Colloquium in Faculty of Physics, Weizmann Institute of Science, February 14, 2013

Wave Compression in Plasma

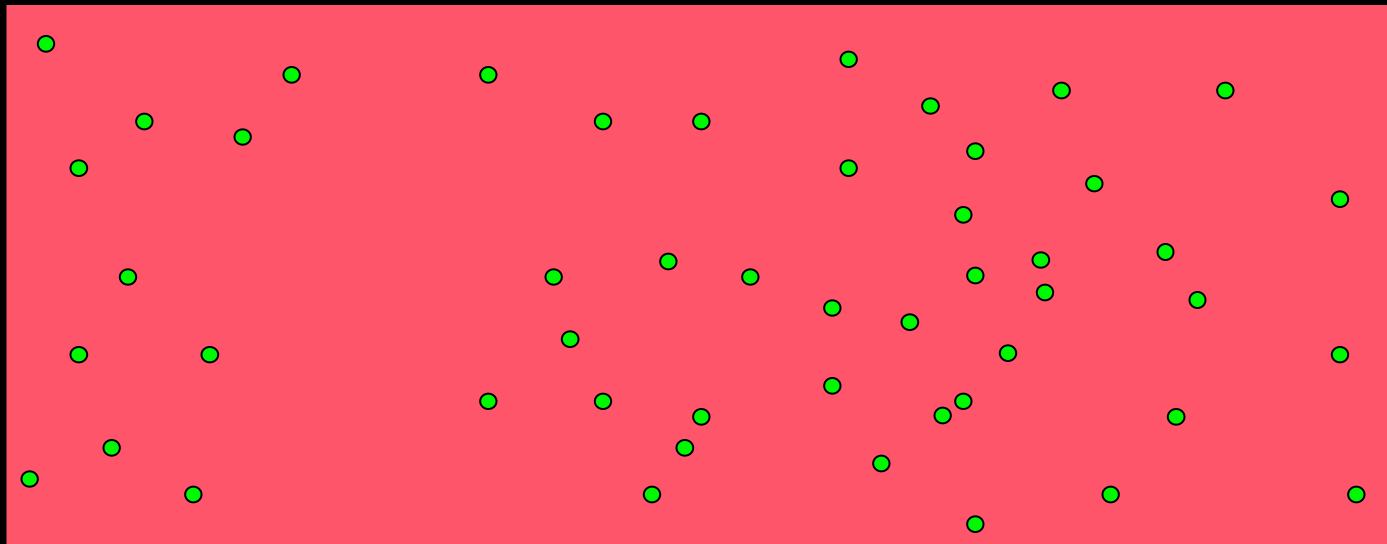
Nathaniel J. Fisch

Permanent position: Department of Astrophysical Sciences
Princeton University

Temporary position: Weston Visiting Professor
Department of Particle Physics and Astrophysics
Weizmann Institute of Science

Simple wave oscillations in plasmas can produce enormous effects. This talk first explains what is a plasma wave. I then focus on two recently discovered, curious, and potentially useful effects, both mediated by the plasma wave, and both involving wave compression. One effect is resonant Raman backscattering, whereby a long moderately intense laser beam loses its energy to a short counter-propagating beam, producing a much shorter and much more intense pulse. This effect might overcome the material limitations of present technology, enabling the next generation of laser intensities. A second compression effect occurs when the plasma itself is compressed; not only does its temperature increase, but any embedded waves might also increase in energy. For adiabatic changes in time of the density of the plasma medium, the coherent wave energy grows, but, importantly, might then very abruptly lose this energy.

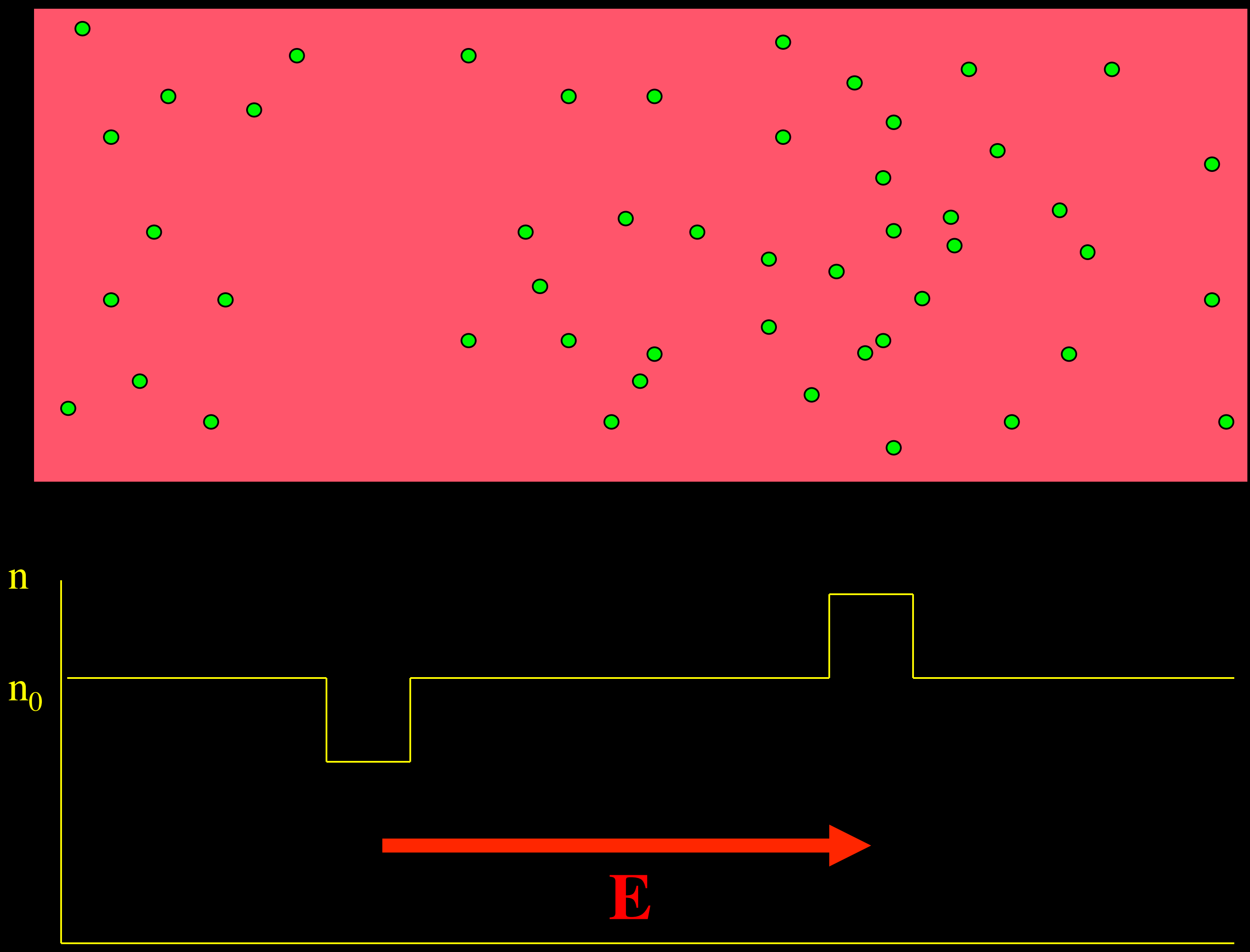
Plasma



In 1927, Irving Langmuir coined the term *plasma* for an ionized gas, because an electrified fluid carrying ions and electrons reminded him of how blood plasma carried red and white corpuscles.

Irving Langmuir (1881-1957); Nobel ‘32

Set up plasma oscillation (Langmuir wave)



Plasma Oscillations

$$\nabla \bullet \vec{E} = 4\pi e(n_0 - n_e) = -4\pi e\tilde{n}$$

Poisson's equation

$$n_e = n_0 + \tilde{n}$$

$$\frac{\partial}{\partial t} n_e + \nabla \cdot n_e \vec{v} = 0$$

Particle conservation

$$\vec{E} = \tilde{\vec{E}}$$

$$\frac{\partial}{\partial t} n_e m\vec{v} + \nabla \cdot n_e m\vec{v}\,\vec{v} = e n_e \vec{E}$$

Momentum conservation

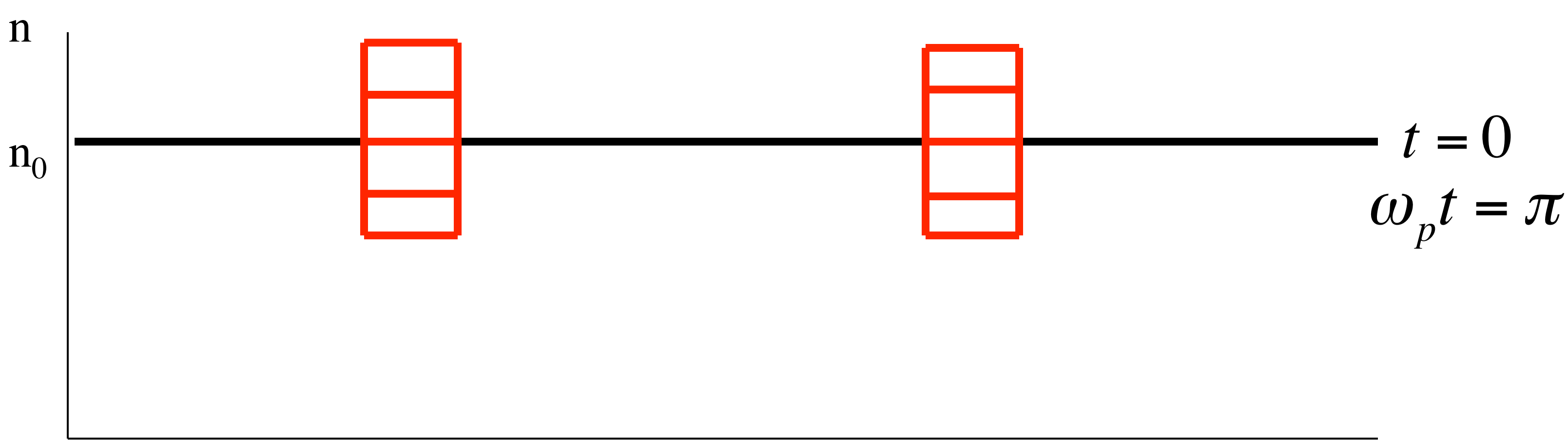
$$\vec{v} = \tilde{\vec{v}}$$

$$\Rightarrow \quad \frac{\partial^2}{\partial t^2}\tilde{n} + \omega_p^2 \tilde{n} = 0$$

$$\omega_p^2 = 4\pi e^2 n_0 / m$$

$$\boxed{\tilde{n} = A(\vec{r})\cos\omega_p t + B(\vec{r})\sin\omega_p t}$$

Plasma Oscillation



Make traveling wave:

$$\boxed{\Phi(x,t) = A(x)\cos[\omega_p\,(t - x/c)]}$$

Resonant Surfers

Not-resonant surfers $V \neq V_{ph}$

Things that a plasma wave can do

1. Toroidal current in tokamaks (3 MA)
2. High-gradient accelerators (100 GeV)

Significant Progress

1. Mediate resonant Raman compression of optical lasers
2. Compression of x-rays, short wavelength optical

Promising

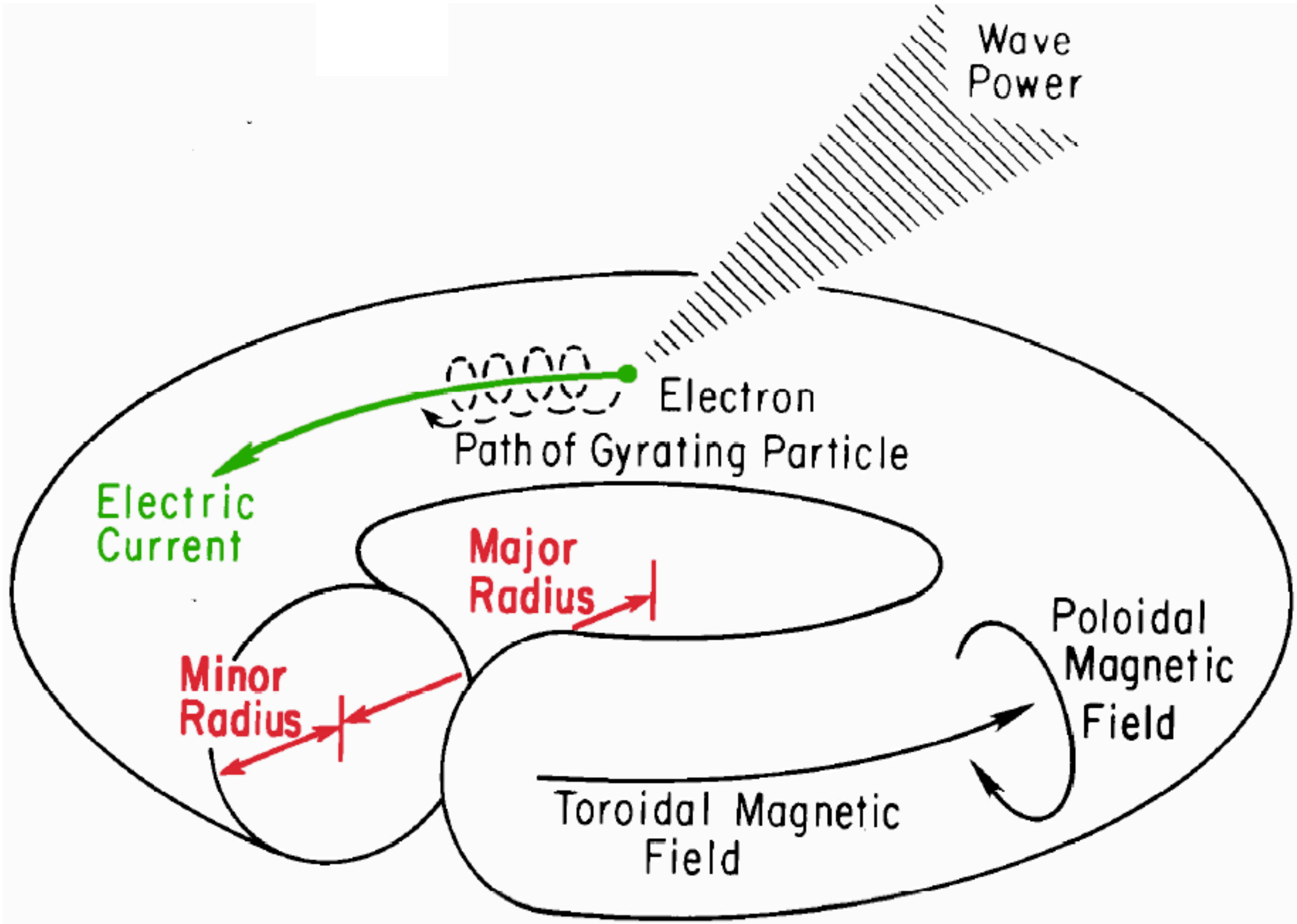
Goal: Achieve next generation of light intensities

