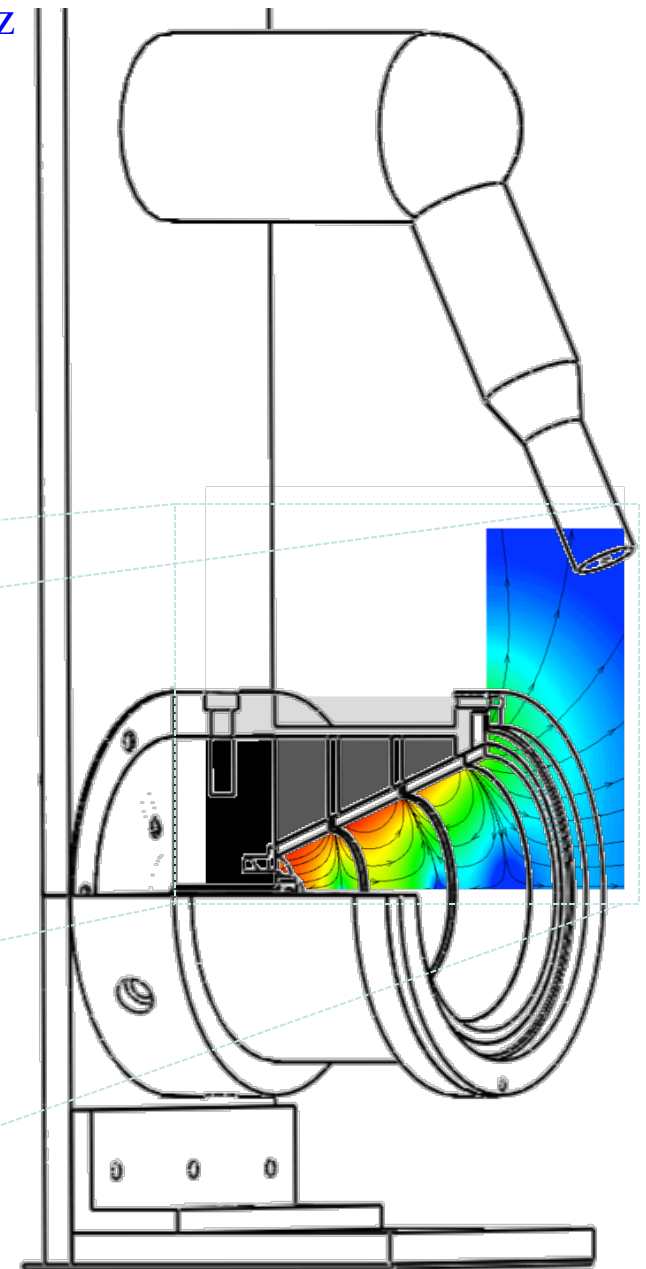
- closed azimuthal electron drift
- Ion acceleration is mainly axial
- short annular high density region
- Length of the annular region $\sim \lambda_{\text{ion}}$

Y. Raitses and N.J. Fisch, Phys. Plasmas **8**, 2579 2001.

MIT Diverging Cusped Field Thruster

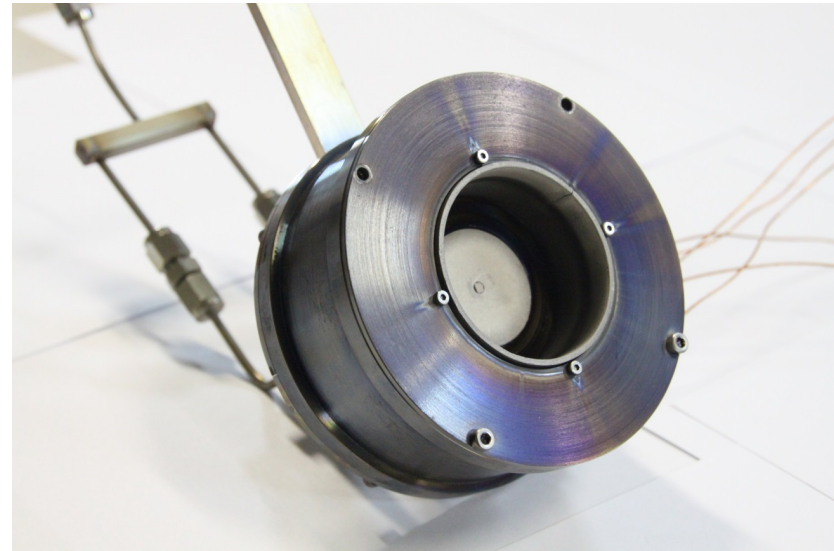
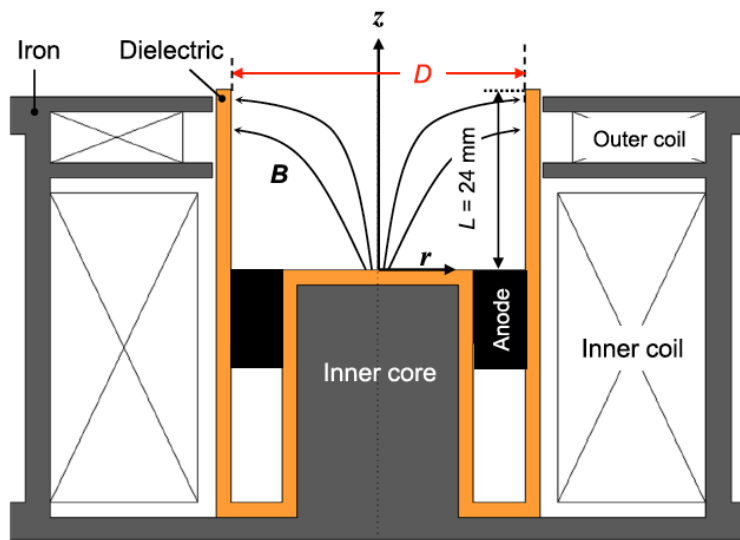
Martinez-Sanchez

- Low power (200 W) – designed for long life
- Max. efficiency measured 44% at 240 W
- Plasma loss to walls only at cusps
- Max. erosion rate (at C2) $1.9\mu\text{m/h}$ in oscillatory mode
- Measure majority of potential fall downstream of exit



KAIST Cylindrical Thruster

Choe et al



KAIST KCHT-50

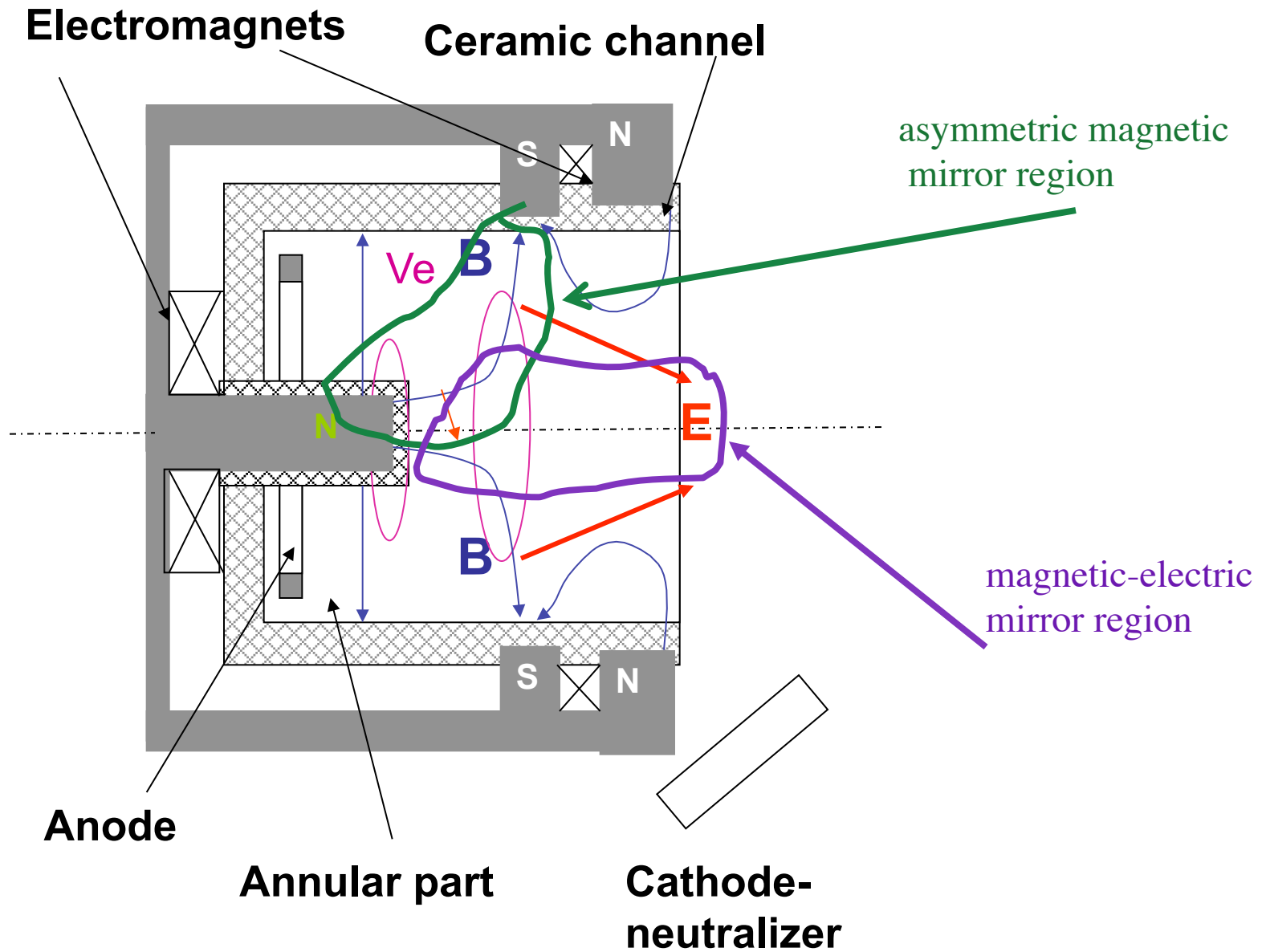
Appl. Phys. Lett. 103, 133501 (2013)

Radial scale effect on the performance of low-power cylindrical Hall plasma thrusters

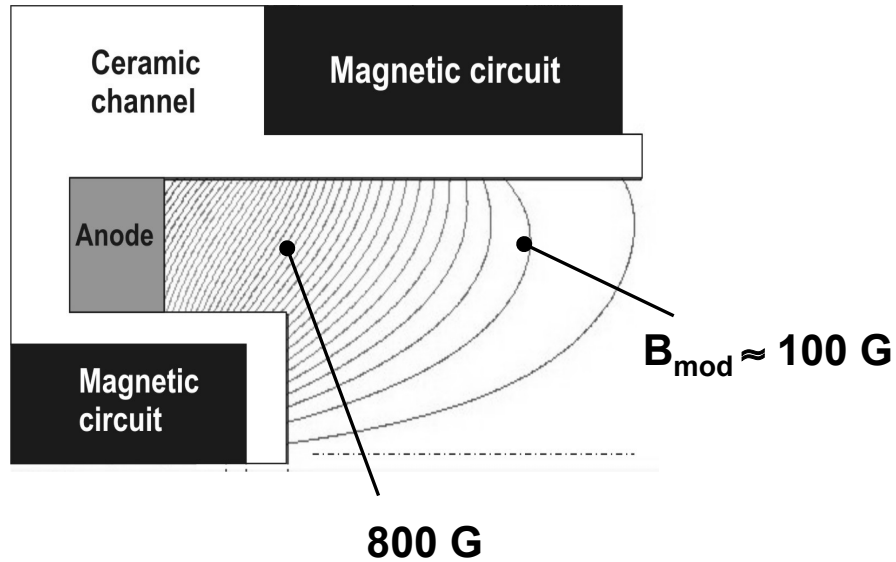
M. Seo, J. Lee, J. Seon, H. J. Lee, and Wonho Choe

Comparison of beam characteristics between annular type and cylindrical type low power Hall thrusters, IEPC-2013-221

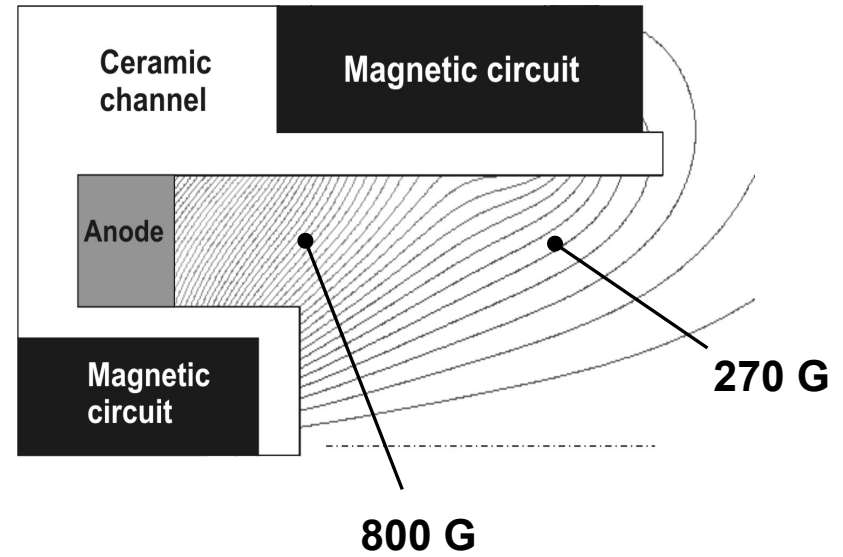
Cylindrical Hall Thruster with Cusp Fields



Cylindrical Configurations



Cusp Geometry

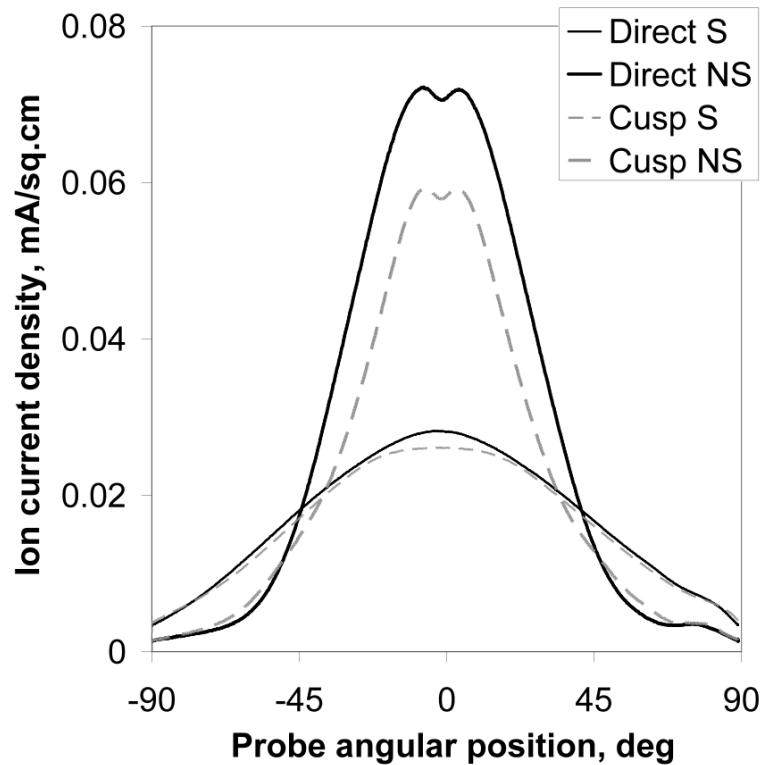


Direct Geometry

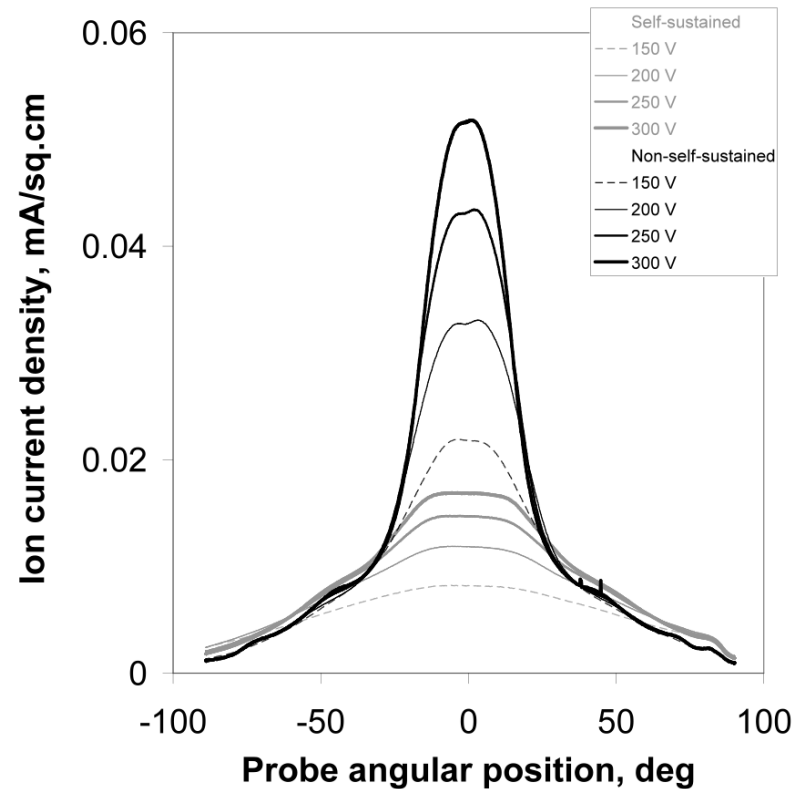
Cusp Geometry was thought important to produce axial thrust

Surprises!

Raitses, Smirnov, Fisch, APL, 2007



Cylindrical thruster

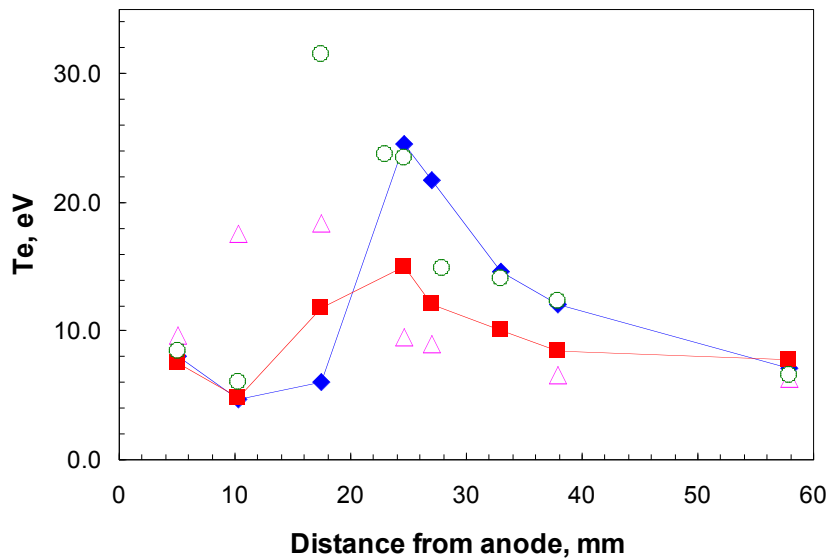
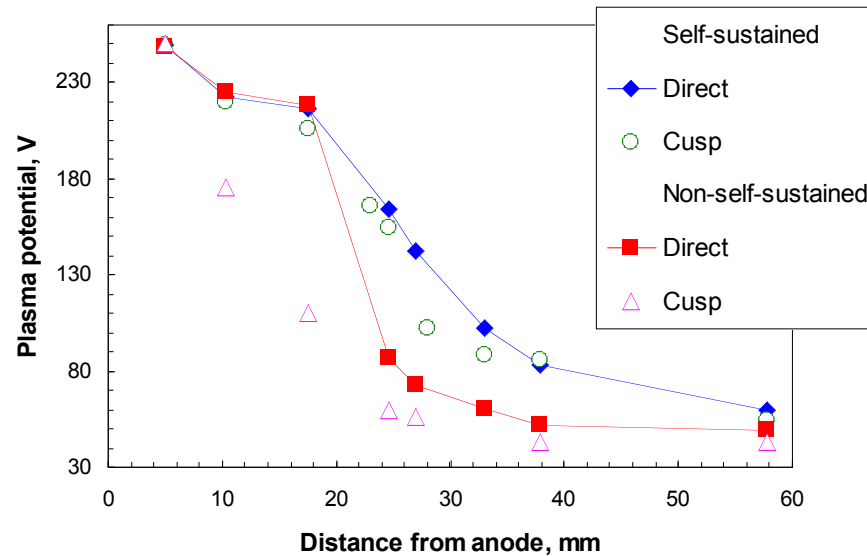


Annular thruster

1. With enhanced cathode electron supply, plume is narrowed.*
2. Direct geometry is just about as efficient as cusp geometry!

**current overrun regime (NS)*

Temperature Anomaly

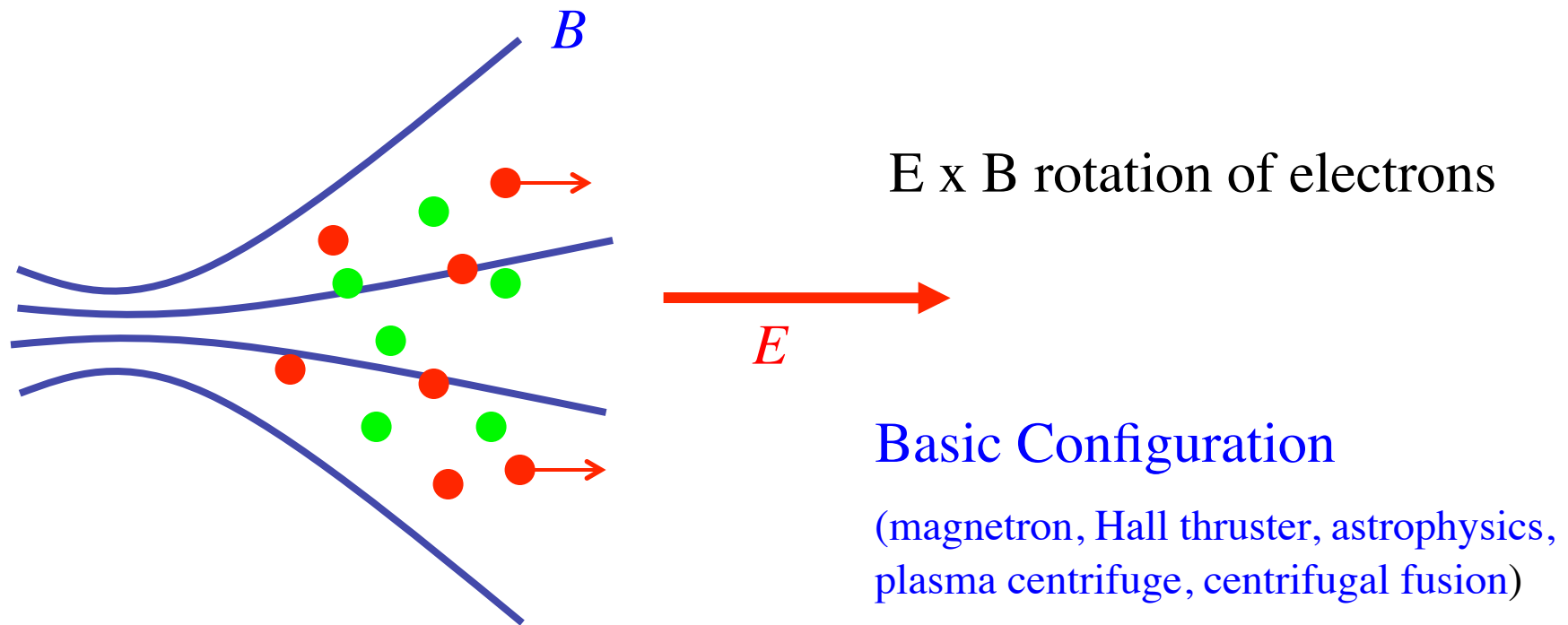


Observables

1. Plume is narrowed by $\sim 30\%$ in non-self-sustained (current-overrun) regime.
2. Electron density “peaking” on axis.
3. Electron temperature lower by perhaps $\sim 15\%$ compared to self-sustained regime.
4. Voltage drop is steepened and moves towards anode.
5. Electron temperature decreases axially near anode.

Acceleration in Rotating Electron Plasma

Basic question: How does the plasma potential self-organize in rotating magnetized electron plasma (thereby e.g. to propel un-magnetized ions)?



Working assumption:

Magnetic surfaces are nearly equipotential surfaces

Isorotation Theorem

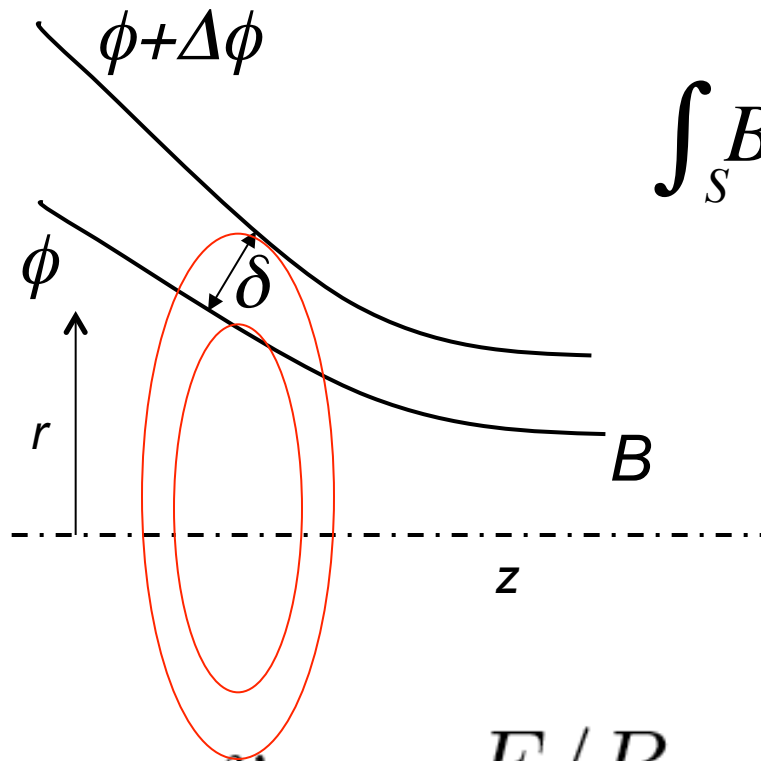
Flux surface: $rA = \text{const}$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{A} = A_\theta(r, z) \hat{\theta}$$

$$\int_S \mathbf{B} \cdot d\mathbf{s} = \int_S \nabla \times \mathbf{A} \cdot d\mathbf{s} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

$$\Delta(2\pi r A) = 2\pi r B \delta,$$

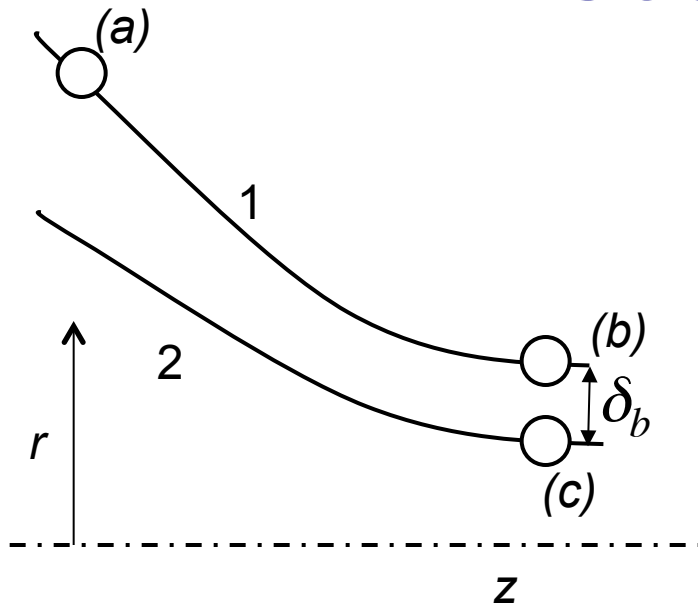


Since, $E \perp B$, $E = -\Delta\phi/\delta$

$$v_{d\theta} = E/B = r\Delta\phi/\Delta(rA)$$

But both ϕ and rA are constant on the flux surfaces, so their differences are retained as well. QED

Cooling corollary



$$\Delta(2\pi r A) = 2\pi r B \delta,$$

$$\begin{aligned} P_\theta &= r (m v_\theta + q A_\theta) \\ q \Delta[r A_\theta] &= -m (r_a v_{\theta a} - r_c v_{\theta c}) \\ &= -m (r_a E_a / B_a - r_c E_c / B_c) \\ &= -m (r_a E_a / B_a - r_b E_b / B_b) - \Delta_{bc}, \\ &= m (E_b / B_b r_b) (r_b^2 - r_a^2) - \Delta_{bc}, \\ \Delta_{bc} &\equiv m \left(\frac{r_b E_b}{B_b} - \frac{r_c E_c}{B_c} \right) \simeq m \delta \left[\frac{\partial}{\partial r} \left(\frac{r E}{B} \right) \right]_{r=r_b} \\ \delta_b &= \frac{m E_b}{B_b r_b} \frac{(r_b^2 - r_a^2)}{\left[q r B + m \delta \frac{\partial}{\partial r} \left(\frac{r E}{B} \right) \right]_{r=r_b}} \end{aligned}$$

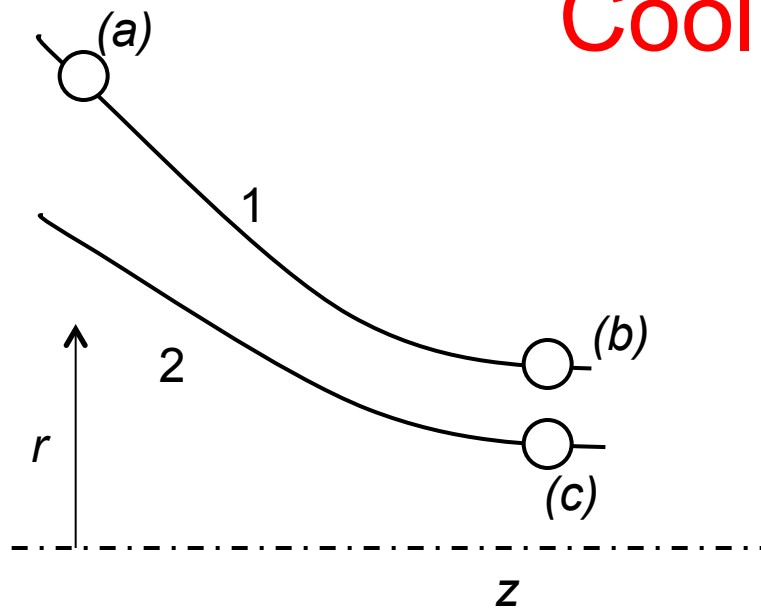
Neglect shear term, for scale length of $E/Br \sim r$; Note $\Omega/\Omega_r = (v_t/v_D)(r/\rho)$



$$\begin{aligned} q \Delta \phi &\simeq q \delta_b E_b = m \left(\frac{E_b}{B_b} \right)^2 \left(1 - \frac{r_a^2}{r_b^2} \right) \\ &= m \left(\frac{E_b}{B_b} \right)^2 - m \left(\frac{E_a}{B_a} \right)^2 \end{aligned}$$