1. Switch-like mechanisms in compression of plasma waves
2. Couple diffusion in space to diffusion in energy – cooling effect

Speculative

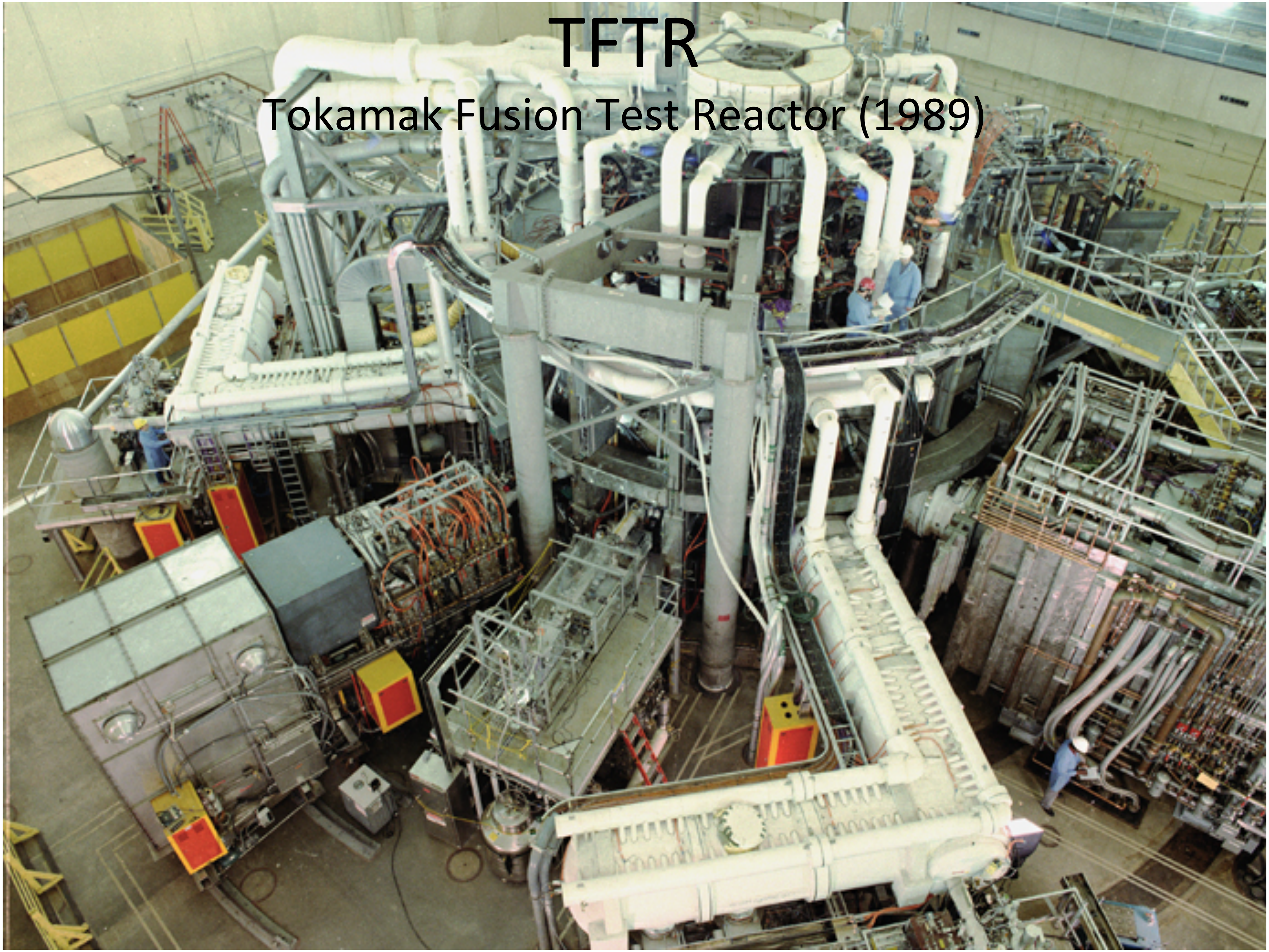
Goal: Realize new effects in new facilities for highly compressing plasma

Generating Current with Waves

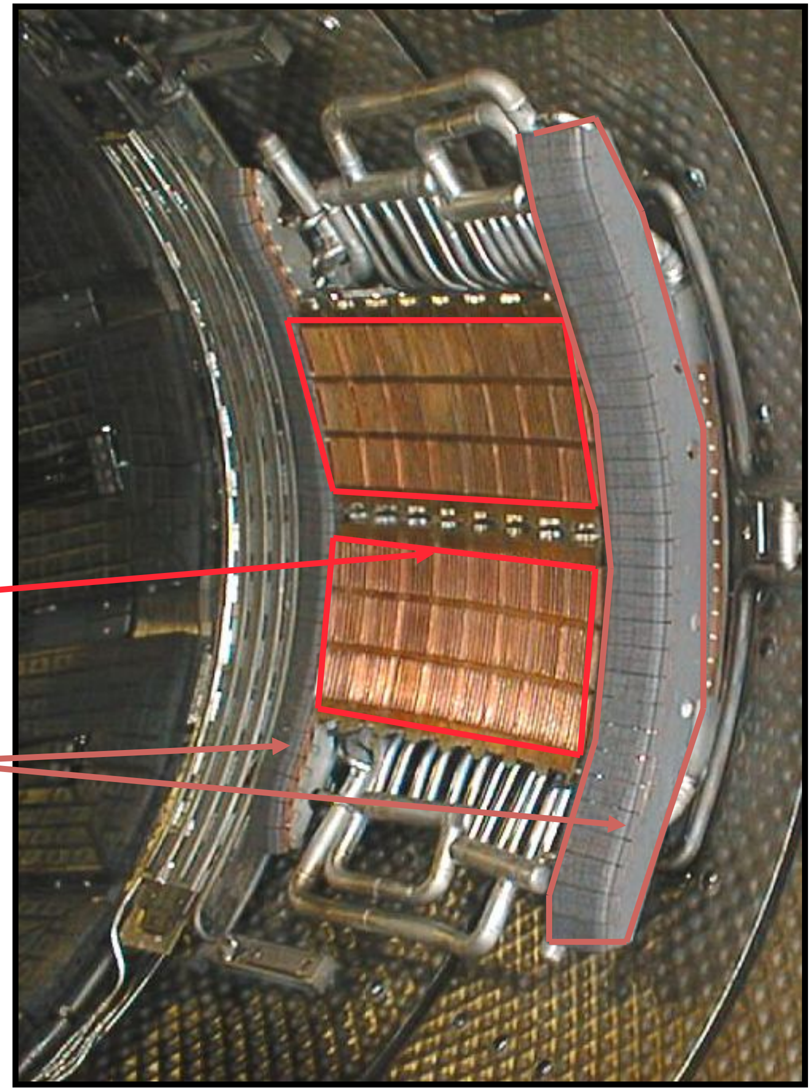


TFTR

Tokamak Fusion Test Reactor (1989)



Tore Supra: LH coupler

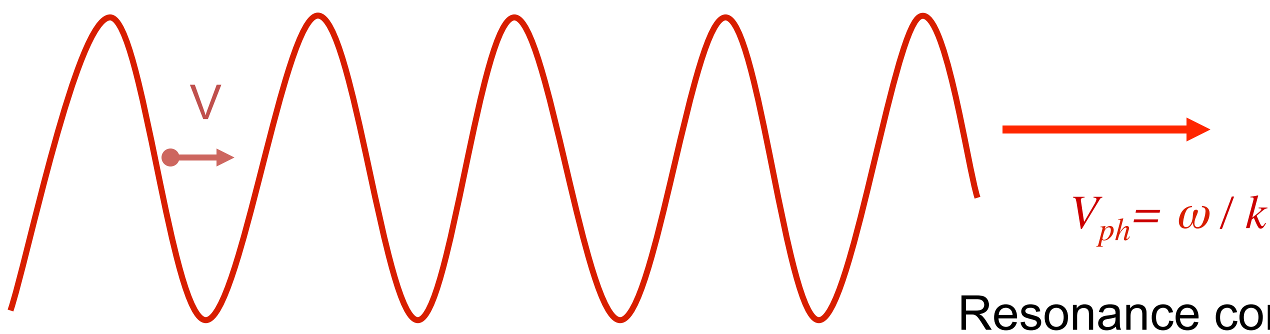


Antenna for long pulse operation:
4 MW, 1000 s, 3.7 GHz
(25 MW/m² and $n_{//0} = 2$)

Actively cooled side limiter
(exhaust capability:10 MW/m²)

48 active, 9 passive waveguides

Radio Frequency (RF) Current Drive Effect



Resonance condition

$$\omega - \vec{k} \cdot \vec{v} = 0$$

$$\mathrm{v} \longrightarrow \mathrm{v} + \Delta \mathrm{v}$$

$$J = en\Delta \mathrm{v}$$

$$\Delta E = mn\mathrm{v}\Delta \mathrm{v}$$

$$P_{\mathrm{D}} = \nu \, \Delta E$$

$$\nu(\mathrm{v}) \approx \mathrm{v}^{-3}$$

$$\Longrightarrow \quad \boxed{J / P_{\mathrm{D}} = e / m\mathrm{v}\nu(\mathrm{v})}$$

$$\omega - k_{\parallel}\mathrm{v}_{\parallel} = 0 \Longrightarrow$$ LHCD, FWCD, MCIBW

Fisch (1978)

$$\omega - k_{\parallel}\mathrm{v}_{\parallel} - n\Omega = 0 \Longrightarrow$$ ECCD

Fisch and Boozer (1980)

Examples: RF Current Drive

JT-60 and JT60-U (Japan) -- 3MA LHCD 800 kA ECCD, ITB sawteeth stabilization (2001)

JET (England) -- 3 MA LHCD, ITB with LHCD, Minority Species CD. ITB

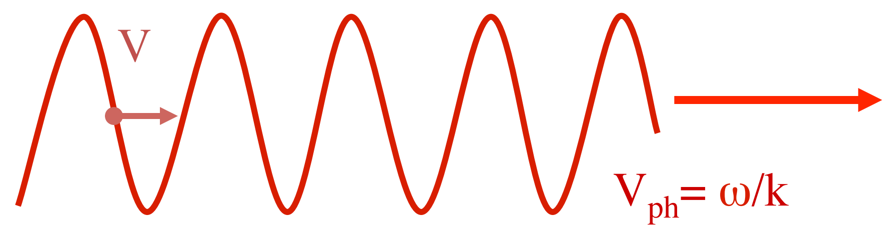
Tore Supra (France) -- 1000 s LHCD, ITB; 330 s, 1 GJ, LHCD (2004), ECCD Synergy

C-Mod tokamak (MIT) : LHCD
TRIAM (Japan): several hours LHCD
T-10 (Russia): ECCD , sawteeth
TCV tokamak --- ECCD steady state, sawteeth
ASDEX (Germany): ECCD stabilization of tearing modes
Wendelstein 7-AS Stellarator: ECCD
Frascati FT-U (Italy): LHCD, ECCD stabilization of sawteeth, tearing modes
General Atomics DIIID tokamak; ECCD, ITB, mode suppression
Princeton spherical torus: NSTX (HHFWCD)

New Steady-State Lower-hybrid driven Superconducting Tokamaks
SST (India)
KSTAR (Korea)
East (China)

Accelerating Gradient in Plasma

Conventional Accelerator
20 MeV/m at 3 GHz
Limited by breakdown
1 TeV Collider requires 50 km



$$\nabla \bullet \vec{E} = -4\pi e\tilde{n}$$

$$\tilde{n}_{MAX} \approx n_0$$

$$k = \frac{\omega_p}{c}$$

$$eE_{MAX} \approx \sqrt{n_0}\, GeV/cm$$

Plasma Accelerator
High field gradients (10^4)