Northrop (1963)

Cooling Corollary



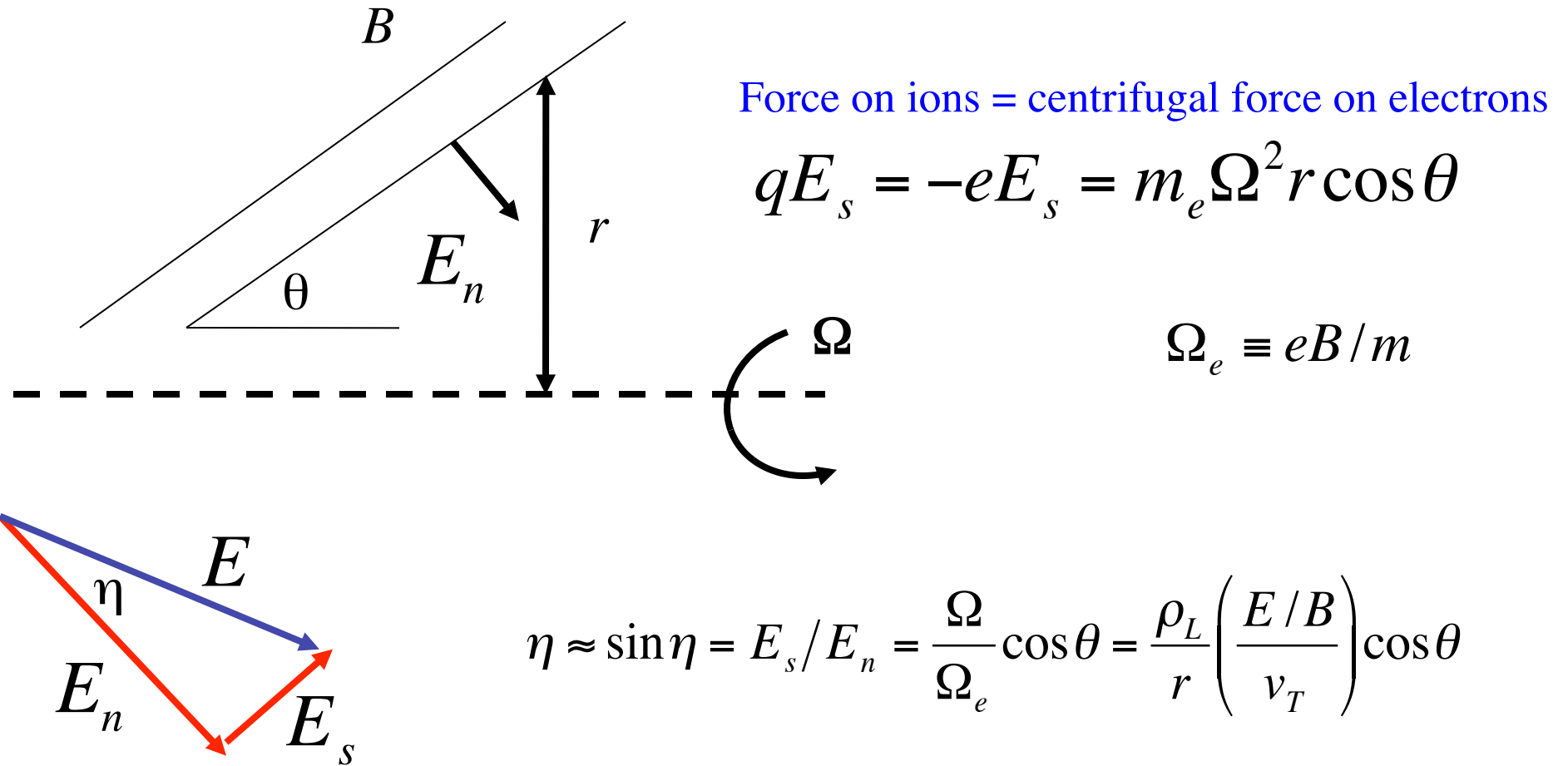
Particles moving along equipotential surfaces, regardless of the shape of the surface so long as it is azimuthally symmetric, gain in potential energy exactly twice the kinetic energy lost in azimuthal drift energy, so as to climb up the electric potential.

That means that particles at (c) will have less kinetic energy than do particles at (a) by exactly twice the difference in rotation energies.

Note that *in contrast* in the absence of the vector potential term (which dominates the angular momentum for magnetically confined particles), particles at smaller radius need higher kinetic energy to conserve angular momentum; hence particles are heated rather than cooled in going to smaller radii!

Same effect gives centrifugal fusion extra axial confinement

A. Rotation of Force Vector by Supersonically Rotating Electrons



Example: $T_e = 20 \text{ eV}$,

$E_n = 200 \text{ V/cm}$, $a=L=1 \text{ cm}$

$$\rho_L \cong T_{20}^{1/2} / B_{100} \text{ mm}$$

$$r \sim 10 \text{ mm}$$

or 12 degrees for $r = 5 \text{ mm}$

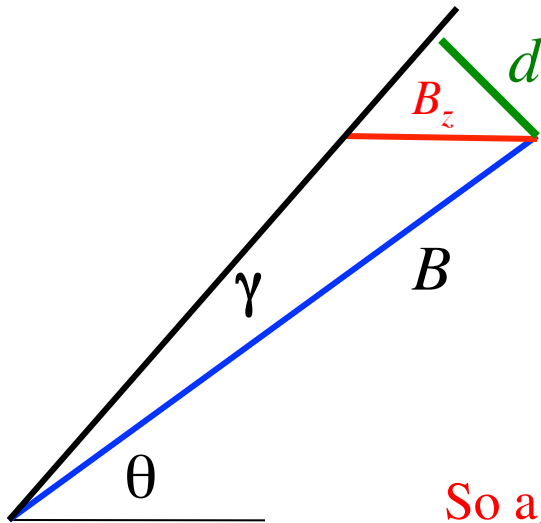
B. Deflection of Magnetic Surface by Hall Field

$$B_z = \mu_0 r J_\varphi / 2 = \mu_0 r n E / 2 B$$

$$\frac{B_z}{B} = \frac{1}{2} \left(\frac{r}{\rho_e} \frac{\rho_L}{\rho_e} \frac{E/B}{v_T} \right)$$

$$\rho_L \equiv v_T / \Omega_e = \frac{T_{20}^{1/2}}{B_{100}} \text{ mm}$$

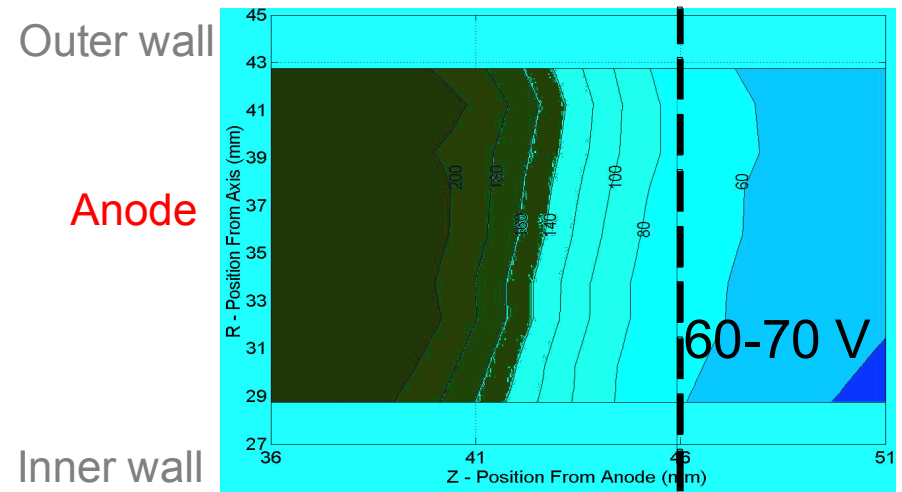
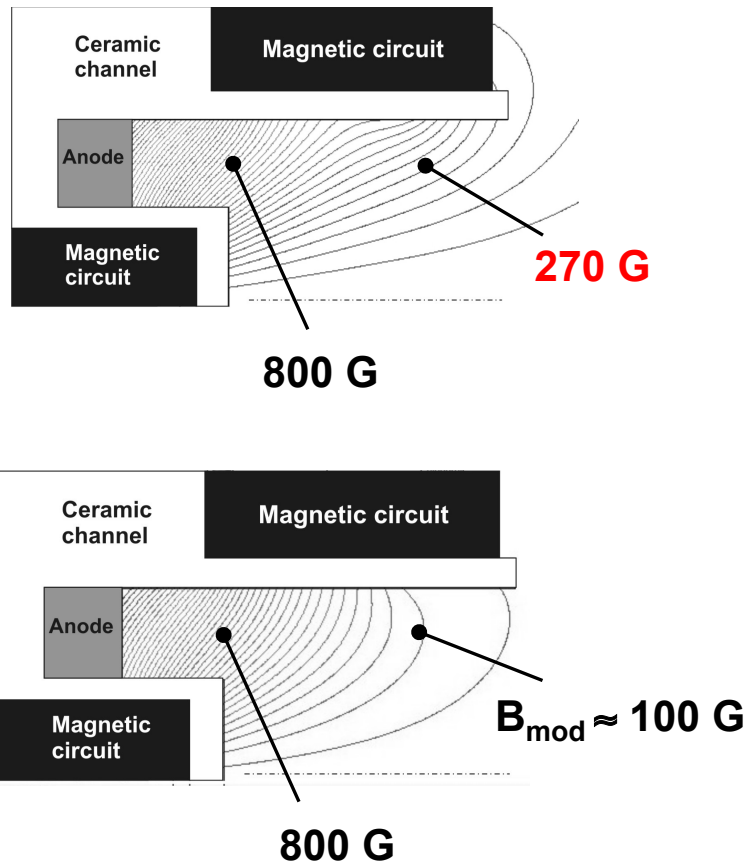
$$\rho_e \equiv c / \omega_p = 5 n_{12}^{1/2} \text{ mm}$$



$$\gamma \cong \frac{d}{B} = \frac{B_z \sin \theta}{B} = \frac{1}{2} \left(\frac{r}{\rho_e} \frac{\rho_L}{\rho_e} \frac{E/B}{v_T} \right) \sin \theta$$

So again rotate γ by about 12 degrees for sonic rotation.

Rotation Speeds



Equipotential surfaces
Annular Thruster
(note convergent geometry \rightarrow useful adjustment)

Also called “magnetic lens” geometry

Example:

$E \sim 200\text{-}300$ V/cm (along axis)

$B \sim 270$ G

$E/B \sim 10^8 (E_{200}/B_{200})$ cm/sec

Compare to Thermal:

$v \sim 4.2 \times 10^7 T_e^{1/2}$ cm/sec

Note: roughly sonic!

Summary of Rotations of Force Vector

$$\eta \cong \frac{\rho_L}{r} \left(\frac{E/B}{v_T} \right) \cos \theta$$

Centrifugal Rotation

$$\gamma \cong \frac{1}{2} \left(\frac{r}{\rho_e} \frac{\rho_L}{\rho_e} \frac{E/B}{v_T} \right) \sin \theta$$

Hall-field Rotation

1. Force rotation tends to straighten (make axial) the force on ions.
2. Supersonic electron rotation enhances the effect.
3. Effects are additive and can be in the range of ten degrees.
4. Effects are inner-outer-wall asymmetric.

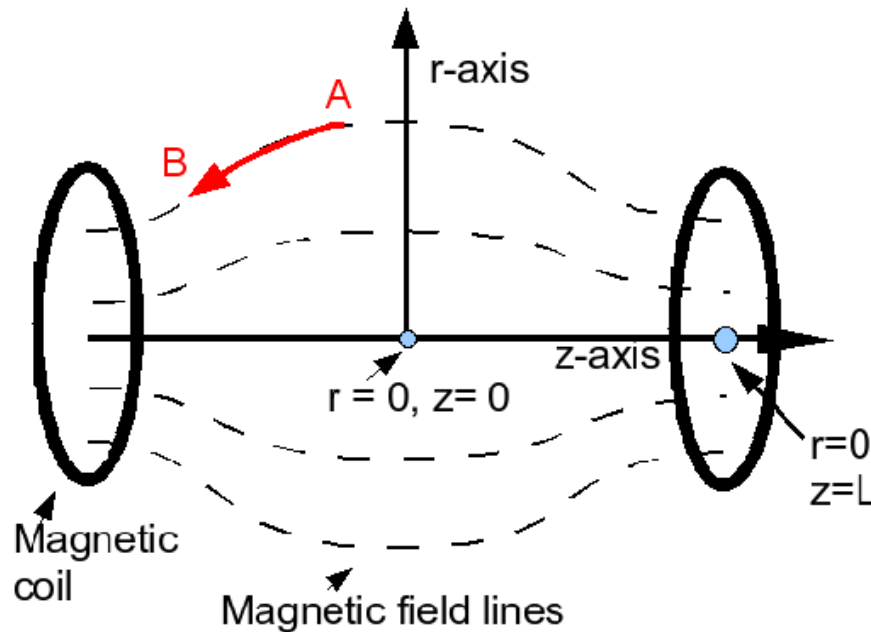
Summary of Thrust-Straightening Physics

- A. Identified curious, surprising, but fundamental effects in $E \times B$ rotating plasma

- B. Identified mechanisms of deflecting E-field from (generalized) B-surface normal
 1. Generation of E-field to restrain supersonically rotating electrons
 2. Generation of axial B from Hall current (only electrons $E \times B$ rotate)

- C. Related mechanisms of deflecting E-field to experimental data
 1. Act to straighten *convergent* thrust vectors!
 2. Deflection increases with radius – no need for cusp.
(Could be counterproductive!)
 3. Electron-starved discharge likely has these effects less pronounced.
 4. Mechanism for apparent cooling of electrons towards anode.

Mirror Confinement Fusion



Reflection of Particle moving from A to B

$$\mu = \frac{mv_{\perp A}^2}{2B_A} = \frac{mv_{\perp B}^2}{2B_B}$$

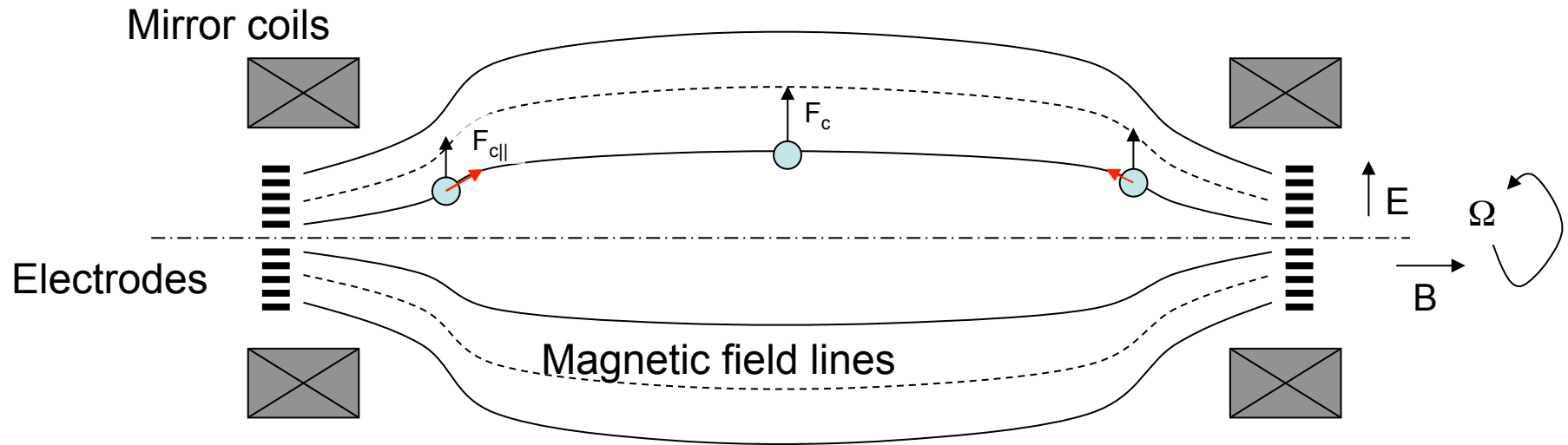
Conservation of Magnetic Moment

$$\frac{1}{2}mv_{\perp B}^2 = \frac{B_B}{B_A} \frac{1}{2}mv_{\perp A}^2$$

$$\frac{1}{2}mv_{\parallel A}^2 + \frac{1}{2}mv_{\perp A}^2 = \frac{1}{2}mv_{\parallel B}^2 + \frac{1}{2}mv_{\perp B}^2$$

Conservation of Energy

Centrifugal Confinement Fusion



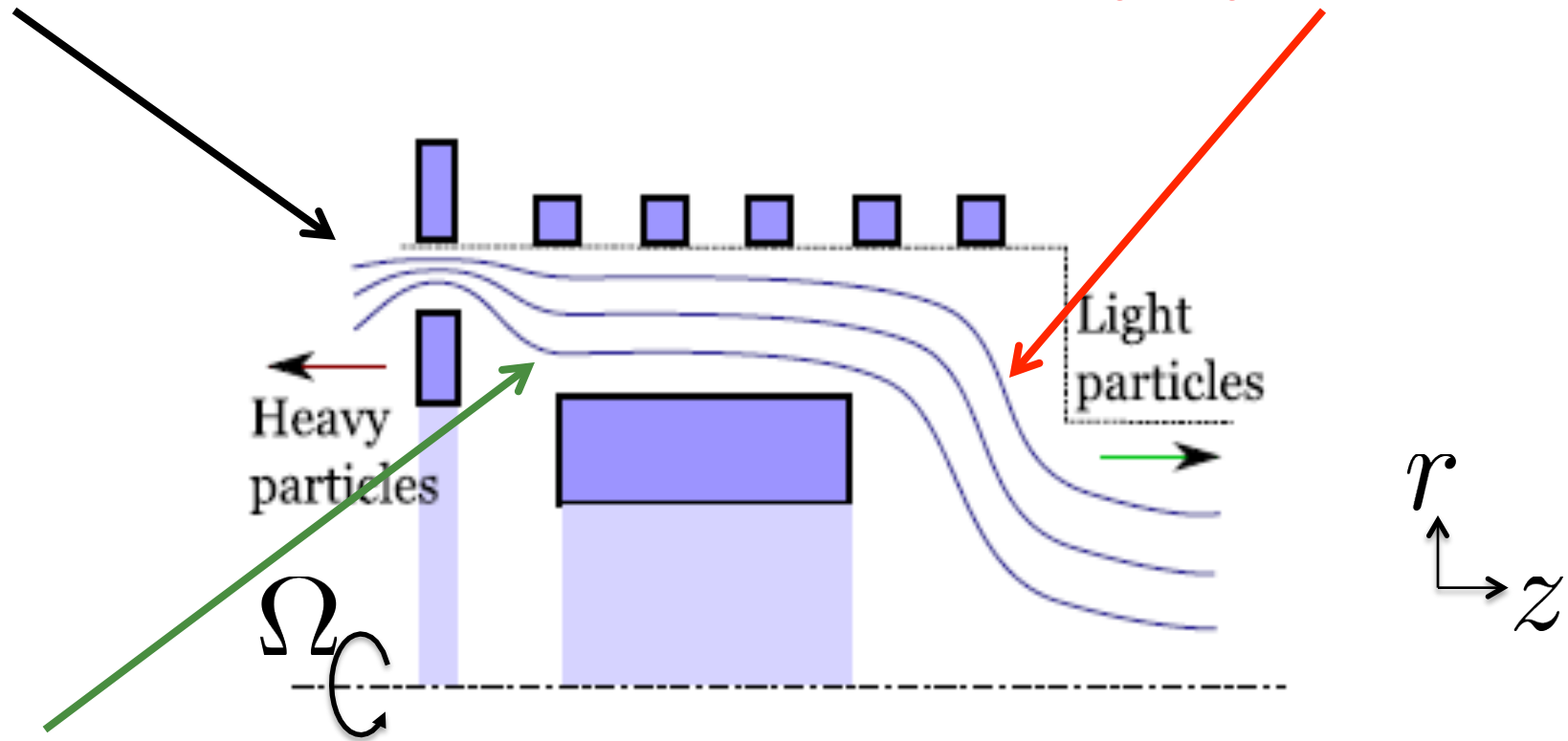
$$W_{\parallel 0} < W_{\perp 0} (R_m - 1) + W_{E0} (1 - R_r^{-1}).$$

$$R_m = B_m/B_0 \quad R_r = r_0^2/r_m^2 \quad W_{E0} = m\Omega_E^2 r^2/2 \quad \Omega_E = -E_r/r B_z$$

Magnetic Centrifugal Mass Filter

Centrifugal force on heavy ions is sufficiently strong to overcome the $\mu\nabla B$ force.

Centrifugal force is not sufficient to confine energetic light ions.



Ring electrodes enforce radial voltage drop (with magnetic surfaces are equipotential)

Opportunity for plasma separation techniques

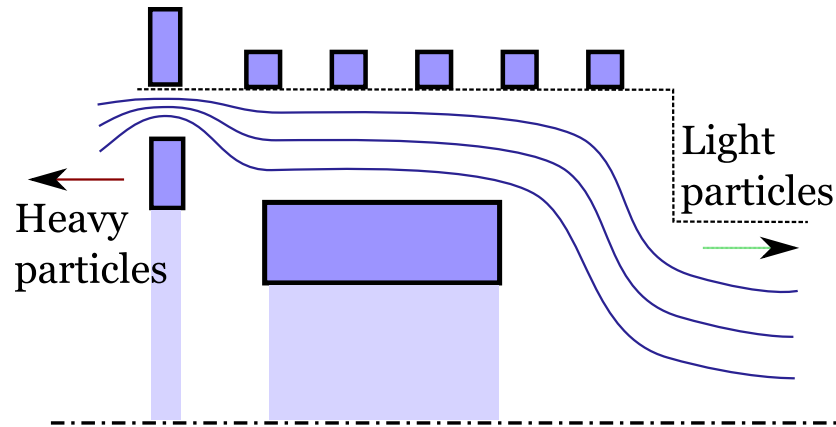
- Archimedes planned to process 800 MT of solid HLW per year (1/2 Hanford).
- Other waste problems may be addressed with plasma mass filters.
 - Waste similar to Hanford at the Savannah River Site or other US sites
 - Waste from reprocessing in other countries
 - Nuclear plant de-commissioning, nuclear accidents
 - Hanford site cleanup > 100B\$.
- Ionization costs alone are around \$10/kg. Archimedes estimated \$50/kg total cost. Chemical separation is approximately \$1000/kg.

Total energy cost (assume 800 MT sodium, 1 keV/atom ionization, \$0.15/kWh):

$$800 \text{ MT/yr} \times 10 \text{ yr} \approx 1.3 \times 10^9 \text{ \$}$$

$$(2.6 \times 10^{28} \text{ atoms/MT}) \times 1 \text{ keV/atom} \times (1.6 \times 10^{-25} \text{ eV/MJ}) \times (0.04 \text{ \$/MJ})$$

MCMF Advantages



Confinement condition only depends on mass

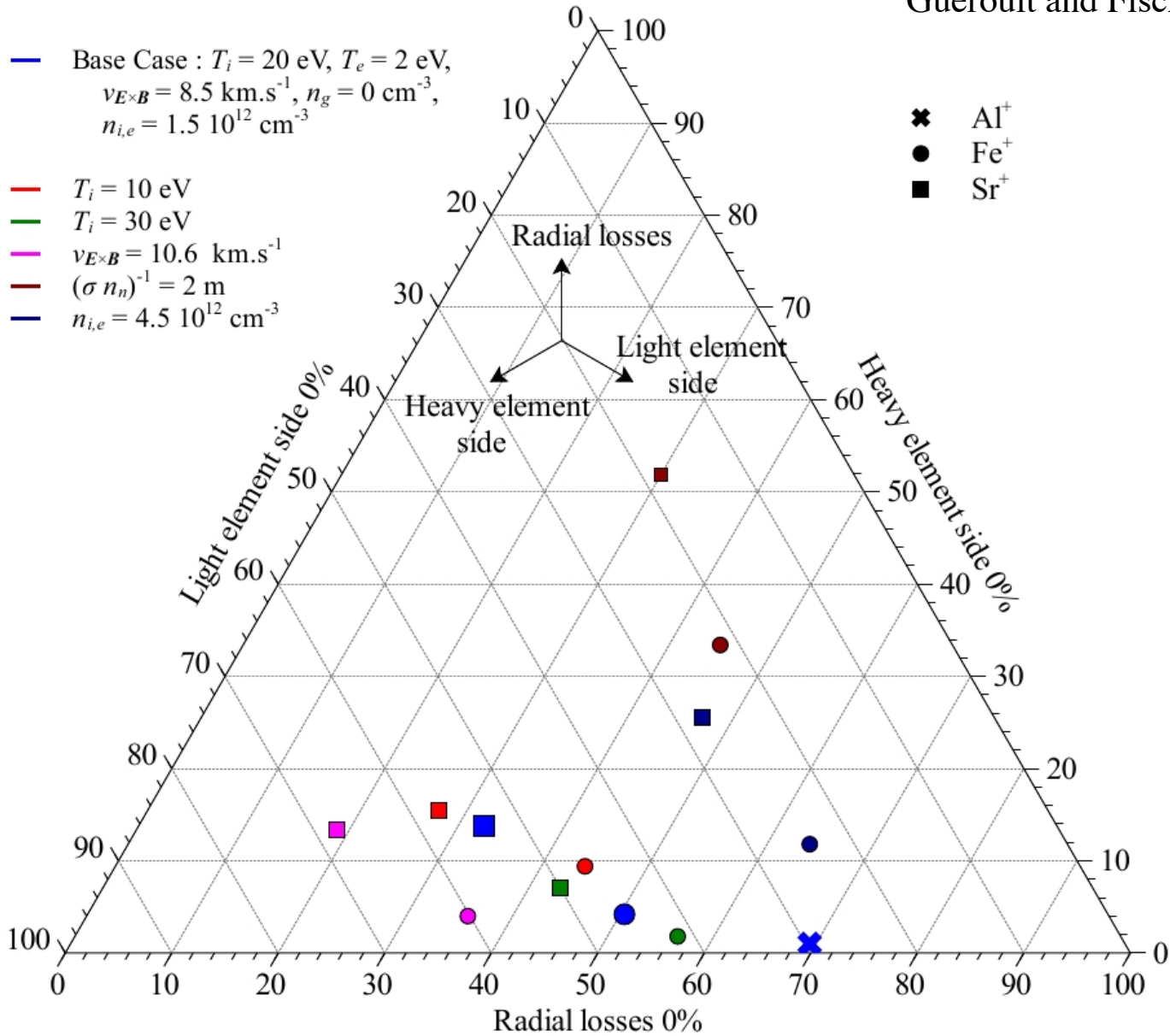
$$W_{\parallel 0} < W_{\perp 0} (R_m - 1) + W_{E0} (1 - R_r^{-1}) .$$

$$R_m = B_m / B_0 \quad R_r = r_0^2 / r_m^2 \quad W_{E0} = m \Omega_E^2 r^2 / 2 \quad \Omega_E = -E_r / r B_z$$

1. Output streams collected axially over a smaller area
2. Plasma source can be on field lines
3. Works on large mass differences (less proliferative)

Simulations of Separation Effect

Gueroult and Fisch (POP, 2012)



Very Interesting Result!

JPL Reduced-Erosion Tests on Magnetically-Shielded Hall Thrusters

Wear test of a magnetically shielded Hall thruster at 3000 seconds specific impulse

IEPC-2013-033

Hofer, Jorns, Polk, Mikellides, and Snyder

Erosion rates reduced by orders of magnitude.

113 h wear test.

specific impulse 2000-3000 s; power density 6 to 9 kW (same thruster).

Extends results of De Grys et al (2010); Mikellides et al (2011); Hofer et al (2012).