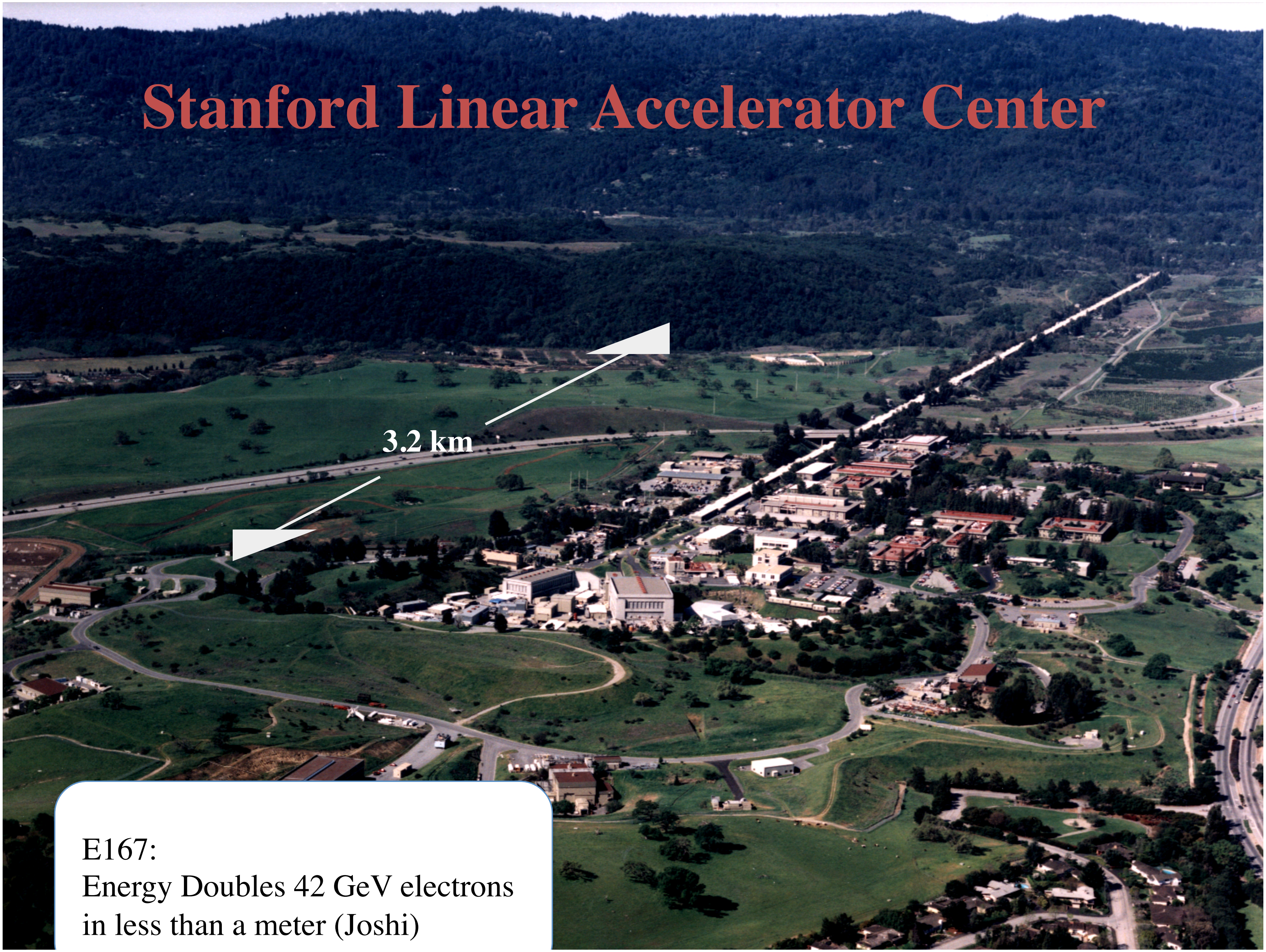
$n_0 = 10^{18}$ cm^{-3}

eE=100 GeV/m

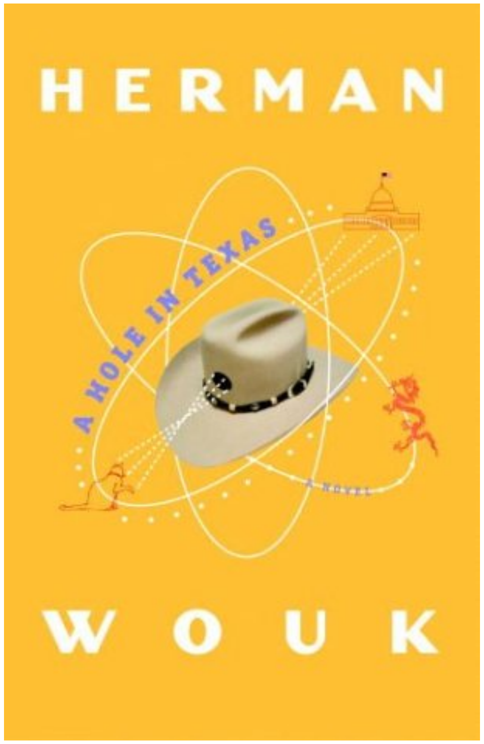
Note: For v << c , $\frac{v_{osc}}{c} \approx \frac{\tilde{n}}{n_0}$

Particles accelerated to relativistic energies, even as plasma motion is not

Stanford Linear Accelerator Center



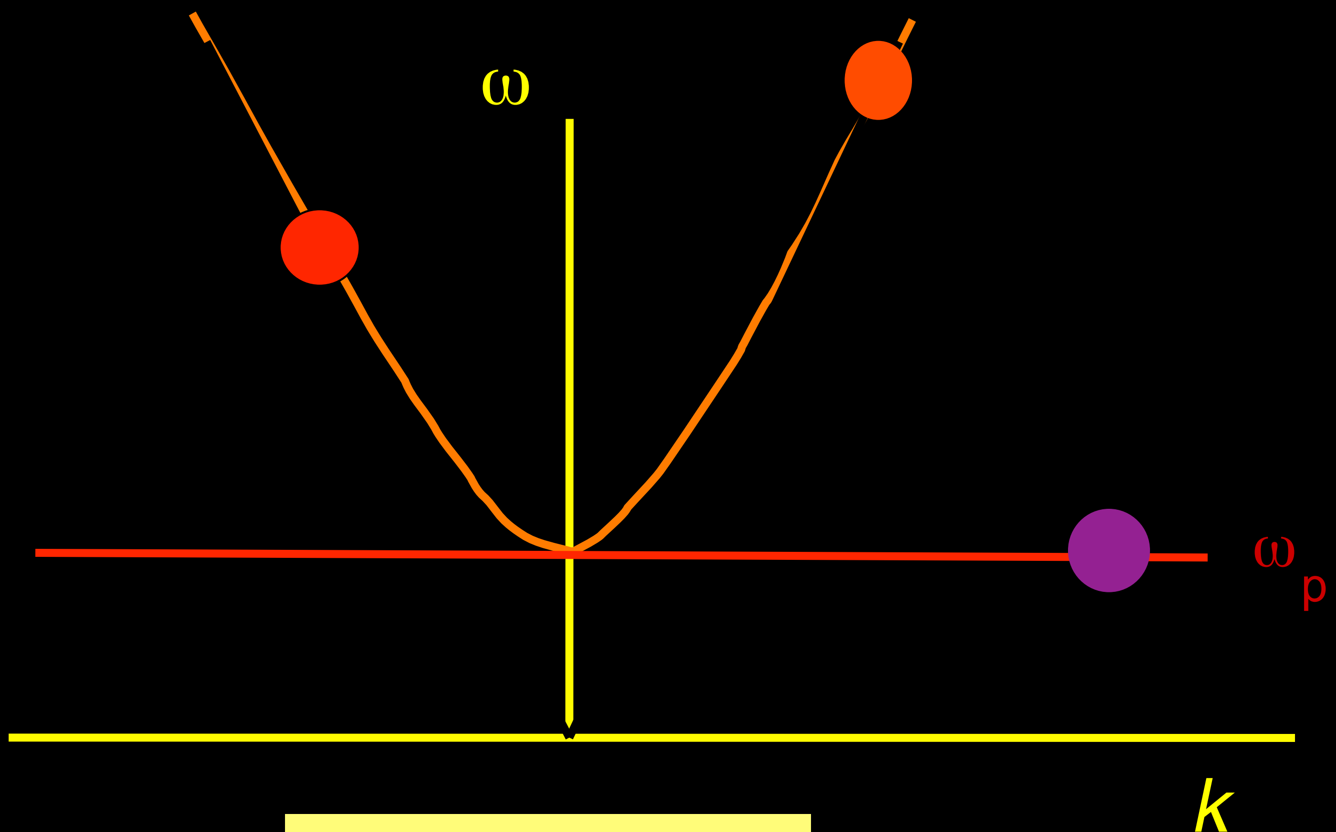
E167:
Energy Doubles 42 GeV electrons
in less than a meter (Joshi)



A Hole In Texas is a novel by Herman Wouk. Published in 2004, the book describes the adventures of a high-energy physicist following the surprise announcement that a Chinese physicist (with whom he had a long-ago romance) had discovered the long-sought Higgs boson. Parts of the plot are based on the aborted Superconducting Super Collider project.

The novel, The Caine Mutiny (1951), went on to win the Pulitzer Prize. A huge best-seller, drawing from his wartime experiences aboard minesweepers during World War II, The Caine Mutiny was adapted by the author into a Broadway play called The Caine Mutiny Court Martial, and was later made into a film, with Humphrey Bogart portraying Lt Commander Philip Francis Queeg, captain of the fictional DMS Caine. Some Navy personnel complained at the time that Wouk had taken every twitch of every commanding officer in the Navy and put them all into one character, but Captain Queeg has endured as one of the great characters in American fiction.

Raman Decay in Plasma

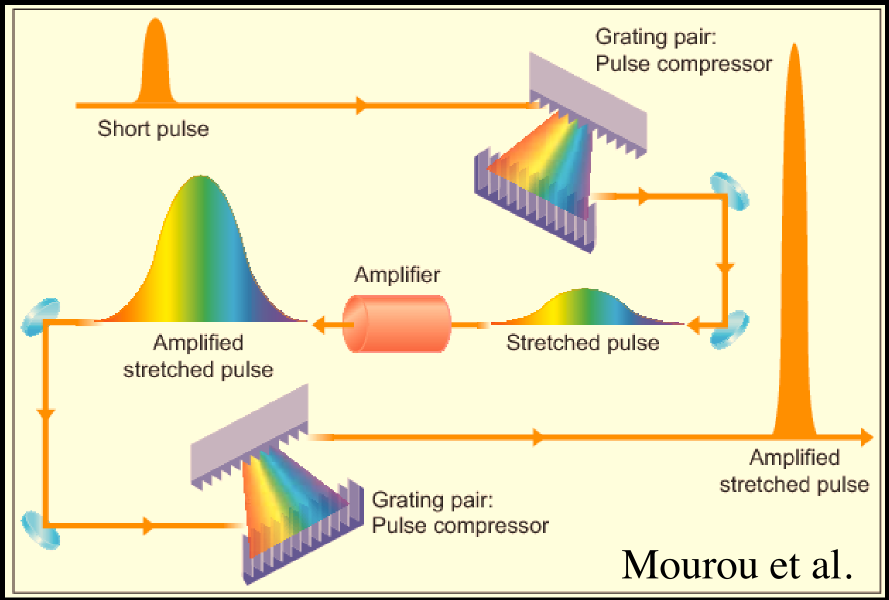


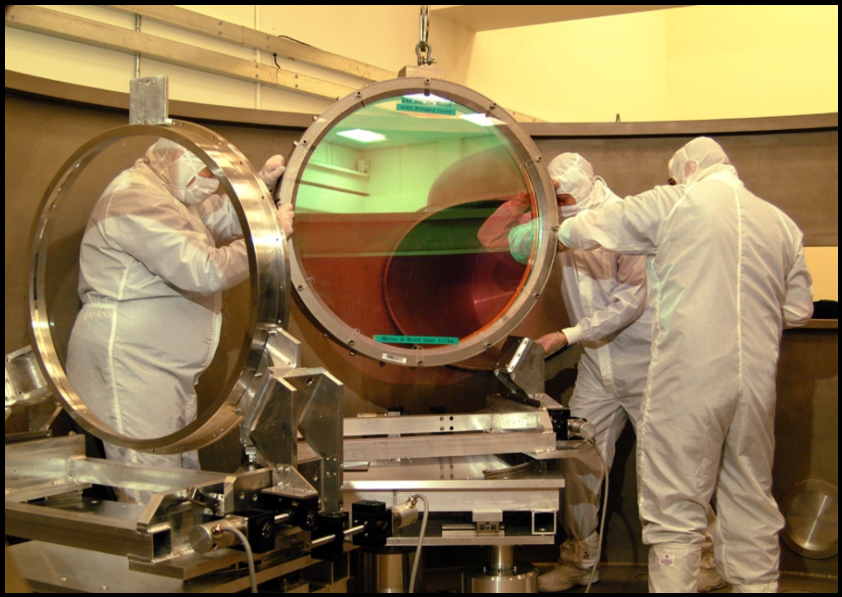
resonance condition

$$\omega_a - \omega_b = \omega_p$$

$$\vec{k}_a - \vec{k}_b = \vec{k}_p$$

Chirped Pulse Amplification: stretch, amplify, then recompress





Gratings for Petawatt (10^{15}W) Laser

Limitations of CPA

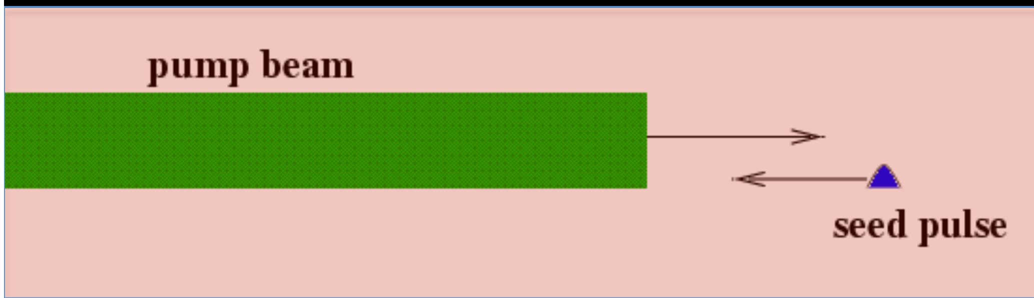
- Thermal damage to expensive gratings
- Requires broad-bandwidth high-fluence amplifiers

For PW

10^3 compression

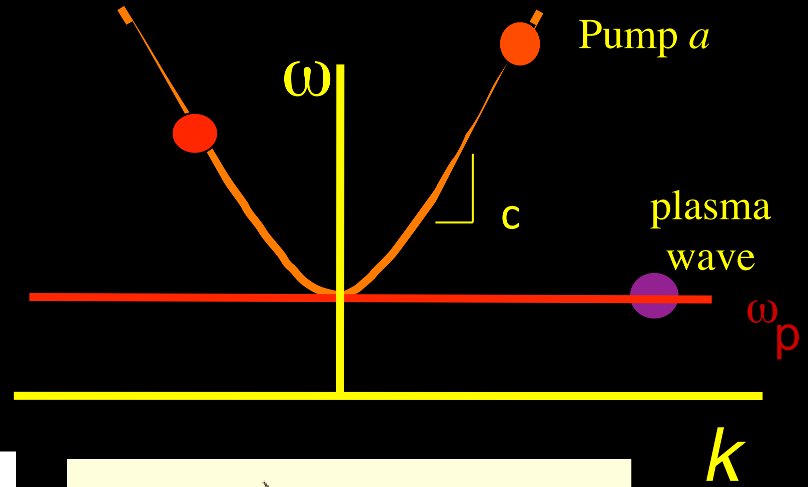
GW/cm^2 in amplifier → TW/cm^2 (< 10 ps) → 10^3 cm^2 gratings