Magnetic Shielding in Hall Thrusters

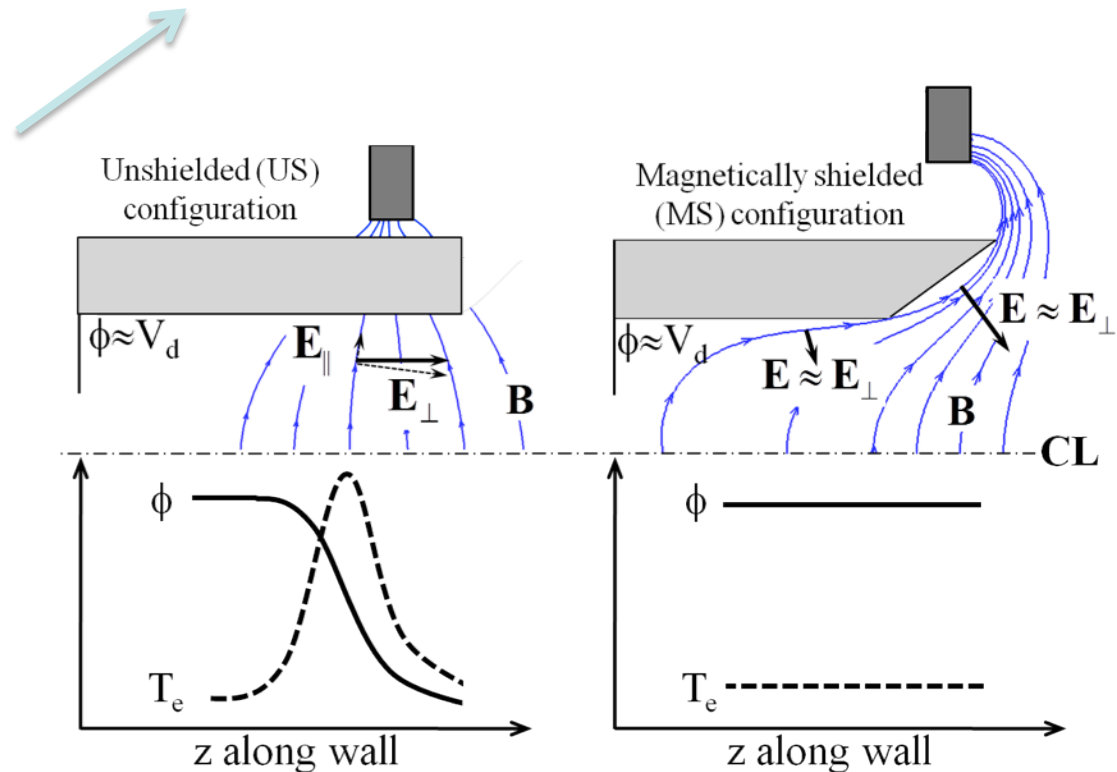
from Hofer 2013

- **What does it do?** It eliminates channel erosion as a failure mode by achieving adjacent to channel surfaces:
 - high plasma potential
 - low electron temperature
- **How does it do it?** It exploits the isothermality of magnetic field lines that extend deep into the acceleration channel, which marginalizes the effect of $T_e \times \ln(n_e)$ in the thermalized potential.
- **Why does it work?** It reduces significantly ALL contributions to erosion: ion kinetic energy, sheath energy and particle flux.
- **Status?** Physics-based modeling and laboratory experiments have demonstrated at least 100X reductions in erosion rate.

$$T_e = T_{eo} \approx \text{constant}$$

$$\phi_o = \phi - \frac{kT_{eo}}{e} \ln\left(\frac{n_e}{n_o}\right) = \text{constant}$$

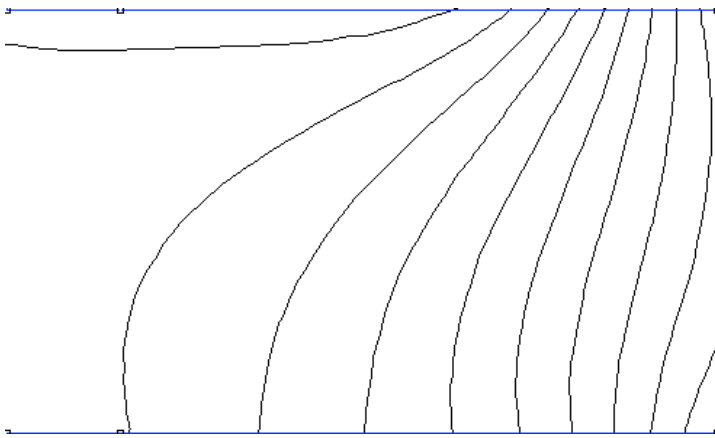
Isothermal field lines
Thermalized potential



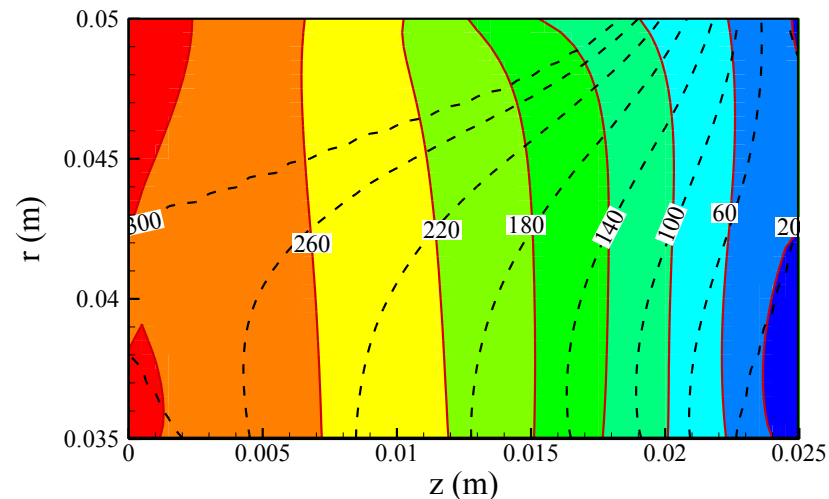
Mikellides, I. G., Katz, I., Hofer, R. R., and Goebel, D. M., "Magnetic Shielding of Walls from the Unmagnetized Ion Beam in a Hall Thruster," Applied Physics Letters 102, 2, 023509 (2013).

Other Effects near Plasma- Material Boundary

J. Geng, "On the potential solver in Hall Thrusters", IEPC-2013-371



(a) Magnetic field lines.



(b) Comparison of equipotential contours of plasma potential with “thermalized potential” (black dashed lines).

“ ... if the magnetic field is not uniform or the electron temperature is not constant along the magnetic field lines, the *thermalized potential* is not accurate.”

Plasma-Wall Interaction in Presence of Intense Electron Emission from Walls
IEPC-2013-132

Kaganovich, Sydorenko, Khrabrov, Campanell, Wang, and Raitsev

Near-wall conductivity SEE-induced cross-field current

Wall collisionality - exchange of primary magnetized electrons by non-magnetized SEE electrons

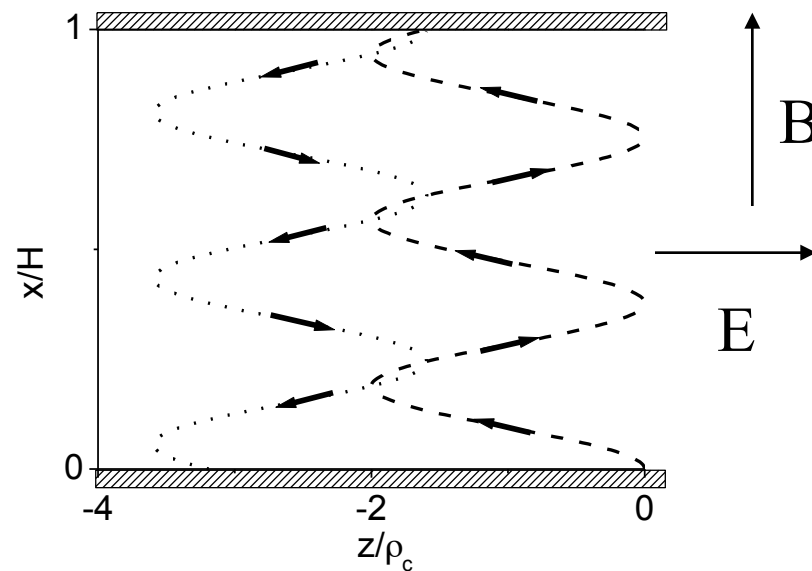
The displacement, $\rho_c = v_{\perp} / \omega_c$, $v_{\perp} = u_d = \frac{E_z}{B_x}$ during the flight time H/u_{bx}

gives average velocity

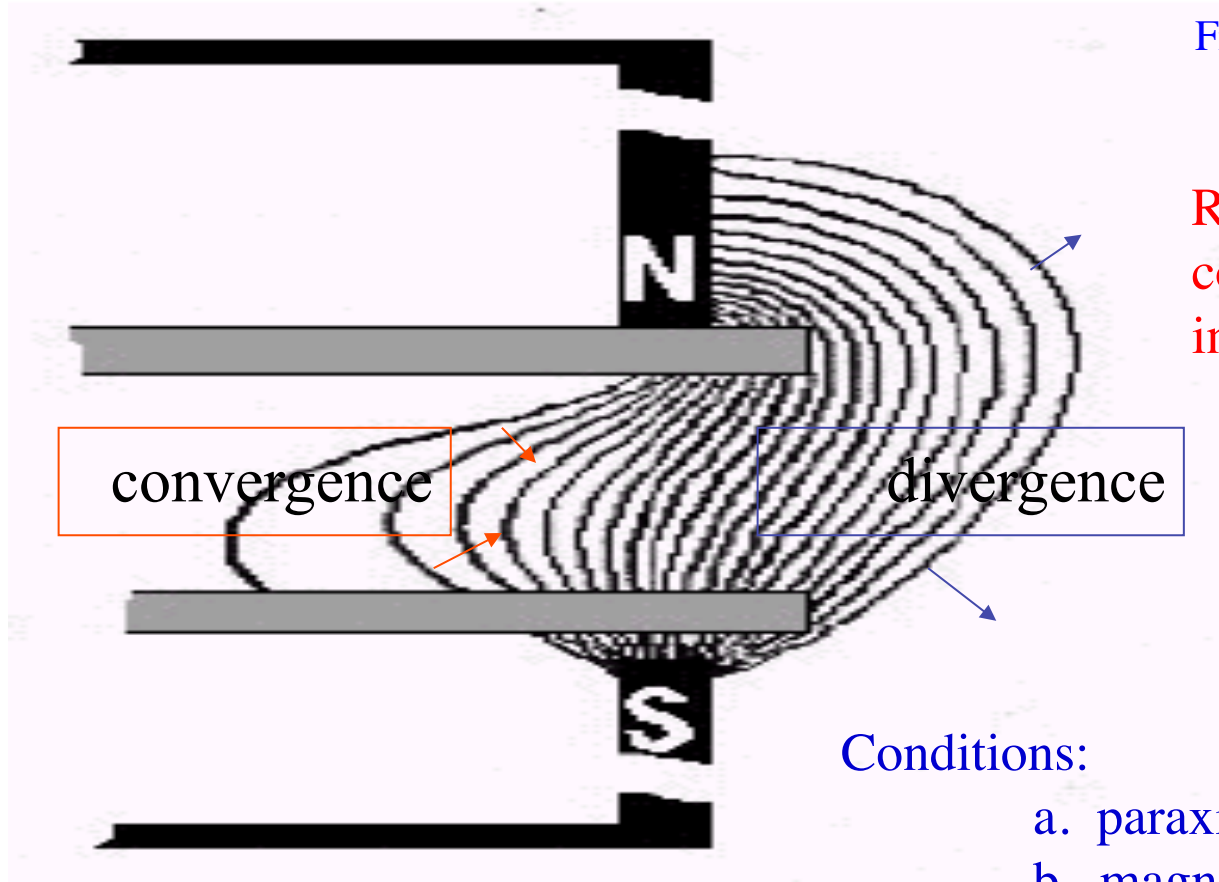
$$\langle u_z \rangle \sim u_d u_{bx} / H \omega_c$$

and current

$$J_{bz} \approx \frac{m}{H} \frac{\gamma_p}{1 - \gamma_b} n_e \sqrt{\frac{T_{ex}}{M}} \frac{E_z}{B_x^2}$$



Fruchtman and Cohen-Zur Lens Theorem



Fruchtman and Cohen-Zur , APL, 2006

Radial acceleration by E field in converging region balances that in diverging region.

Conditions:

- paraxial approximation
- magnetic surfaces are equipotential
- magnetic field is curl-free
- electron cross-field mobility is a function of B only

cf. Hofer and Gallimore, JPC, 2002

Thermalized Potential

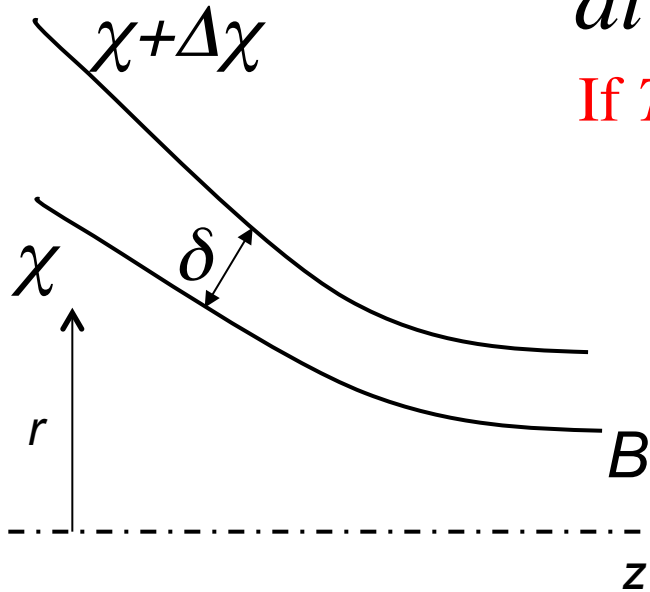
$$\frac{d}{dt} nmv_e = -en(E + v_e \times B) - \nabla P = 0$$

If $T = \text{const}$, then $\nabla P = T\nabla n$ \implies

$$v_e \times B = \nabla[\Phi - (T/e)\ln n] \equiv \nabla\chi$$

$$\implies \nabla\chi \perp B$$

$$\chi = \Phi - (T/e)\ln n, \quad v_e = \frac{-\nabla\chi \times B}{B^2}$$



1. Since both χ and rA are constant on the flux surfaces, the fluid velocity v_e obeys isorotation.
2. Assumptions: P isotropic, T constant. (Neglect centrifugal and $\mu\nabla B$ forces.)
3. Since $\Phi = \chi + (T/e)\ln n$, if n decreases along a field line, then the potential must decrease, but not so much if T small.

Magnetic Shielding in Hall Thrusters

from Hofer 2013

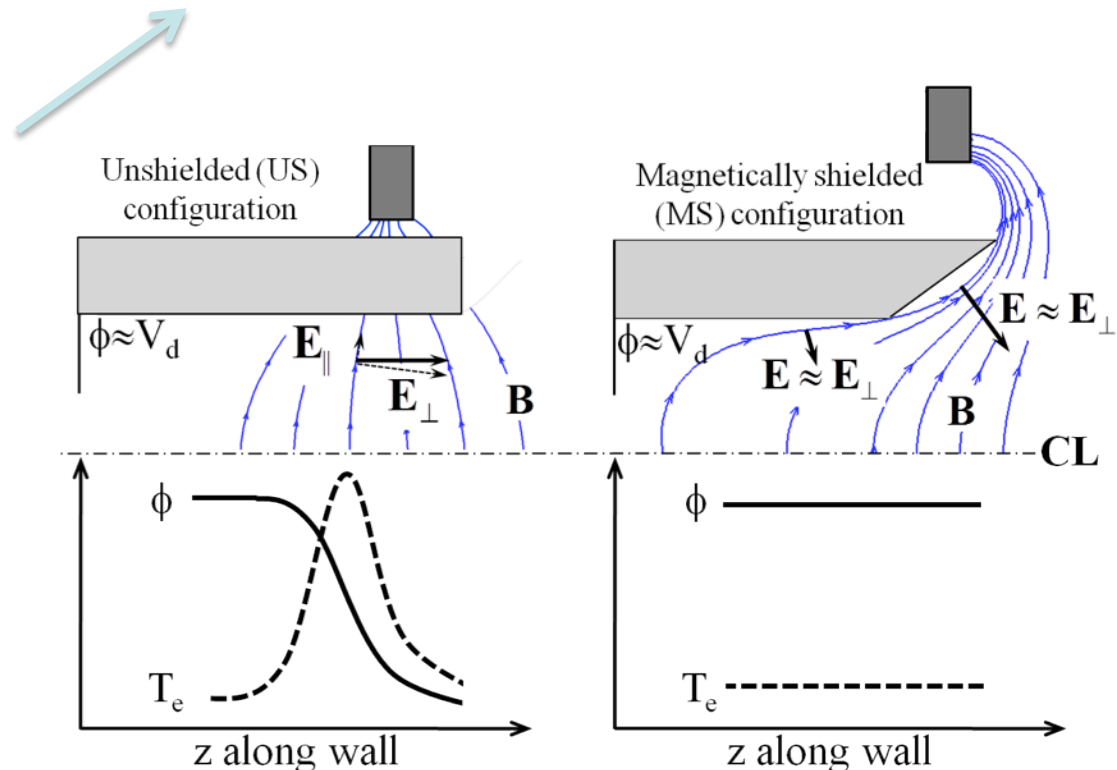
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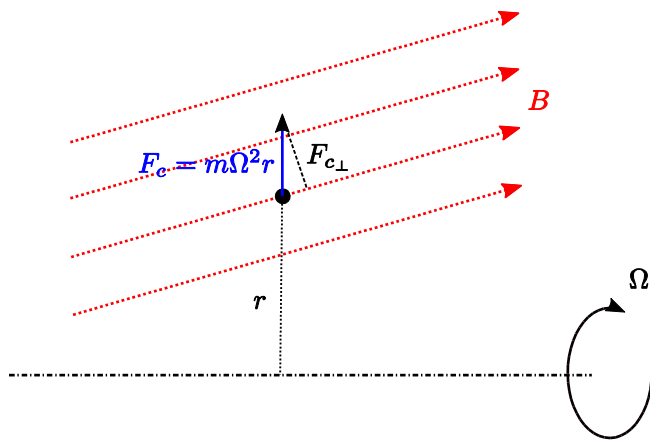
Isothermal field lines

Thermalized potential



Mikellides, I. G., Katz, I., Hofer, R. R., and Goebel, D. M., "Magnetic Shielding of Walls from the Unmagnetized Ion Beam in a Hall Thruster," Applied Physics Letters 102, 2, 023509 (2013).

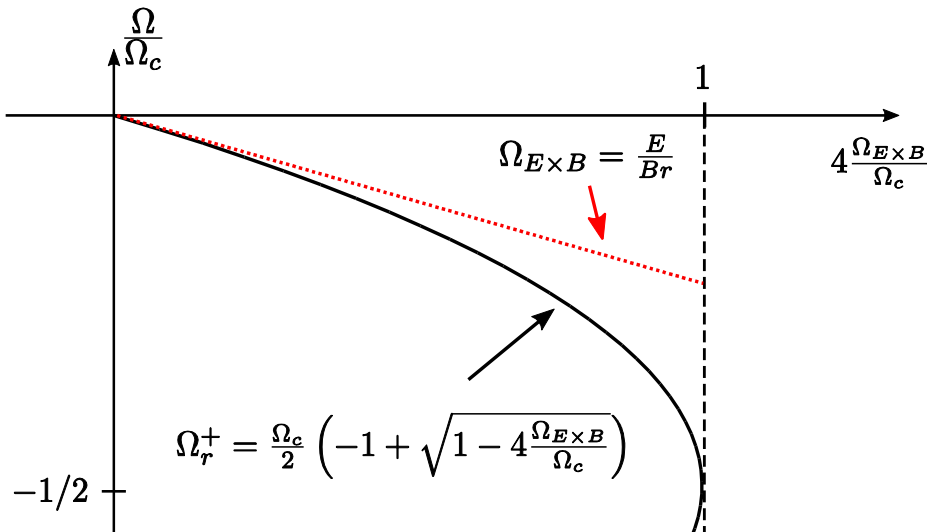
Shielding of magnetic field -- Brillouin rotation mode



additional drift due from centrifugal force

$$F_c \times B \neq 0$$

new rotation frequency



$$\Omega_r^+ = \frac{\Omega_c}{2} \left(-1 + \sqrt{1 - 4 \frac{\Omega_{E \times B}}{\Omega_c}} \right)$$

$\Omega - \Omega_{E \times B}$ increases with p .

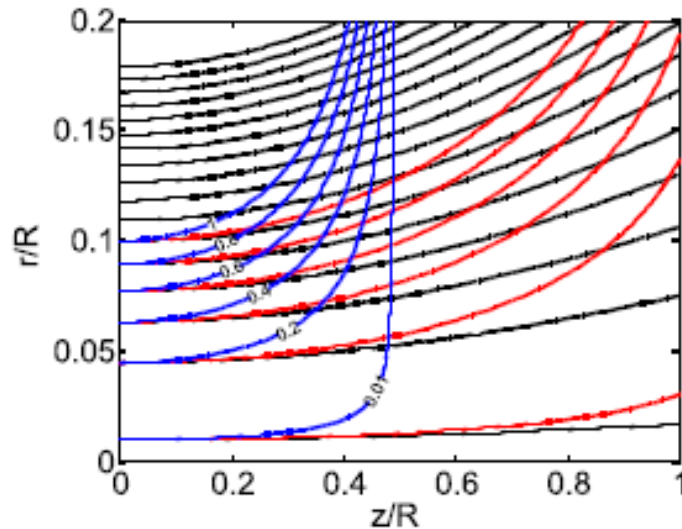
$$p = \frac{mE}{eB^2 r} = \frac{\Omega_{E \times B}}{\Omega_c}$$

$$\Omega \xrightarrow{p \rightarrow 0} \Omega_{E \times B}$$

$$\Omega \xrightarrow{p \rightarrow 1/4} \approx 2\Omega_{E \times B}$$

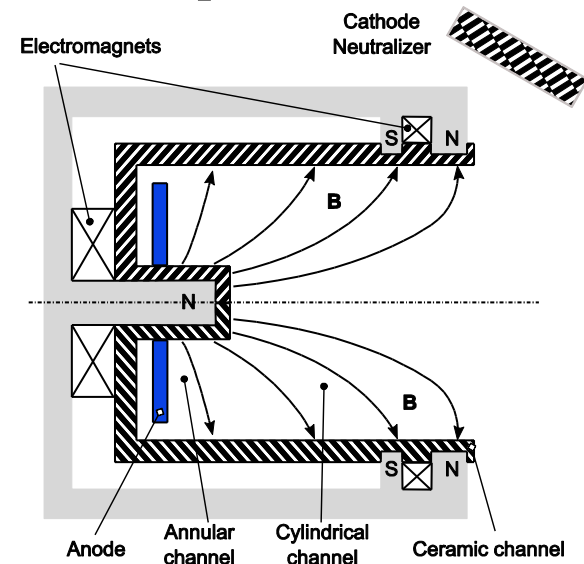
Rotating electron clouds weaken axial magnetic fields, increasing the rotation.

Change in magnetic field topology.



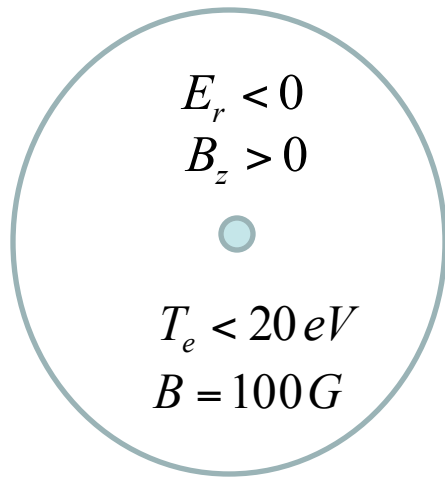
Remapping of the iso-potential lines : vacuum field (black), slow (red) and fast (blue) Brillouin mode

Conditions are expected to be found downstream of a cylindrical Hall Thruster (CHT) plume.

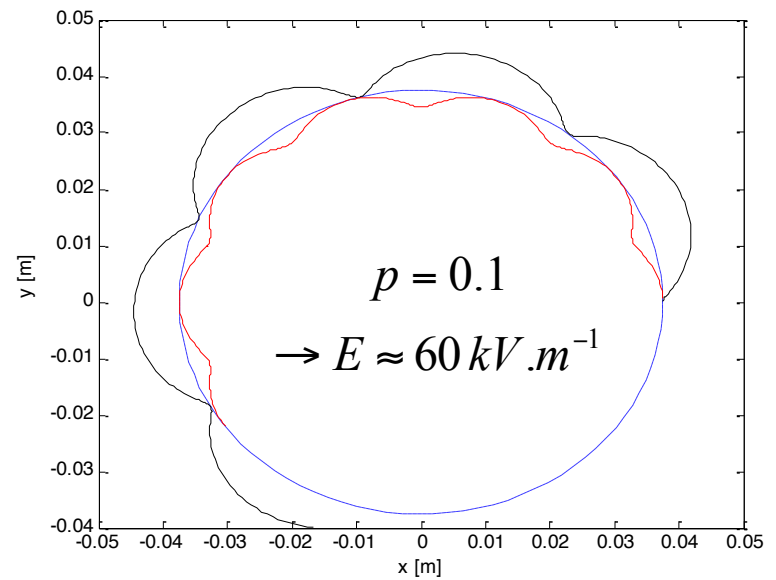
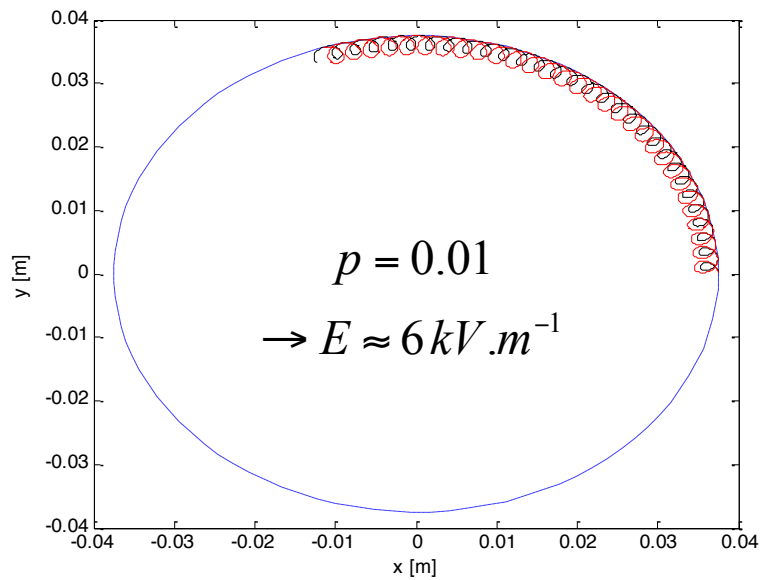
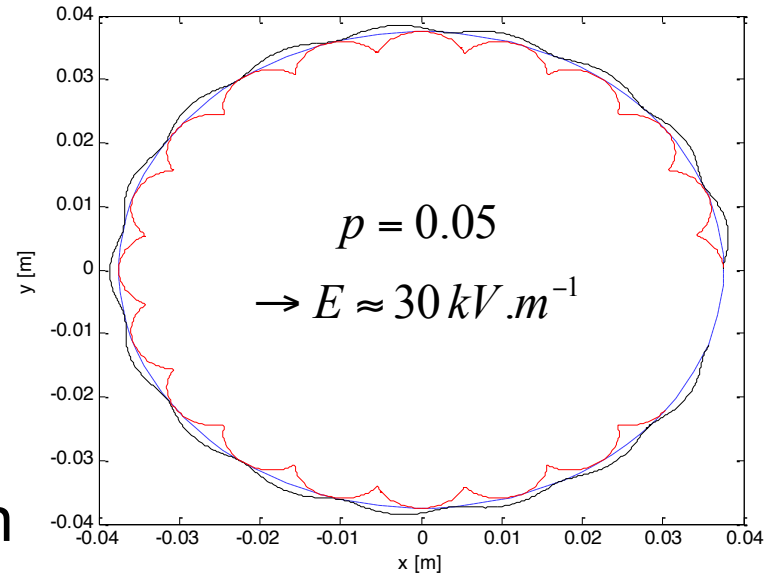


Cylindrical Hall Thruster

Corresponding electron trajectories



— ExB
— Brillouin



TIME

Science & Space

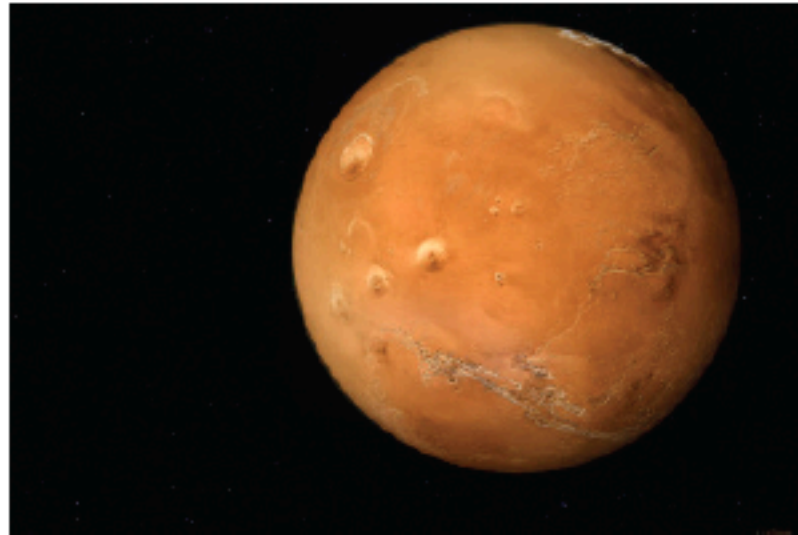
SPACE

Going to Mars via Fusion Power? Could Be

A high-speed, lightweight way to travel in space — provided someone can actually build the thing

By Michael D. Lemonick | Sept. 11, 2018 | 26 Comments

At first, it's hard to know whether to take the company known as [Princeton Satellite Systems](#) (PSS) seriously. For one thing, the PSS offices, a few rooms in a nondescript building in nondescript Plainsboro, N.J., right above the Sugar and Sunshine Bakery, don't exactly suggest the imminent conquest of the final frontier. The company's ambitions, by contrast, certainly do — but those sound so crazy that you have to wonder if they're serious. This team of a half-dozen or so scientists and engineers is determined to send human beings to Mars, launch robotic probes to the outer solar system, send missions to Alpha Centauri and more, and do it all with rockets powered by nuclear fusion.



Getty Images

You heard that right: fusion. It's the energy source that makes stars shine and that plasma physicists have been trying to tame for more than 50 years — so far, despite [ever more gigantic and expensive machines](#), in vain. Controlled fusion could power the entire planet with energy free of carbon emissions and with negligible radioactive waste, but it's proved so difficult to pull off that a commercial reactor won't see the light of day for decades to come at the very soonest. Nonetheless, the folks at PSS think they might be able to build a fusion-powered rocket motor much sooner than that, and they may be onto something.

The advantages of such a breakthrough are easy enough to see: a fusion rocket for a Mars mission, says company founder Michael Paluszek, "would be smaller than a minivan, and you could get there and back in less than a year, compared with more than two years for chemical rockets."