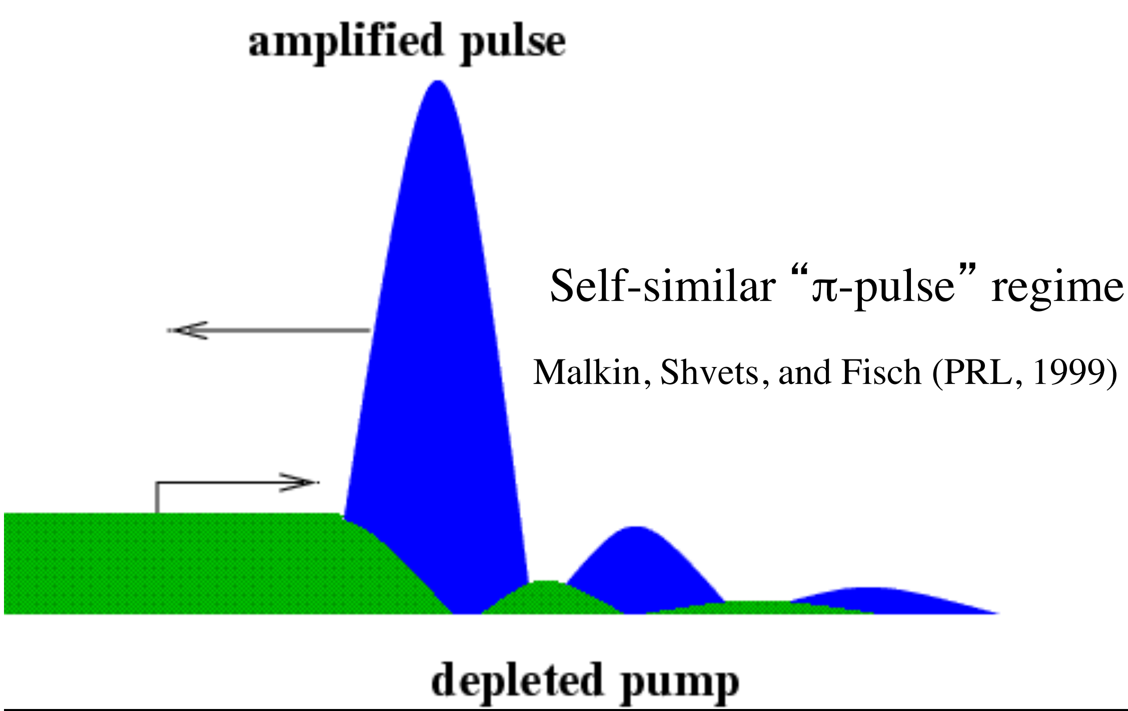
Resonant Raman Amplification and Compression



resonance condition

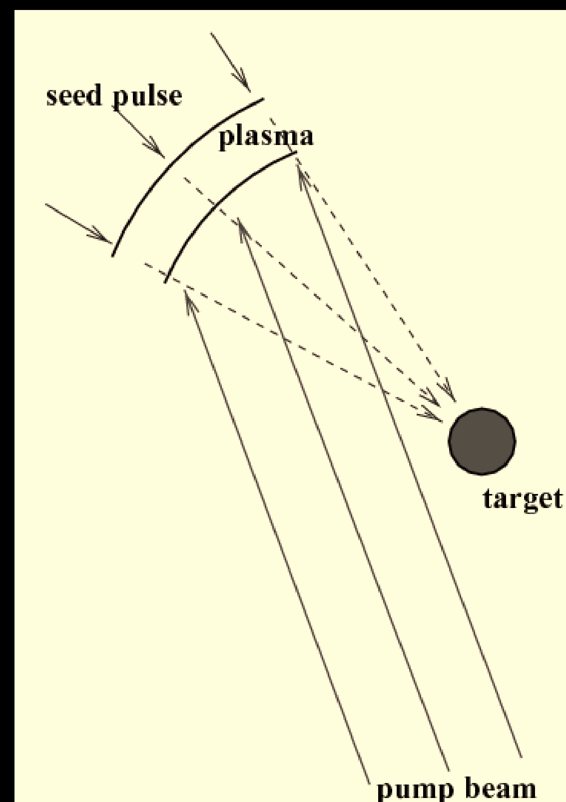
$$\omega_a - \omega_b = \omega_p$$
$$\vec{k}_a - \vec{k}_b = \vec{k}_p$$





Self-similar “π-pulse” regime

Malkin, Shvets, and Fisch (PRL, 1999)



Fast Compression By RBS

$$a_t + c a_z = -V f b \,,$$

$$f_t = V a b^*$$

$$b_t - c b_z = V a f^*$$

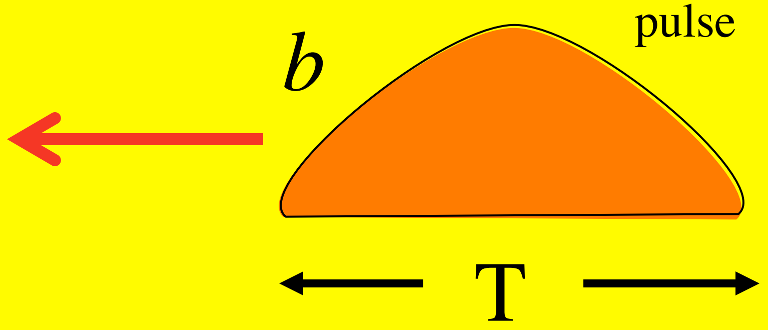
$$V = \sqrt{\omega_p \omega}\,/2$$

Malkin, Shvets, and Fisch, (PRL, 1999)

$$a \equiv \frac{eA_{\text{pump}}}{m_e c^2}, \quad b \equiv \frac{eA_{\text{pulse}}}{m_e c^2},$$

f is normalized plasma wave amplitude

$$\omega >> \omega_p$$



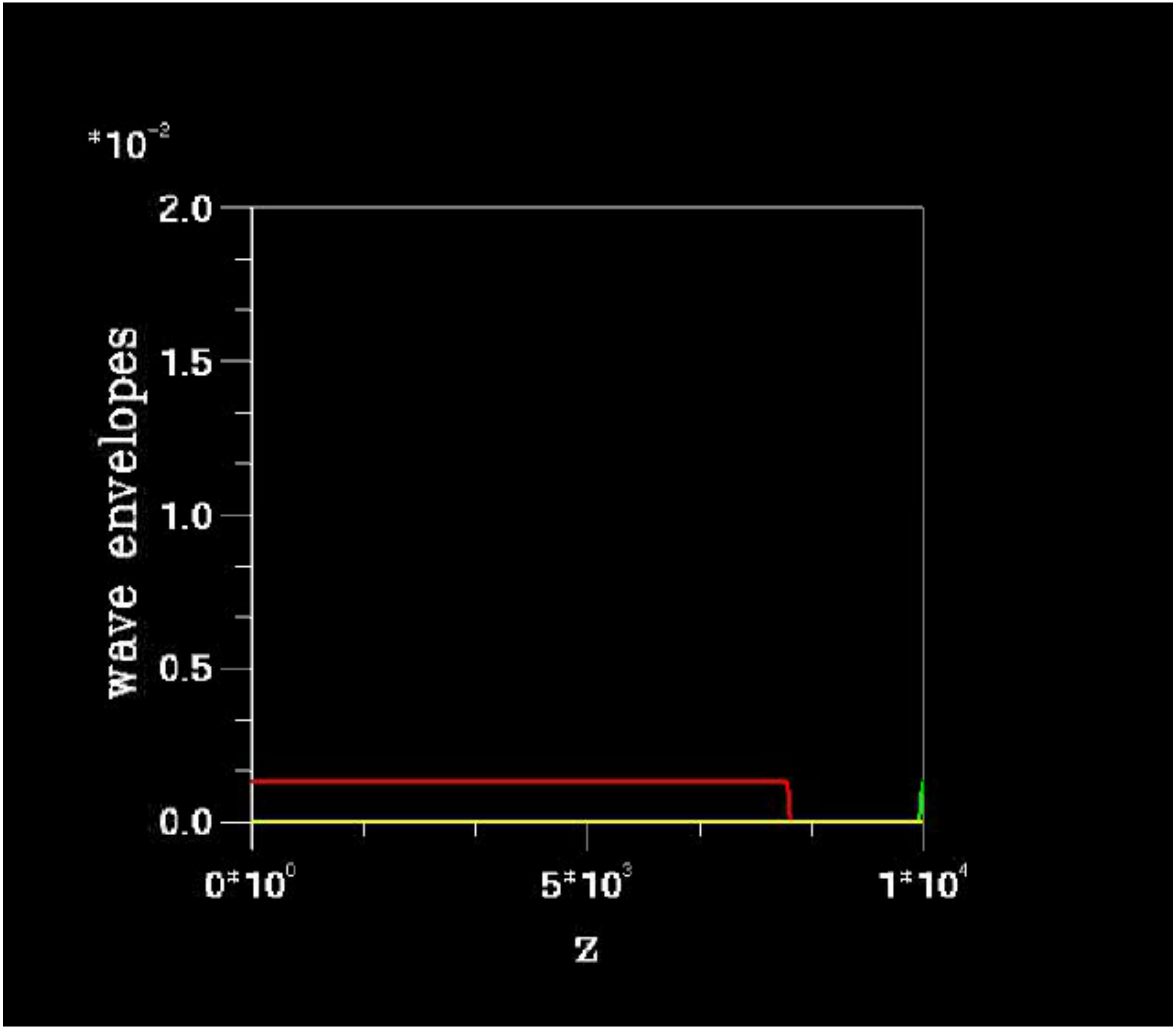
Self-similar solutions:

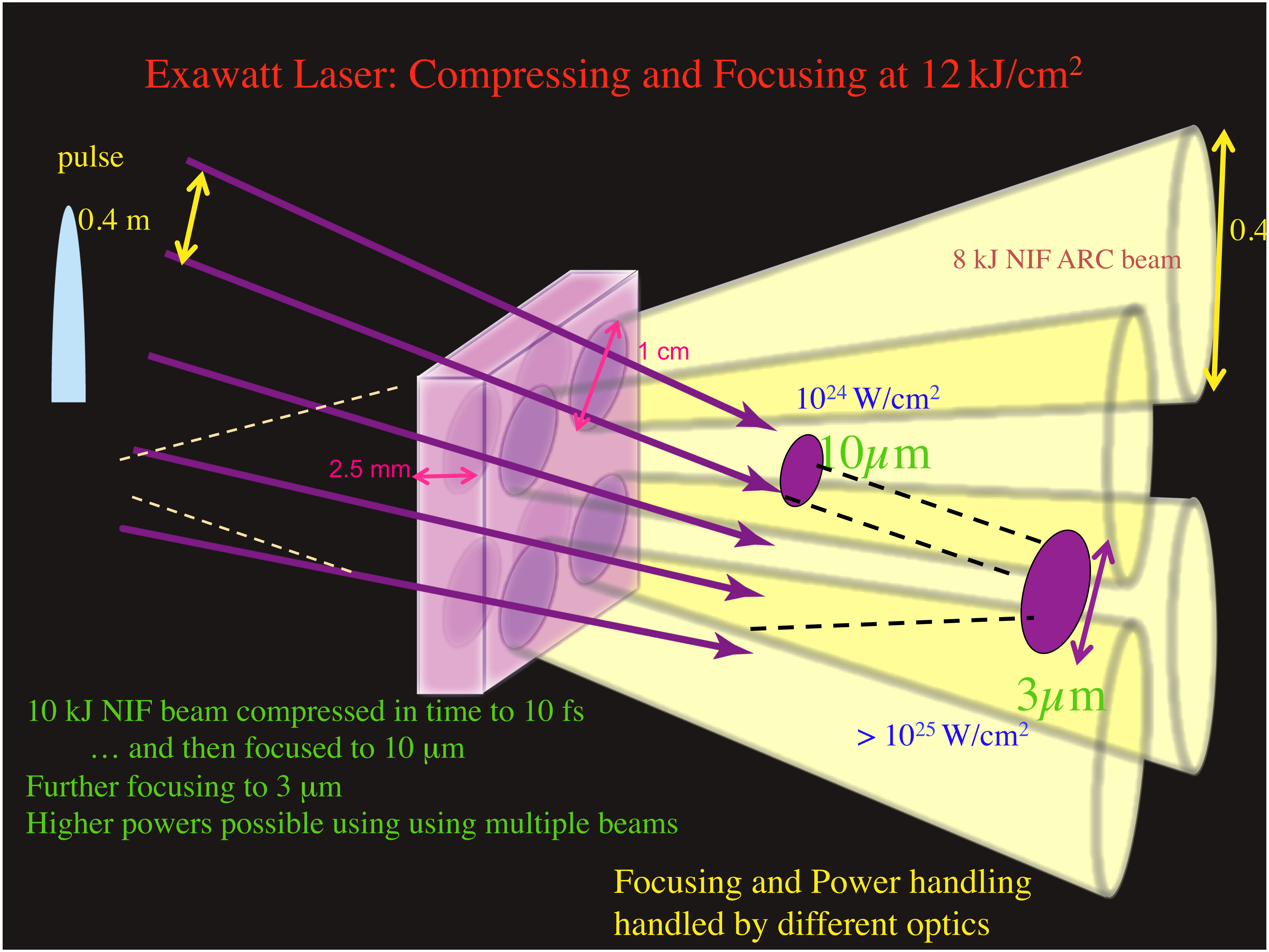
$b \quad \sim \; t$

$\mathrm{T} \quad \sim \; 1/t$

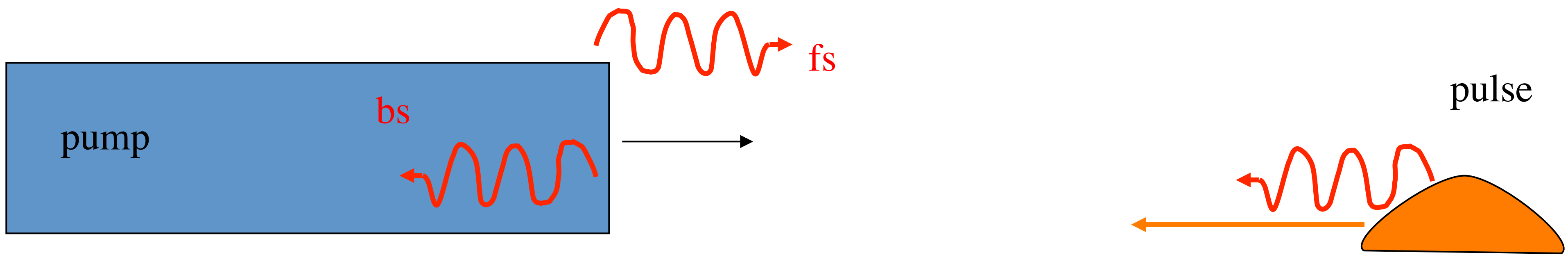
$\mathrm{E} = \mathrm{b}^2\mathrm{T} \quad \sim \; t$