Modular Aneutronic Fusion Engine

G. Pajer, Y. Razin, M. Paluszek, A. H. Glasser and S. A. Cohen

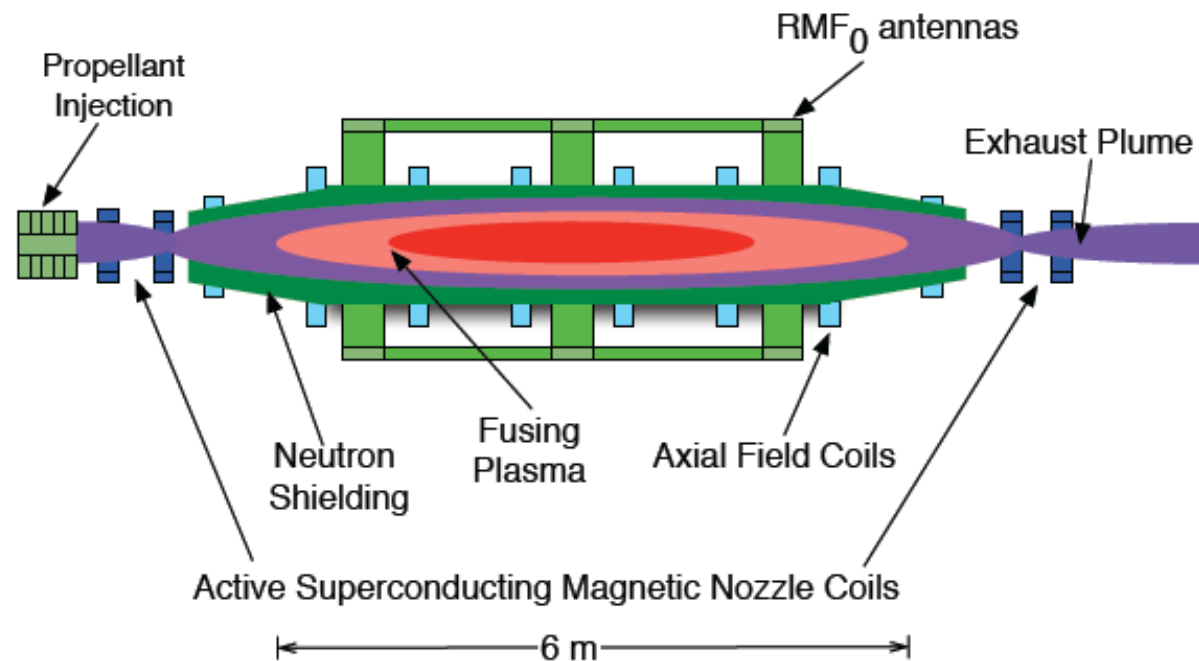


Figure 1. 10 MW reactor diagram

Based on Field-Reversed Configuration (FRC) approach

An Electric Propulsion System Based on Controlled Fusion and Electromechanical Energy Conversion

IEPC-2013-062

P. J. Turchi

Crewed Exploration of Solar System Requires Advanced (neutronless) Fuels

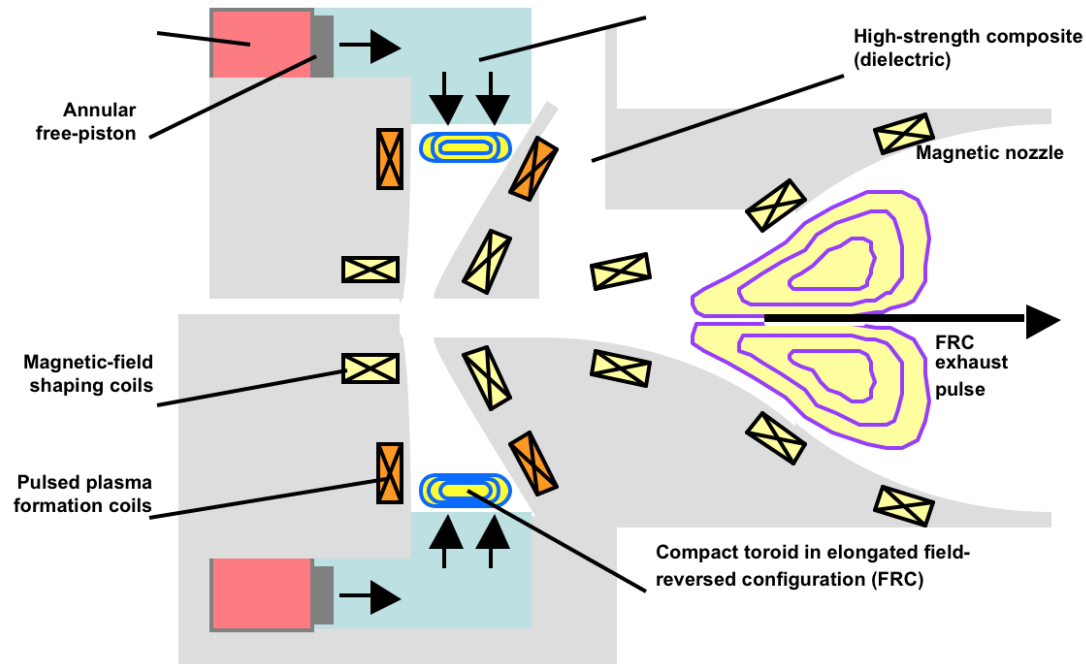
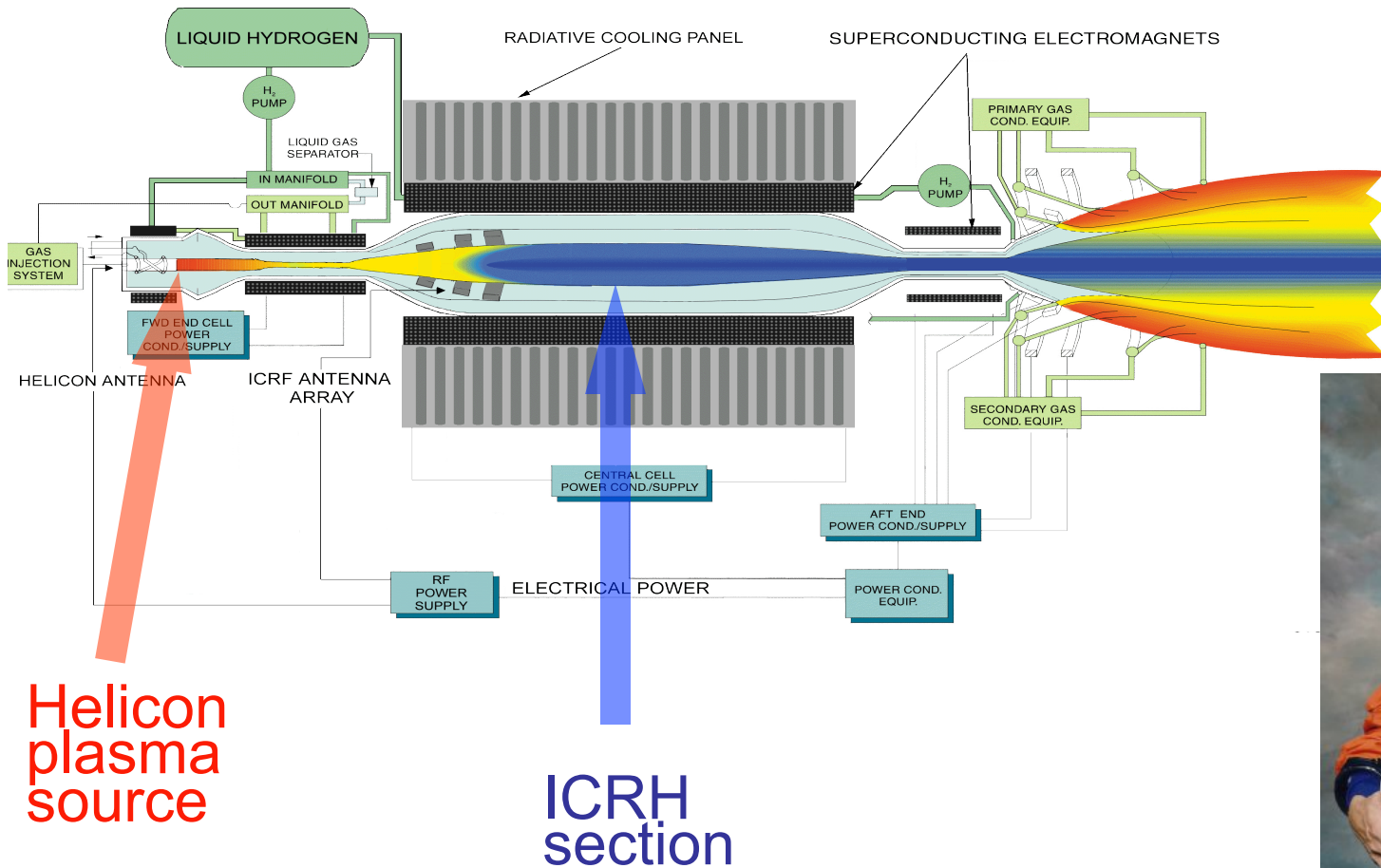


Figure 4. Concept of a liner-driven electric thruster in which a stabilized liner implosion system compresses an FRC adiabatically to high specific energy for a very high speed exhaust pulse.

VASIMR

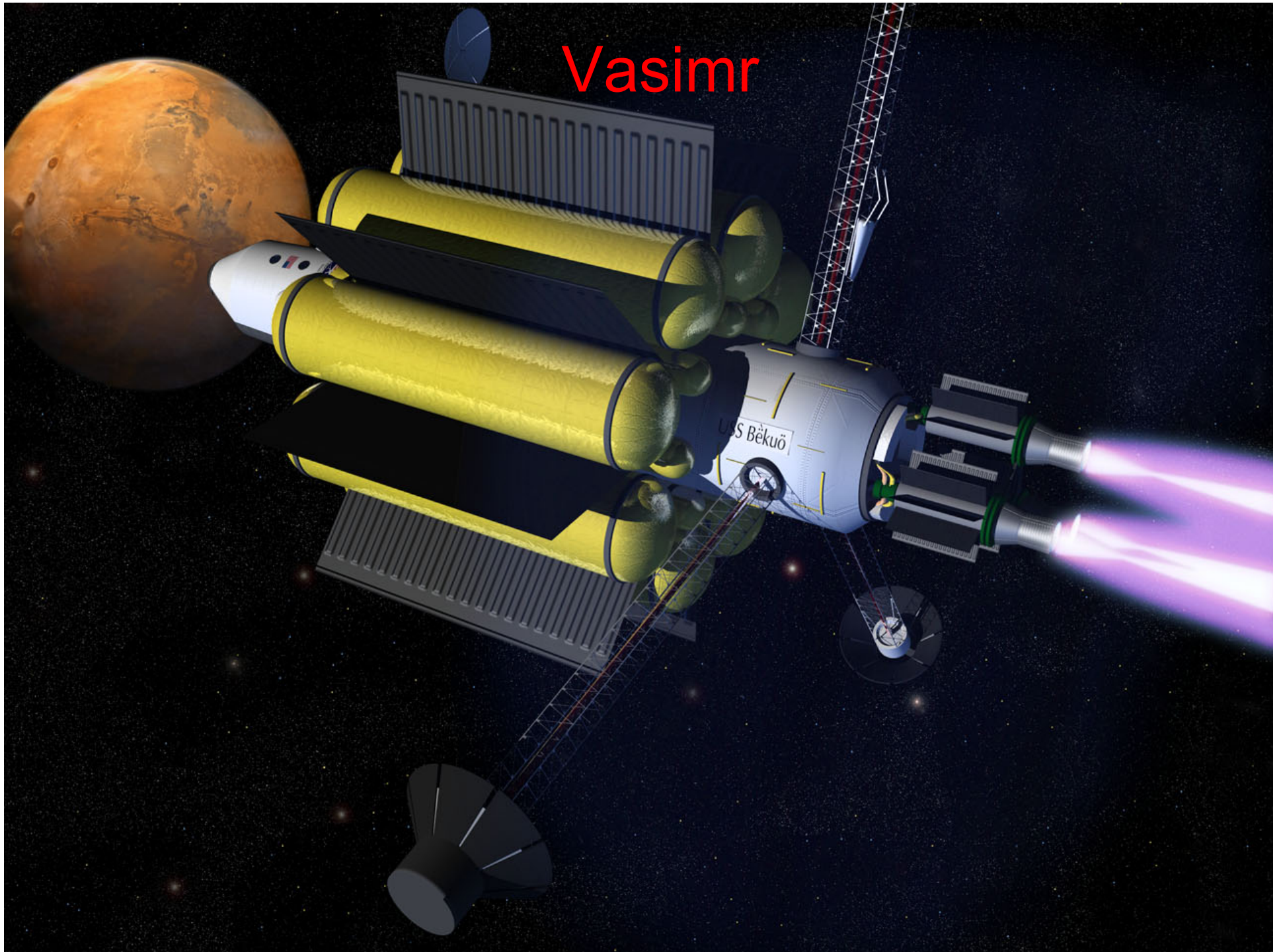
Variable Specific Impulse Magnetoplasma Rocket



Concept by Franklin Chang-Diaz

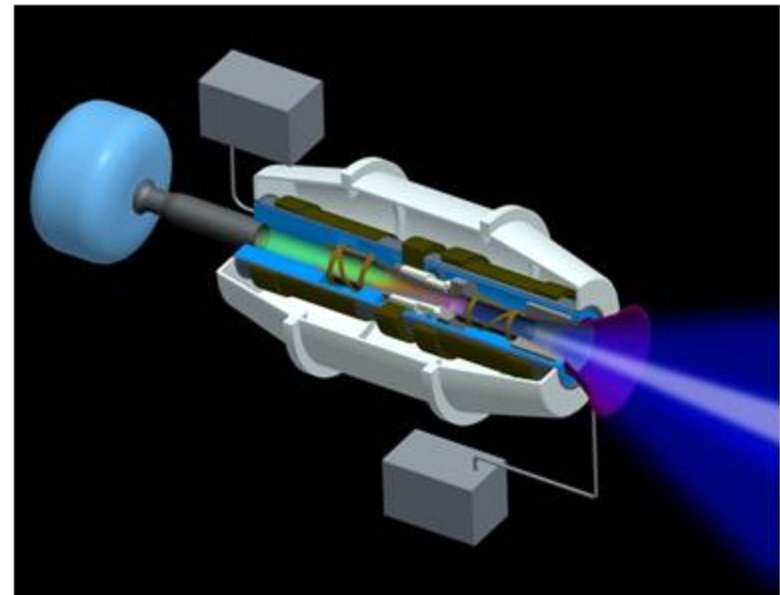


Vasimr



VASIMR[®] Advantages

- **No electrodes**
 - High power density (6 MW/m²)
 - High reliability and long life
 - VX-200 has completed >10,000 high power firings
- **Variable specific impulse**
 - Constant power throttling
 - Adapts to mission/ops requirements
- **Scalable to multi MW power levels**
 - Solar electric (near term)
 - Nuclear electric (future)
- **Competitive mass scaling**
 - Light weight HT SC magnets (cryogen-free)
 - High pwr solid-state RF
- **Use of multiple propellants**
 - Argon, Krypton, Xenon, Hydrogen
 - Propellant mixtures



A superconducting magnetic "duct" guides the plasma with no direct contact with nearby structures

Plasma Source

Dense plasma is created from "feedstock" neutral gas by RF waves

Magnetic Nozzle

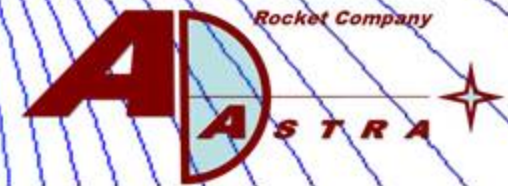
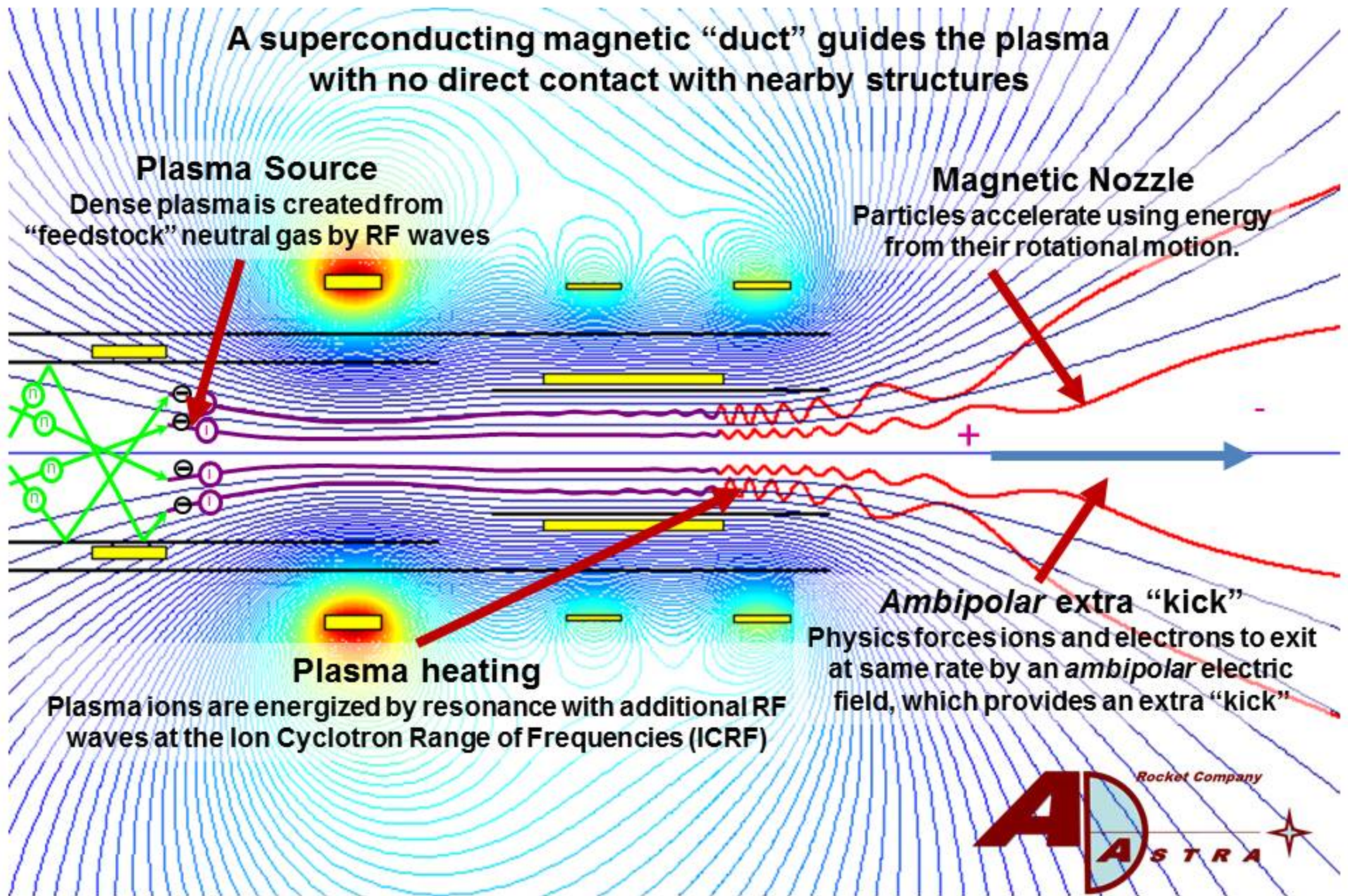
Particles accelerate using energy from their rotational motion.

Plasma heating

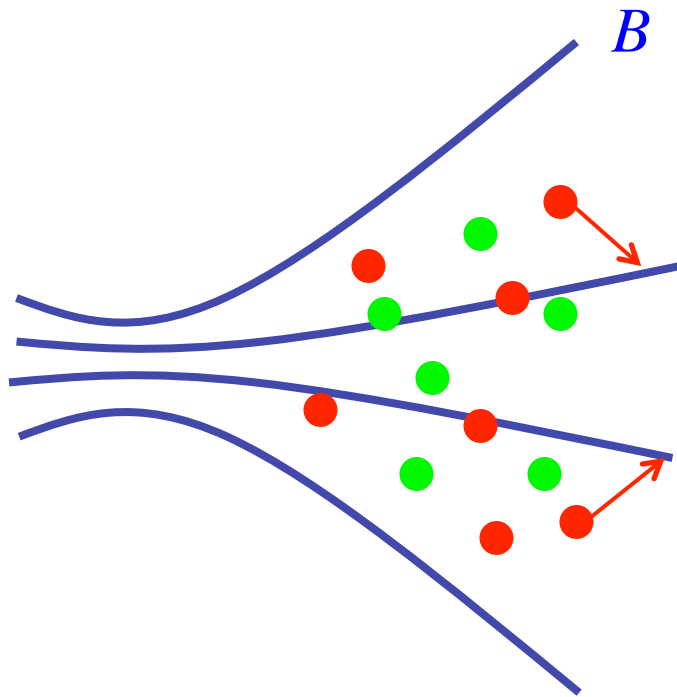
Plasma ions are energized by resonance with additional RF waves at the Ion Cyclotron Range of Frequencies (ICRF)

Ambipolar extra "kick"

Physics forces ions and electrons to exit at same rate by an *ambipolar* electric field, which provides an extra "kick"



Limiting Plume Divergence with Wave Heating (and facilitating *advanced* fuels)



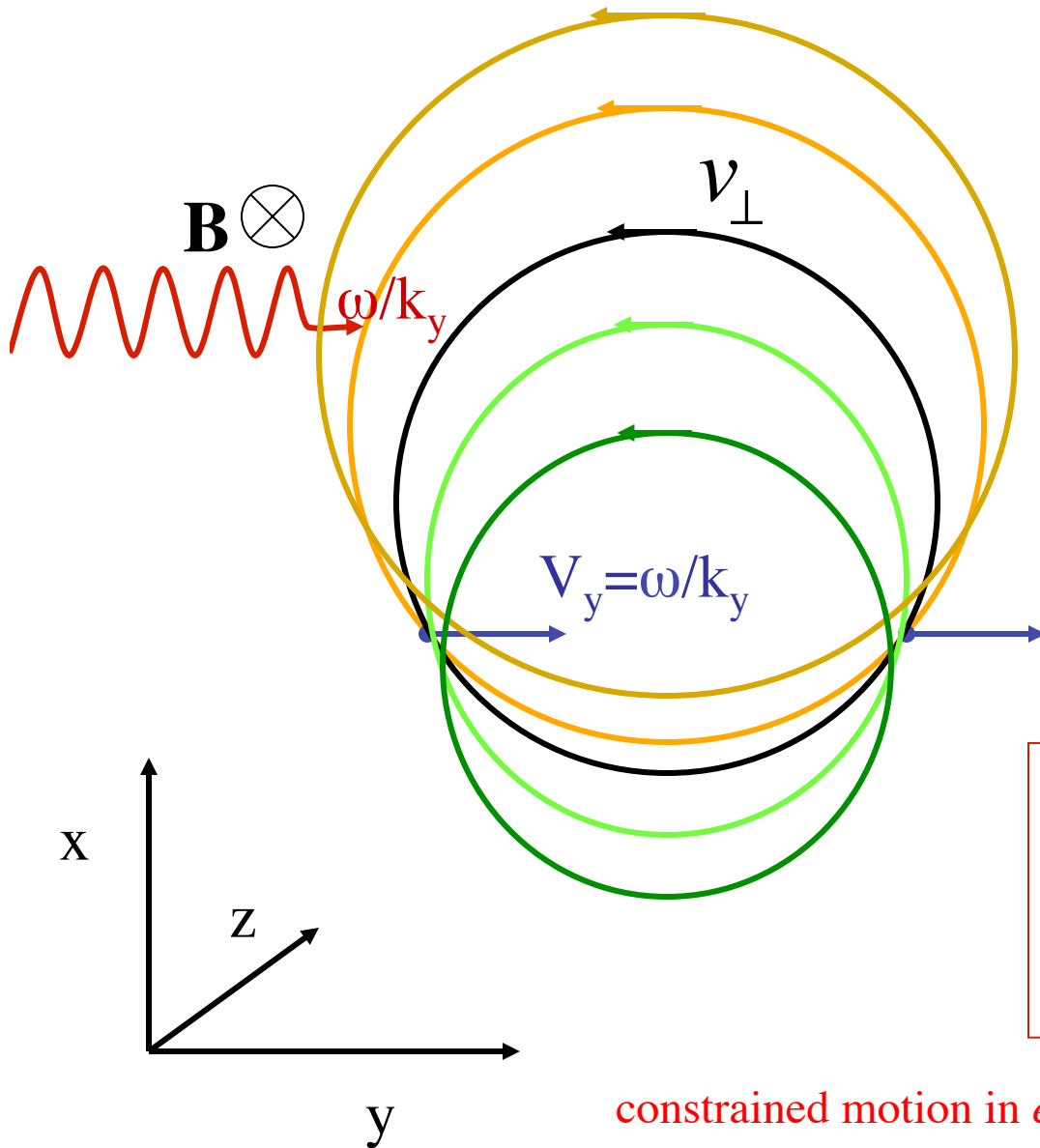
Couple diffusion in energy
to diffusion in space

Thus, ions traveling along outer
field lines will then travel
along inner field lines when heated.

Particles can be pushed by waves in plasma in the direction of the wave momentum. Thus, axial acceleration (or heating) is also possible. But coupling diffusion in space to diffusion in energy can be both stochastic and robust.

Ion Cyclotron Heating

Diffusion paths coupling energy to space



$$v_y \rightarrow v_y + \Delta v_y$$

$$x_{gc} \rightarrow x_{gc} + \Delta v_y / \Omega$$

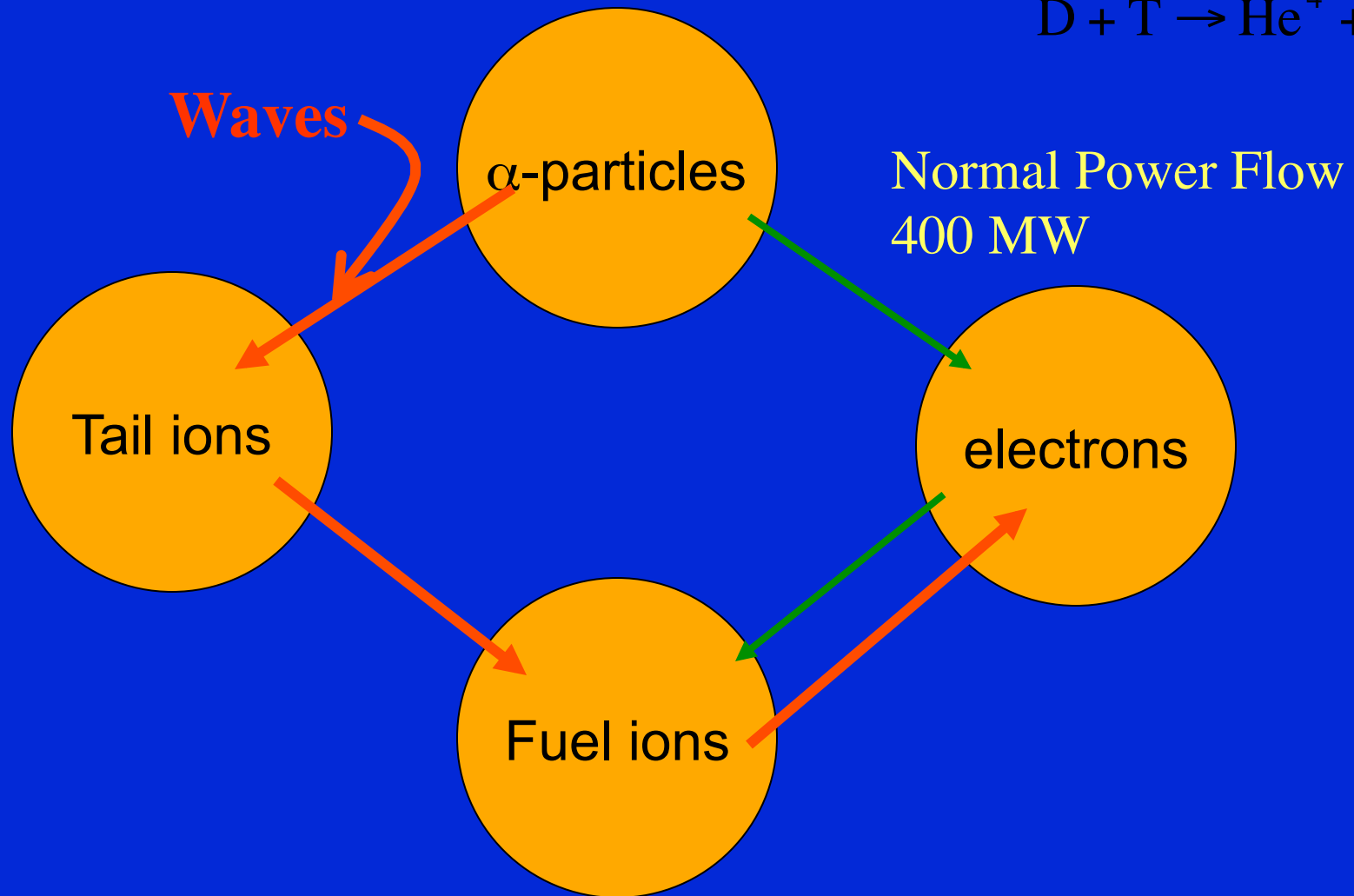
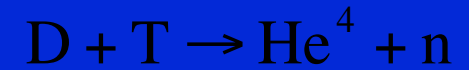
$$\Omega \equiv eB/m$$

$$\Delta E = m v_y \Delta v_y$$

$$x_{gc} \rightarrow x_{gc} + \frac{\Delta E}{m \Omega \frac{\omega}{k_y}}$$

constrained motion in *energy-distance* space

Power Flow in a Self Sustaining Fusion Reactor



For D-T: Get $T_i > T_e$, $P_f \rightarrow 2P_f$

For p-¹¹B: may be essential

Summary

1. Unsolved Problems

- a. Space-charge limit for ion thruster with $V(t)$
- b. B-field limit for Hall thruster

2. Particle Motion in Hall thruster

- a. Radial electric fields give confinement radially and axially.
- b. Cylindrical Hall thruster relaxes axial localization constraint.
- c. Isorotation Theorem and Corollary
- d. Thrust-vector straightening
- e. Thermalized potential – magnetic shielding

3. Related Problems

- a. Centrifugal mirror fusion
- b. Plasma centrifuge – magnetic filter

4. Futuristic opportunities

- a. RF-heated, advanced fuels
- b. Wave diffusion constraints in space and energy

Special Thanks

- Dr. Renaud Gueroult (PPPL)
- Professor Amnon Fruchtman (Holon)
- Dr. Yevgeny Raitses (PPPL)