$z \quad \sim \; t$

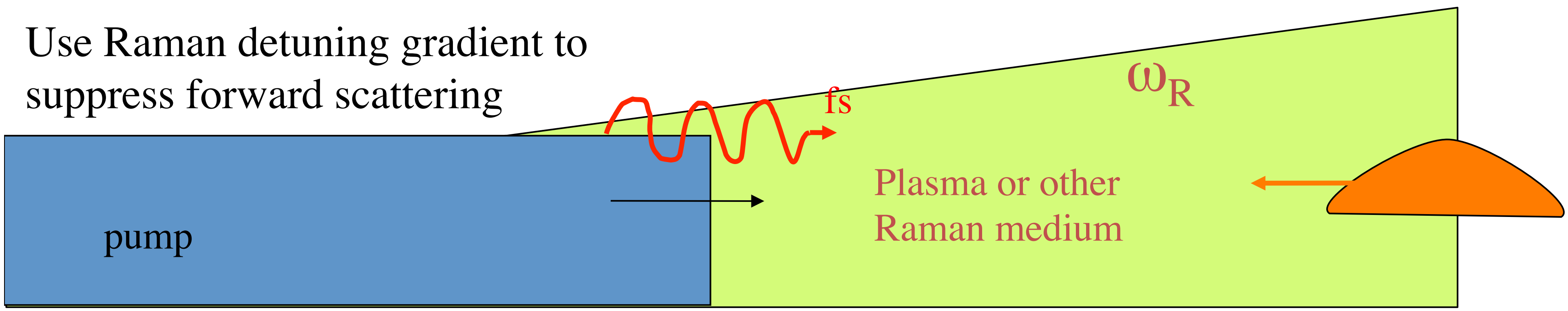

*10^-2
2.0
1.5
1.0
0.5
0.0
wave envelopes
0*10^0
5*10^3
1*10^4
z


Exawatt Laser: Compressing and Focusing at 12 kJ/cm^2
pulse
0.4 m
0.4
8 kJ NIF ARC beam
1 cm
10^{24} W/cm^2
10μm
2.5 mm
3μm
> 10^{25} W/cm^2
10 kJ NIF beam compressed in time to 10 fs
… and then focused to 10 µm
Further focusing to 3 µm
Higher powers possible using using multiple beams
Focusing and Power handling
handled by different optics

Suppression of *unwanted* Raman Scattering

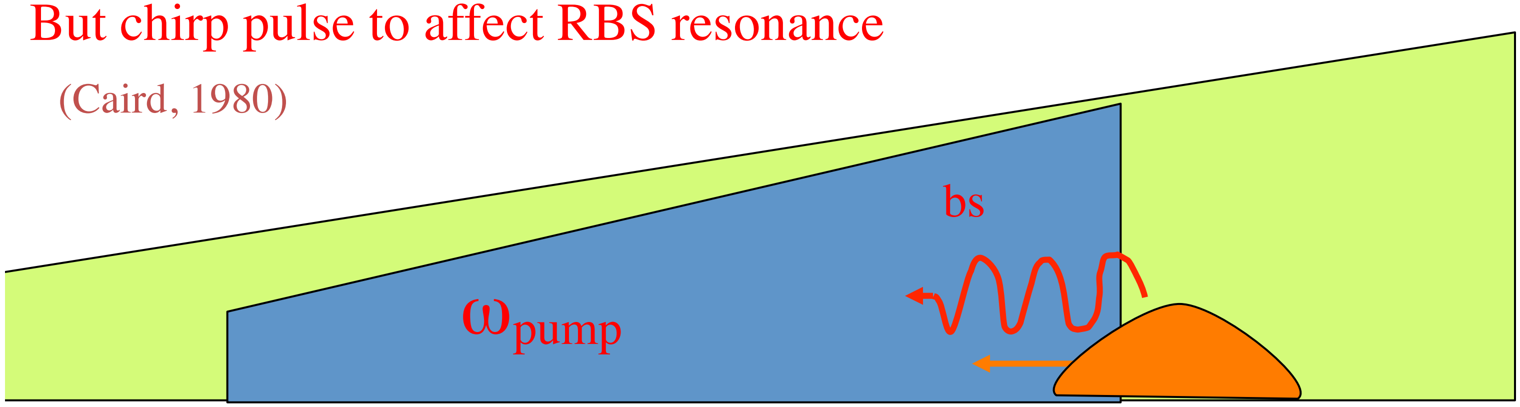


Use Raman detuning gradient to suppress forward scattering



But chirp pulse to affect RBS resonance

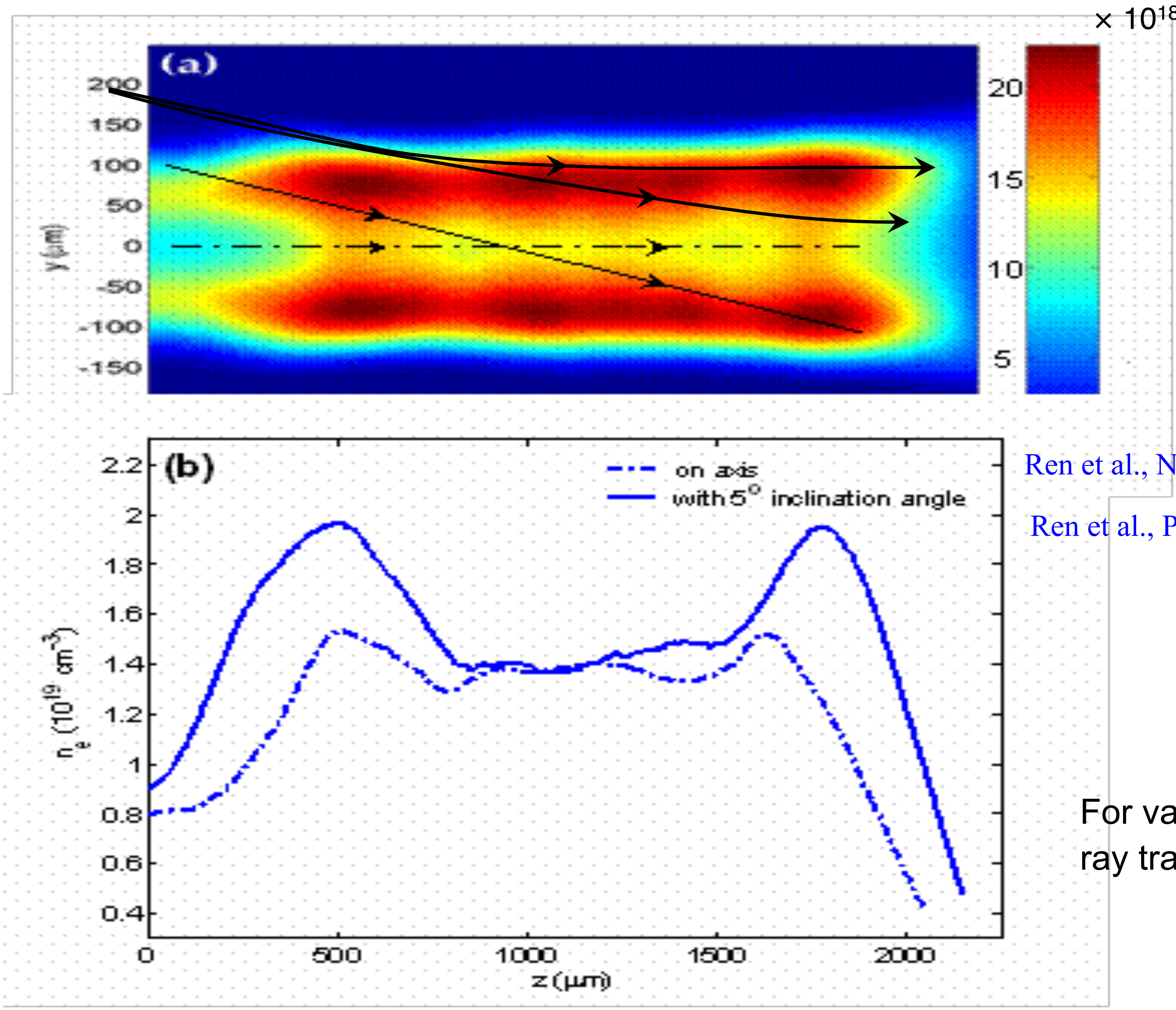
(Caird, 1980)



nonlinear effects
filter noise
but retain signal
(Malkin et al, 2000)

Tilted Laser Experiments

Suckewer (2007)

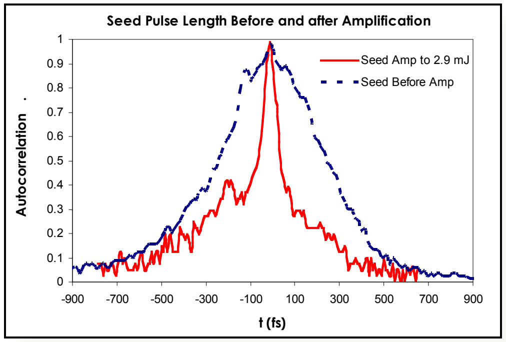


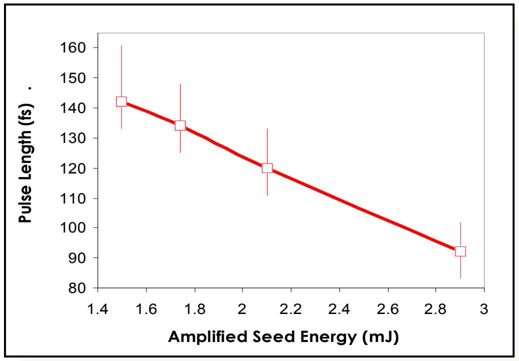
Ren et al., Nature Physics (2007)

Ren et al., POP (2008)

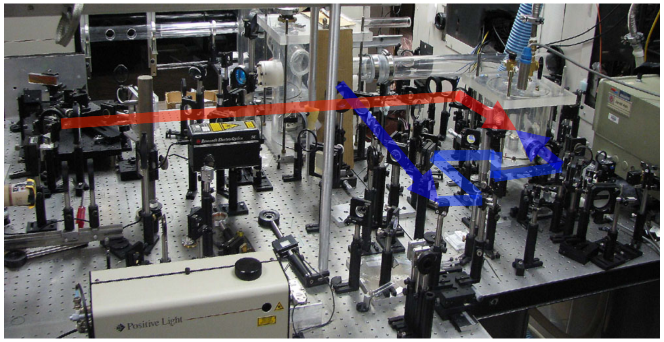
For vacuum
ray trajectory

Signatures of nonlinear self-similar regime





Suckewer Lab



Pump Seed

Factors of 100 in seed intensity over pump intensity

Decreased duration of the amplified pulse (50-90 fs)

Pedestal in the autocorrelation function evidences either precursors or secondary spikes

Duration of the amplified pulse decreases inversely with the pulse energy, as in nonlinear π-pulse solution regime

Large-- but still lower than theoretical maximum efficiency (up to 6.5%, but more adjusting for temporal and spatial overlap)

Amplification tends to saturate at high pump intensity.

Examples

Wavelength of laser μm	1/40	1/4	1	10
Duration of pump ps	1.25	12.5	50	500
Intensity of pump W/cm^2	1.6×10^{17}	1.6×10^{15}	10^{14}	10^{12}
Pump vector-potential a_0	0.006	0.006	0.006	0.006
Laser-to-plasma frequency ratio	12	12	12	12
Concentration of plasma cm^{-3}	1.1×10^{22}	1.1×10^{20}	7×10^{18}	7×10^{16}
Linear e-times growth length cm	.00043	.0043	.013	.13
Total length of amplification cm	.018	.18	.7	7
Output pulse duration fs	1	10	40	400
Output pulse fluence kJ/cm^2	160	16	4	0.4
Output pulse intensity W/cm^2	1.6×10^{20}	1.6×10^{18}	10^{17}	10^{15}

Laser energy vacuum breakdown

Critical electric field $E_c = 1.3 \times 10^{16}$ V/cm^2

Laser compression and focusing to laser wavelength λ

Energy located within volume λ^3

$$W_c \sim \lambda^3 E_c^2/8\pi \sim \lambda^3\ 8 \times 10^{18}\ \mathrm{J/cm^3}$$

λ	1 μm	100 nm	10 nm	1nm	1 Å
W_c	8 MJ	8 kJ	8 J	8 mJ	8 μJ

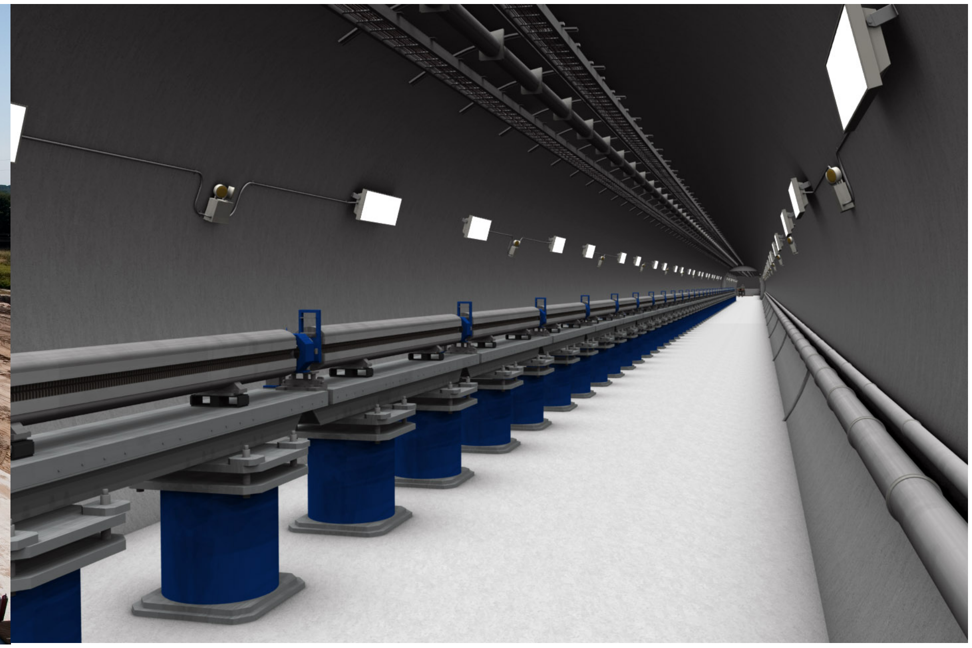
Use MJ optical or mJ x-ray lasers?

Malkin, Fisch and Wurtele (PRE, 2007)

New facilities for intense X-Rays

XFEL construction site: 9/1/09

LCLS Undulator Hall



Laser energy W *vs*. W_c

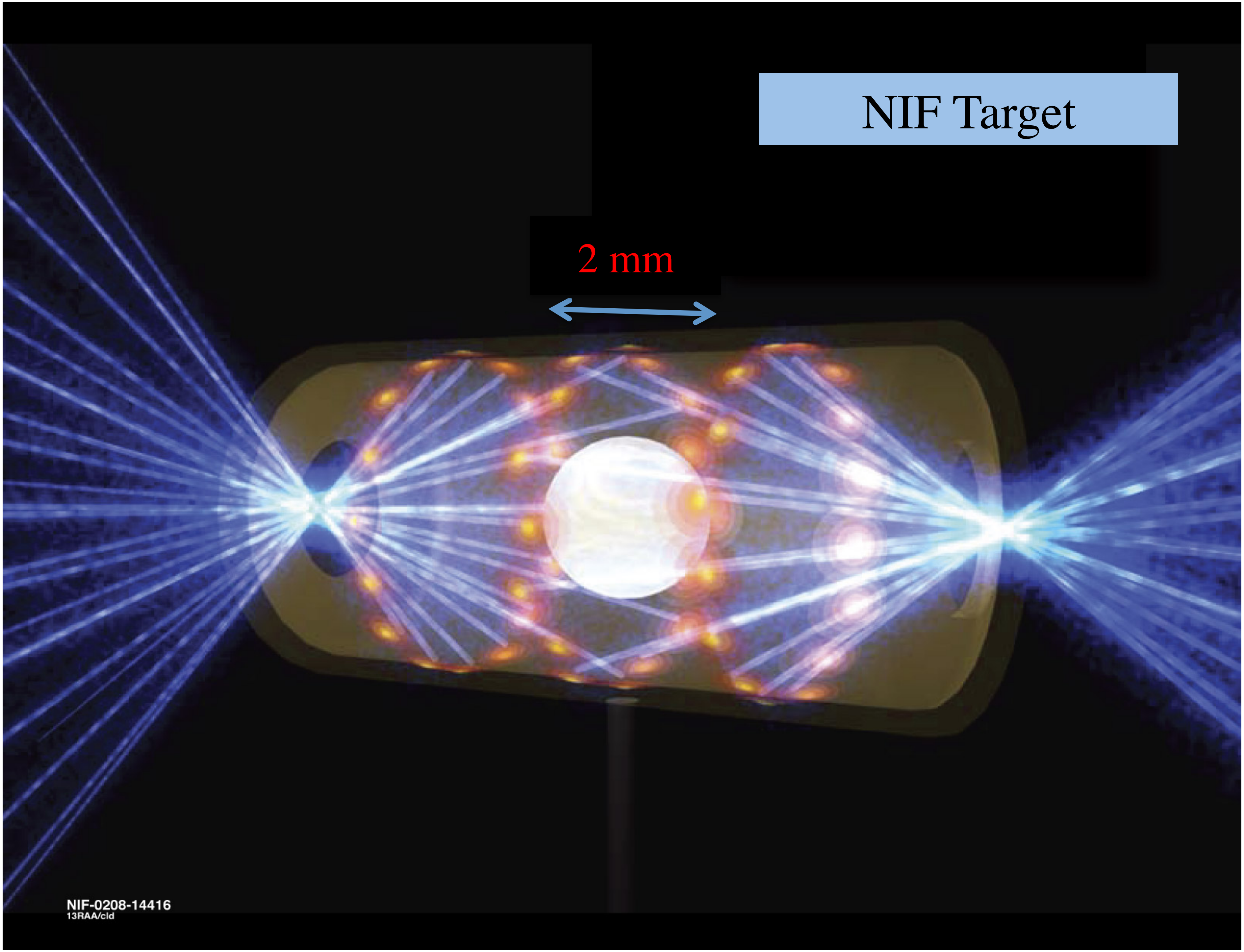
Device	NIF or LMJ	LCLS
λ	0.35 μm	0.15 nm
W	2 MJ	2 mJ
W_c	1/3 MJ	1/40 mJ
W/W_c	6	80

However, current ideas of compressing soft x-rays utilize RBS in the "quasi-transient regime".

Largest Plasma Compression Facilities

NIF -- National Ignition Facility (LLNL)

Z-Machine (Sandia National Laboratory)


NIF Target
2 mm
NIF-0208-14416
13RAA/cld

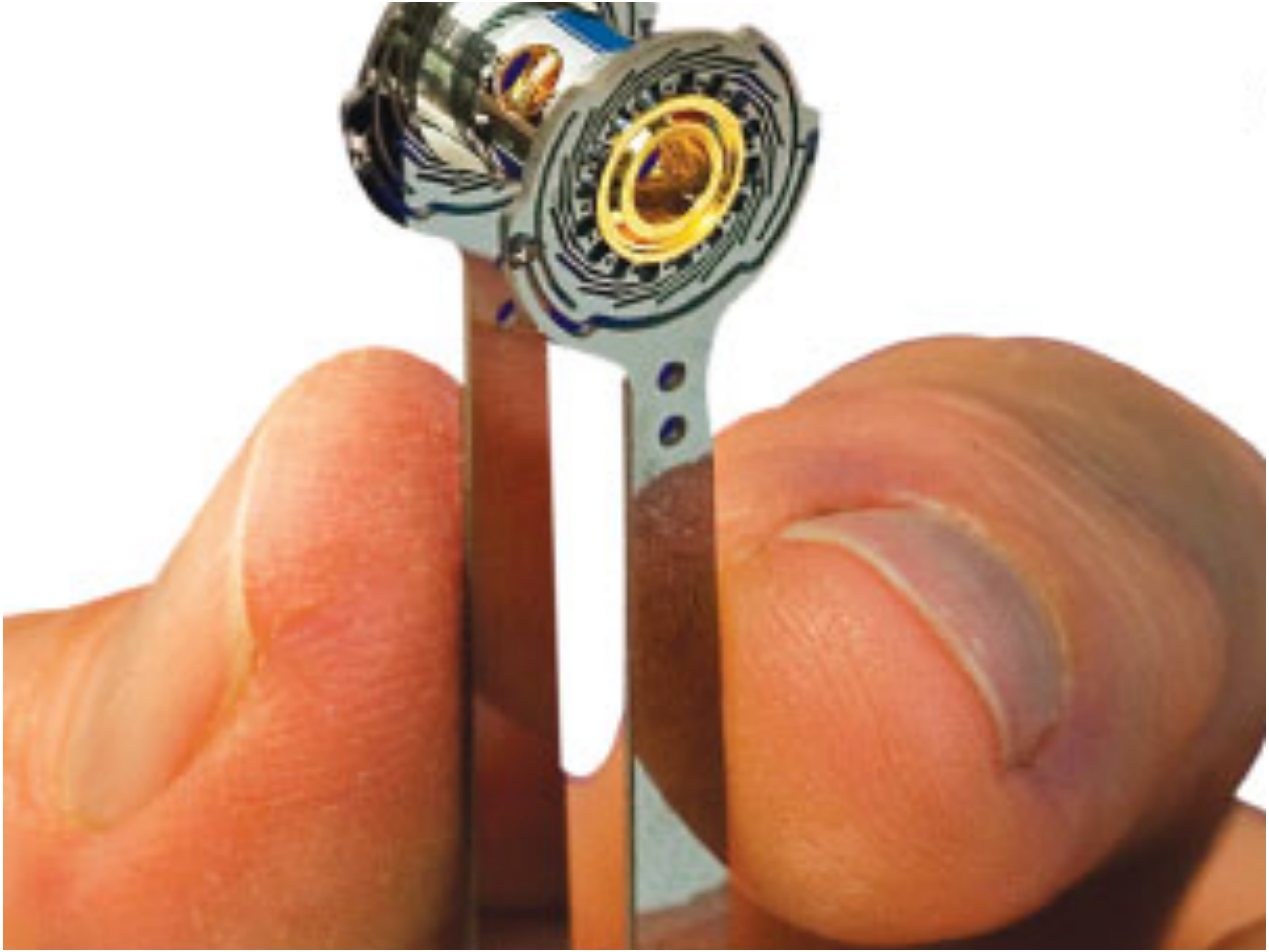
Wave Compression Motivated by New Extreme Compression Facilities

New $B facilities are being built to compress plasma to 1000 times solid density in as little as a nanosecond.

Waves with small group velocity, such as Langmuir waves, can be compressed if the compression time is short compared to the collision time, but long compared to the wave period.

As the imbedded wave grows, the ratio of the field energy to the plasma kinetic energy changes, which can in turn govern a variety of plasma processes.

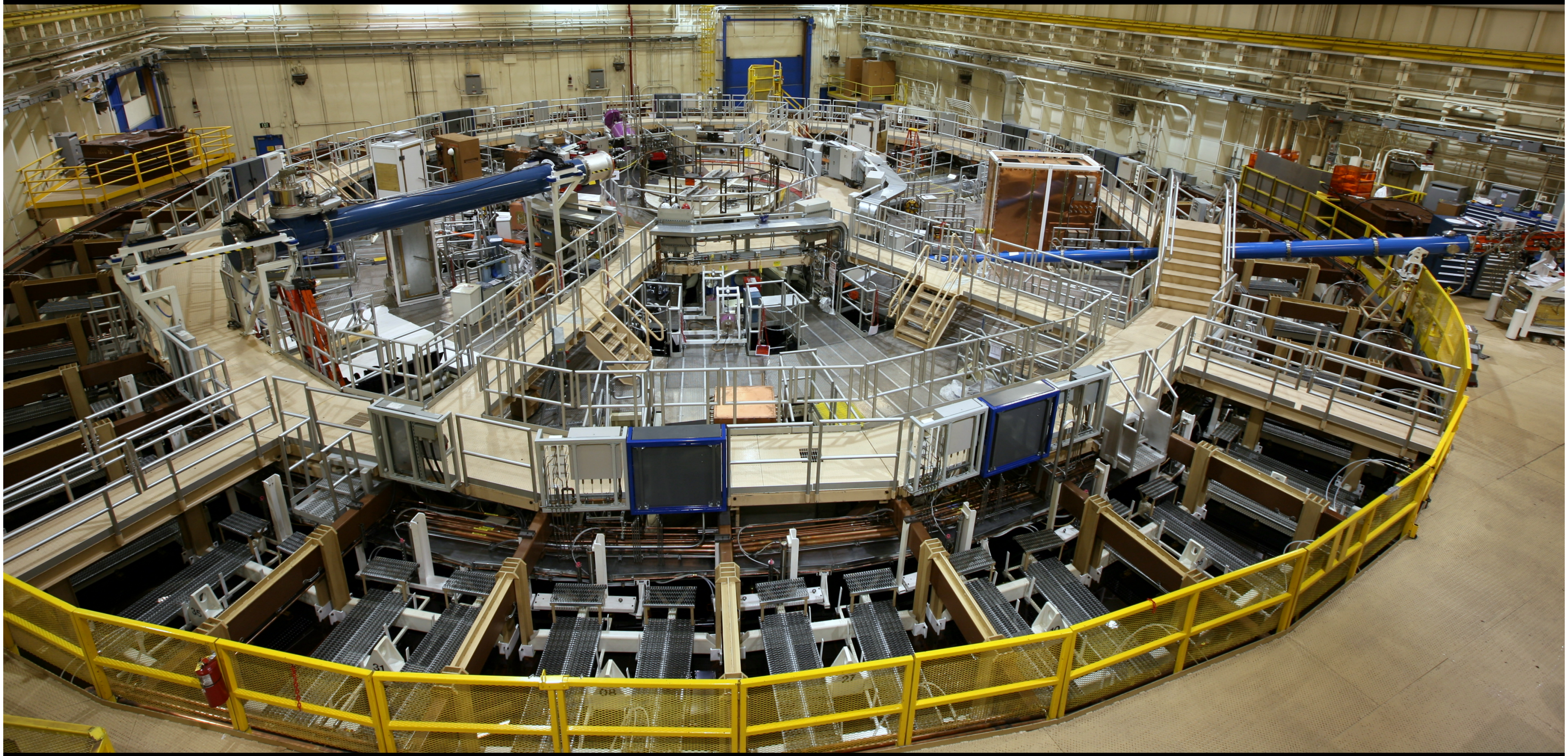
The separate control of wave energy, decoupled from the random particle energy, can be very useful.

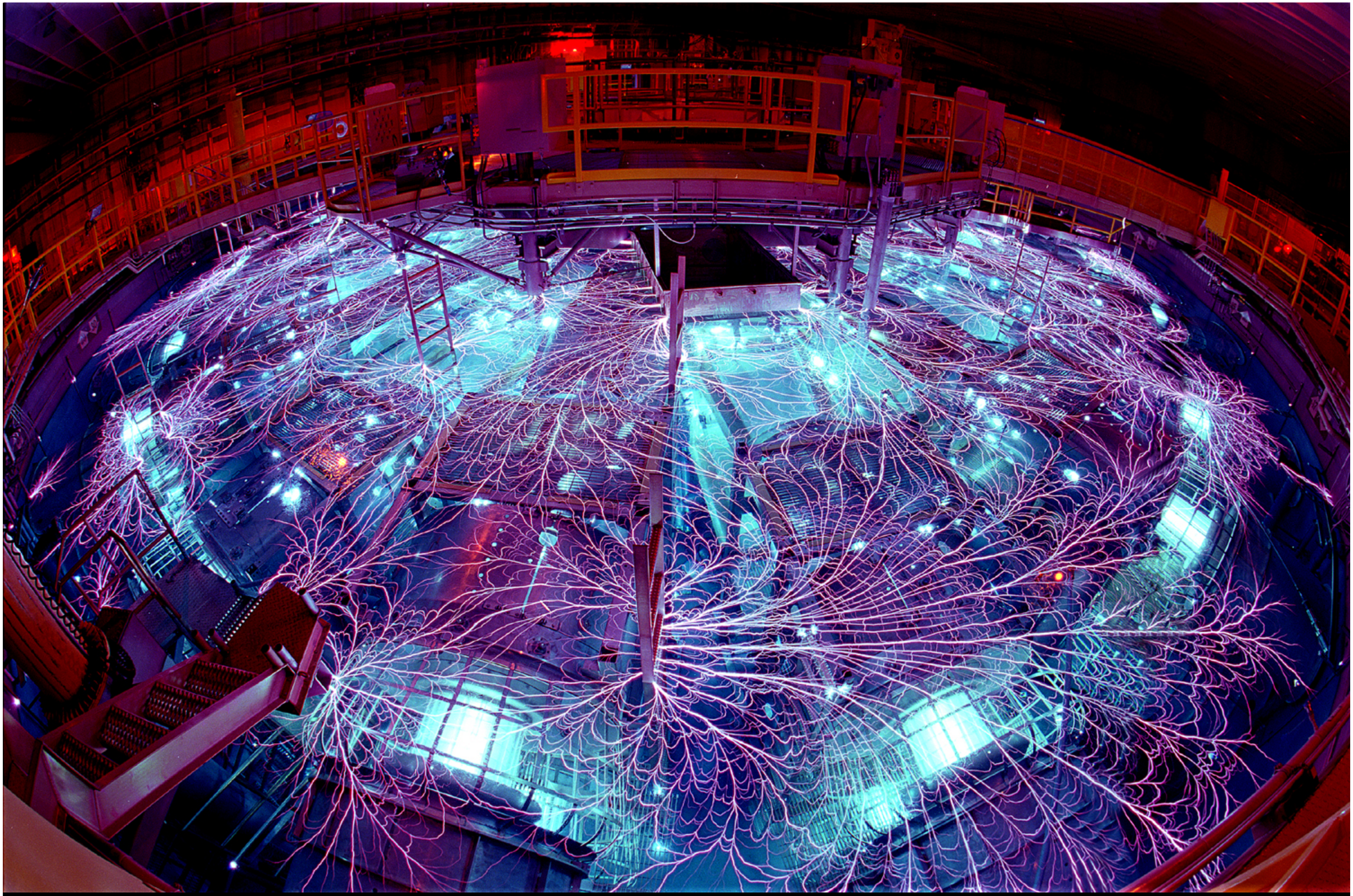


NIF
Target Chamber


lasers.llnl.gov

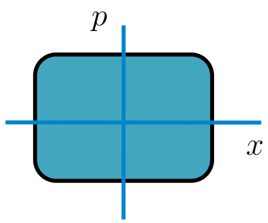
Z-Machine (Sandia)





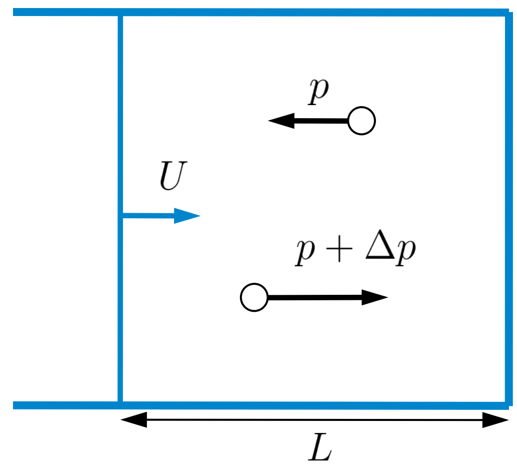
Adiabatic Compression of Waves (action conservation)

$$\Delta p = 2mU = -2m\frac{\Delta L}{\Delta t} = -\frac{\Delta L}{L}\,p$$



$$\boxed{\oint p\,dx = \mathrm{inv}}$$

$$\mathcal{E} = Np^2/(2m) \propto L^{-2} \propto V^{-2}$$



- Wave as a number of quanta:

$$\mathcal{E}/\omega = \hbar N = I$$

$$J = I/\mathcal{V}$$

$$\boxed{\partial_t J + \nabla\cdot(\mathbf{v}_g J) = 0}$$

$$E = \hbar\omega \qquad p = \hbar k$$

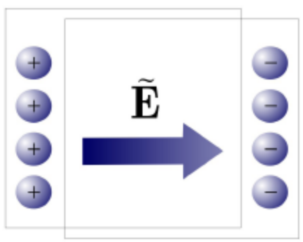
$$\oint p\,dx = \mathrm{inv} \quad \Rightarrow \quad kL = \mathrm{inv}$$

$$\omega = kc$$

$$\mathcal{E} = N\hbar\omega = N\hbar ck \propto L^{-1} \propto V^{-1}$$

Langmuir Wave Compression: Fluid Approach

$\omega_p^2 = 4\pi N e^2/m_e$



- Models vary in EOS, or the expression for $\hat{\mathbf{P}}_e$

$$\partial_t N_e + \nabla \cdot (N_e \mathbf{V}_e) = 0$$

$$\partial_t \mathbf{V}_e + (\mathbf{V}_e \cdot \nabla)\mathbf{V}_e = -(e/m_e)\,\nabla\varphi - \nabla \cdot \hat{\mathbf{P}}_e/(N_e m_e)$$

- Don't *assume* EOS; instead, *derive* it from

$$\partial_t \hat{\mathbf{P}}_e + (\mathbf{V}_e \cdot \nabla)\hat{\mathbf{P}}_e + \hat{\mathbf{P}}_e(\nabla \cdot \mathbf{V}_e) + [(\hat{\mathbf{P}}_e \nabla)\mathbf{V}_e] + [(\hat{\mathbf{P}}_e \nabla)\mathbf{V}_e]^{\mathsf{T}} = 0$$

$$\boxed{\begin{aligned}&\frac{\partial'^2 n}{\partial t^2} + \omega_p^2 n - C_{j\ell}\,\frac{\partial^2 n}{\partial x_j\,\partial x_\ell} + \\ &\qquad + 2\,\frac{\partial' n}{\partial t}\left(\frac{\Omega}{\omega_p}\,\frac{\partial \omega_p}{\partial x_\ell} + k_j W_{j\ell}\right)\frac{k_\ell}{k^2} - \left(\delta_{js} + \frac{k_j k_s}{k^2}\right)\frac{\partial C_{s\ell}}{\partial x_j}\,\frac{\partial n}{\partial x_\ell} = 0\end{aligned}}$$

$$\omega^2 = \omega_p^2 + 3k^2 v_{Te}^2 \approx \omega_p^2 \qquad\qquad \mathcal{E} = |E|^2/(8\pi) \propto N^{3/2}$$

Dodin, Geyko, & Fisch, POP, 2009

What happens to imbedded waves as plasma is compressed?

Regime of adiabatic compression: $\boxed{1/\nu < \tau_{comp} < 1/\omega}$

Action conservation: $\dfrac{VE^2}{\omega} \sim const$

Example:
Plasma Waves

$\omega \sim n^{1/2} \sim V^{-1/2}$ $\longrightarrow$ $\boxed{E \sim V^{-3/4} \sim n^{3/4}}$

$P_{pw} = \dfrac{E^2}{16\pi}$ $\longrightarrow$ $\boxed{P_{pw} V^{3/2} = const}$

compare: $PV^{\gamma} = const$

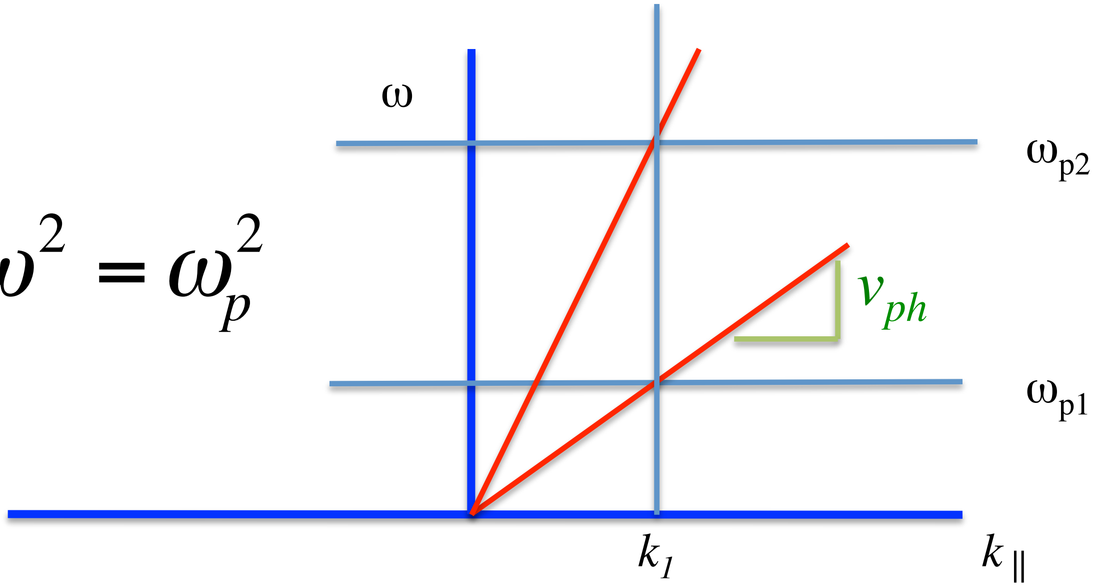
3D: $\gamma = 5/3$

1D: $\gamma = 3$

$$\gamma = \frac{m+2}{m}$$

Compression Perpendicular to *k*

$$\omega^2 = \omega_p^2$$



$\omega_{p1} \longrightarrow \omega_{p2}$

$k_1 \longrightarrow k_1$

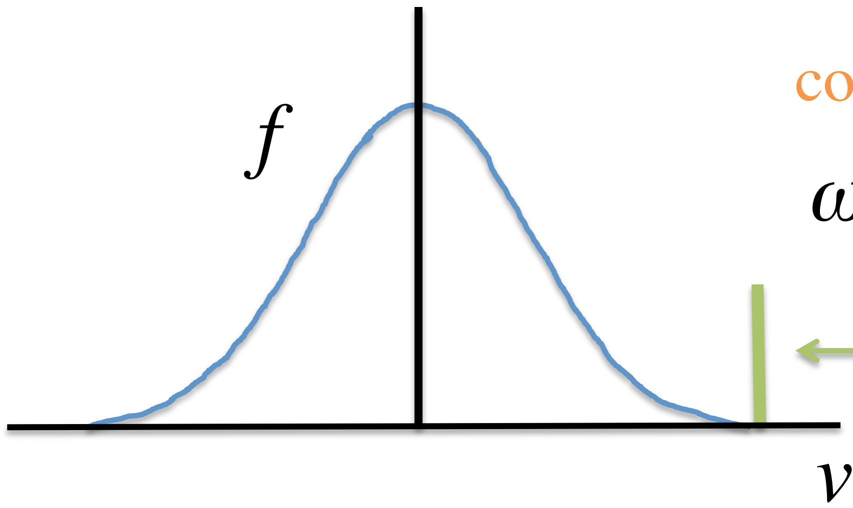
Under compression: Less damping,

if collisionless $\longrightarrow$ $T_\perp > T_\parallel$

Under expansion: More damping, $T_\perp < T_\parallel$

Extra wave energy $\longrightarrow$ $T_\parallel$

Current Drive and Heating

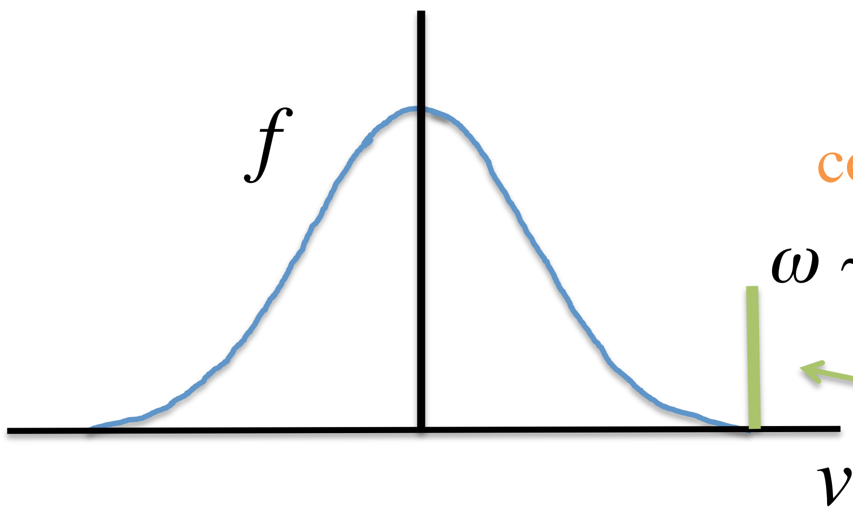


compression perpendicular to *k*

$$\omega \sim n^{1/2} \qquad k \sim v_{\parallel} \sim const$$

$$\boxed{\omega / k v_{\parallel} \sim n^{1/2}}$$

Note: under expansion, $T_{\perp} < T_{\parallel}$



compression parallel to *k*

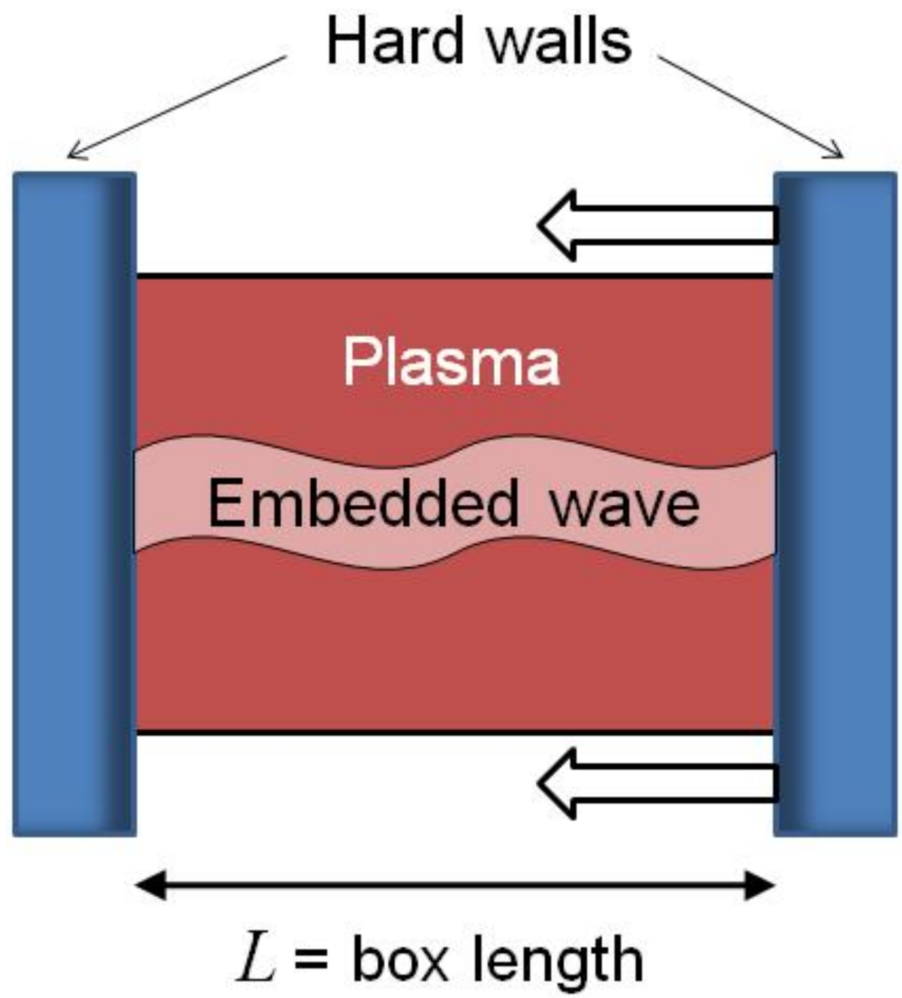
$$\omega \sim n^{1/2} \qquad k \sim 1/L \sim n \qquad v_{\parallel} \sim 1/L \sim n$$

$$\boxed{\omega / k v_{\parallel} \sim n^{-3/2}}$$

Note: under compression, $T_{\perp} < T_{\parallel}$

In either case, extra wave energy can accentuate energy difference

Particle Simulations

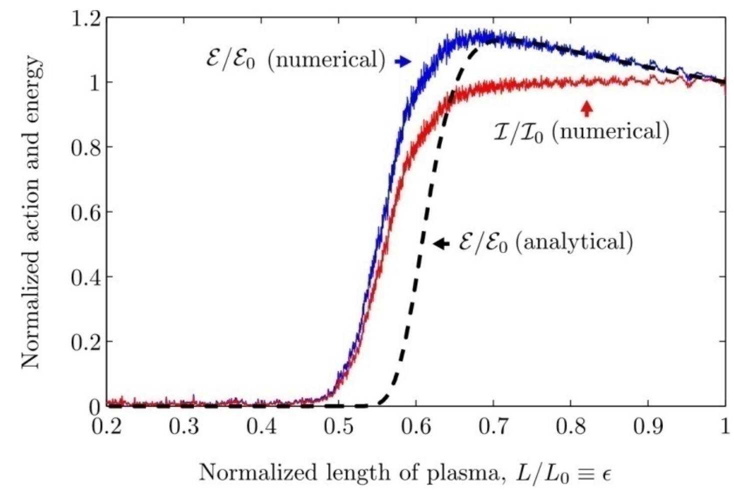


PIC simulation schematic

Schmit, Dodin and Fisch, PRL 2010

Langmuir Wave "Switch"

$$\boxed{\omega^2 = \omega_p^2 + 3k^2 v_{Te}^2 \approx \omega_p^2(n)}$$

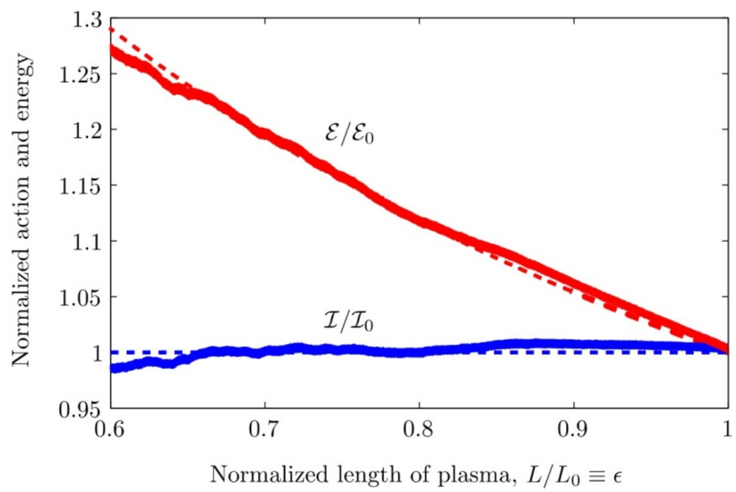


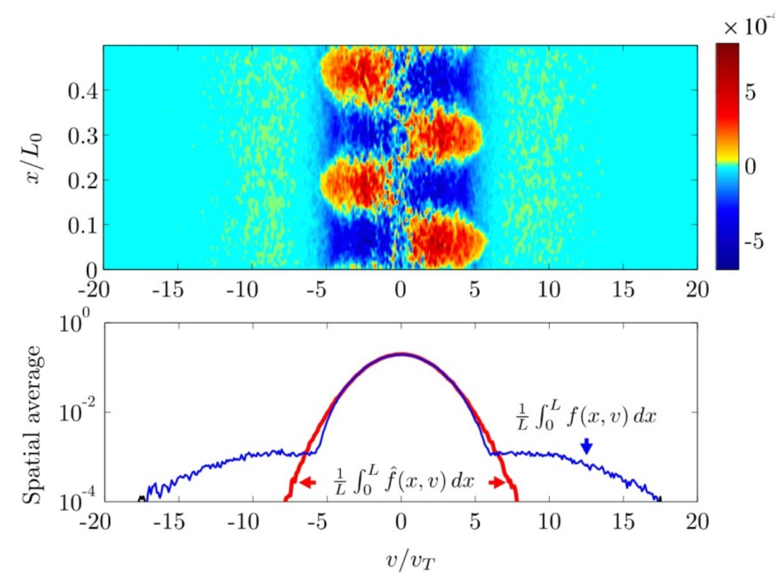
$$|E| \propto n^{3/4}$$

$$k\lambda_{\mathrm{D}} \propto L^{-3/2}$$

Dodin, Geyko, and Fisch, Phys. Plasmas (2010)

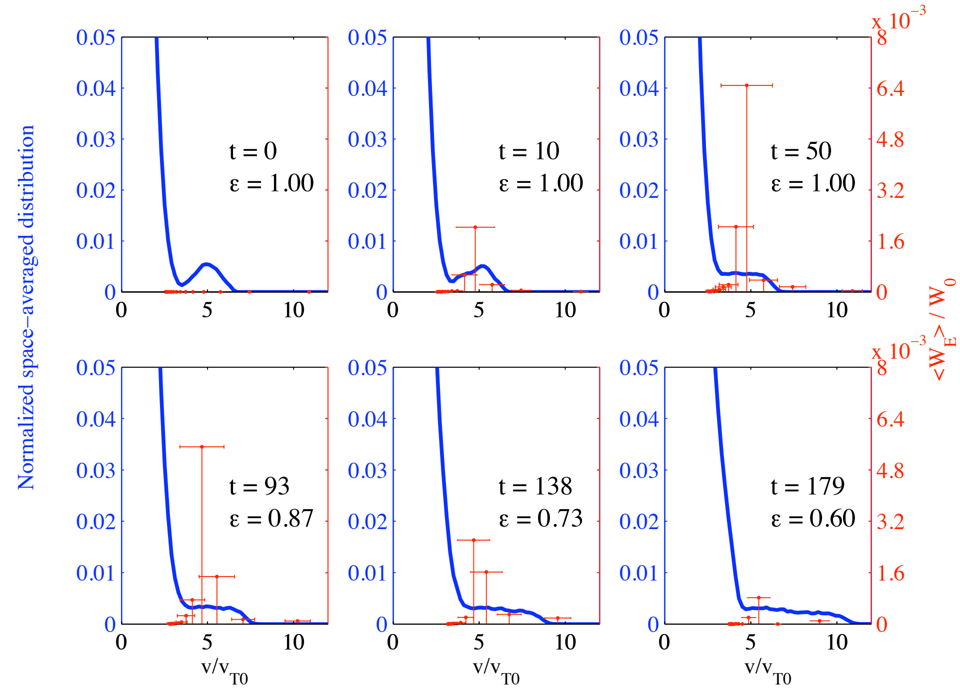
Schmit, Dodin, and Fisch, PRL (2010)





Plasma Compressibility

- Bump-on-tail instability.
- Work done sensitive to wave.
- Increases compressibility.
- Wave energy can be converted back to thermal energy through targeted collisionless damping.



$$P = -\frac{dW}{dV}$$

$$P = \frac{W}{2V}\left(1+3\frac{W_T}{W}\right)$$

$$W = W_w + W_T$$

$$W_w \sim V^{-1/2}$$

$$W_T \sim V^{-2}$$

Processes that increase wave energy at expense of thermal energy reduce plasma pressure

P. F. Schmit, C. R. Mooney, I. Y. Dodin, N. J. Fisch, J. Plasma Phys. **77**, 629 (2011).

Nonlinear wave-particle interactions

Trap particles

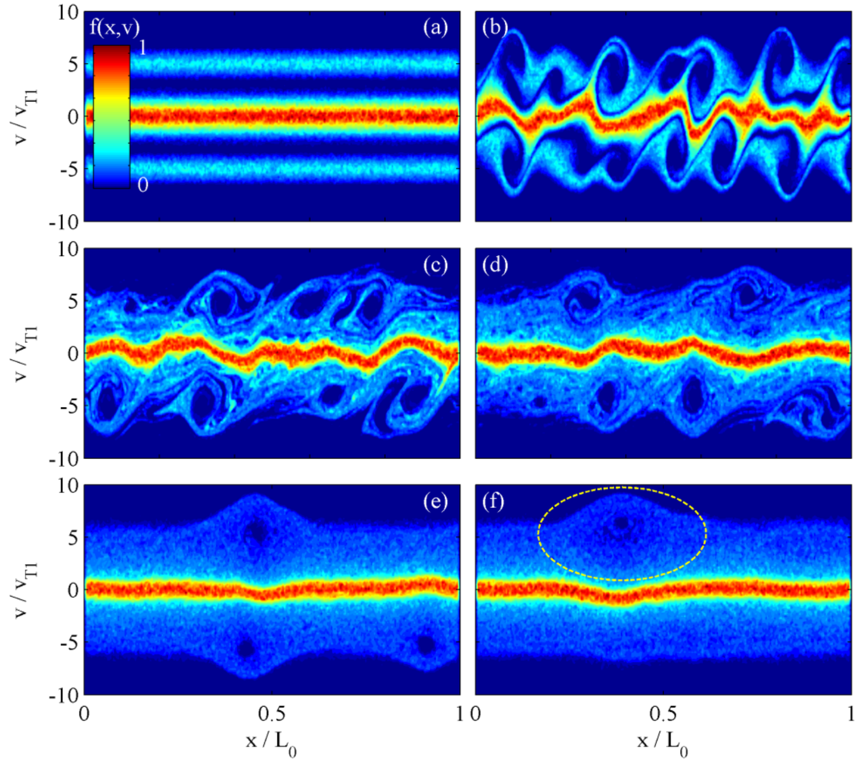
Robust under compression

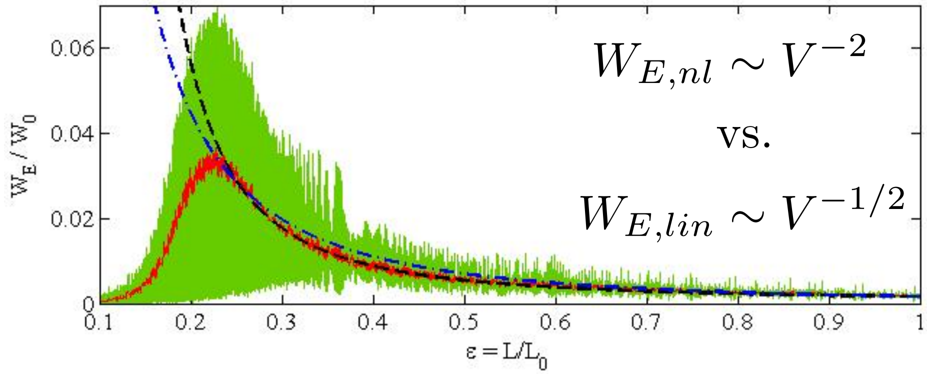
Amplify strongly

Driven to high amplitudes

Resistant to Landau damping

Schmit, Dodin, Fisch (2011)





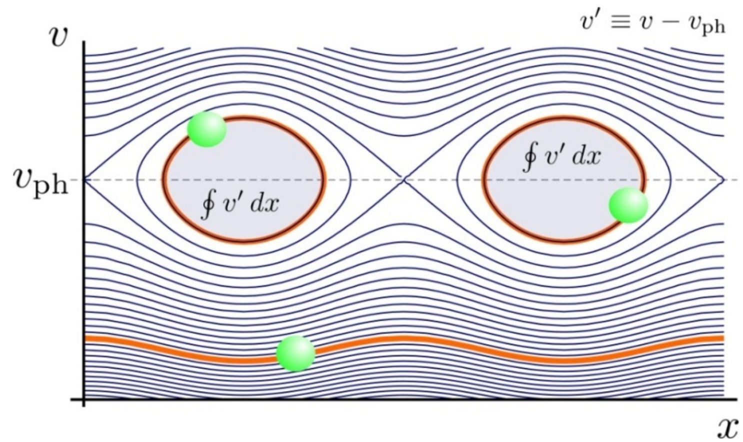
Bernstein-Greene-Kruskal waves (BGK waves)

- Waves with phase-mixed resonant particles

  Bernstein, Green, and Kruskal (1957)

  - Stationary structures, no Landau damping
  - Resonant electrons must follow the trapping islands

$$\boxed{\oint (v - v_{\text{ph}})\, dx = \text{inv}}$$



$$\boxed{\mathfrak{L}(a, \omega, \mathbf{k}) = \frac{\langle E^2 - B^2 \rangle}{8\pi} - \sum_s n_s \langle \mathcal{H}_s \rangle}$$

- dispersion relation: $\mathfrak{L}_a = 0$
- envelope equation: $\partial_t \mathfrak{L}_\omega - \nabla \cdot \mathfrak{L}_{\mathbf{k}} = 0$

Dodin and Fisch, PRL (2011);

Dodin and Fisch, Phys. Plasmas (2012)

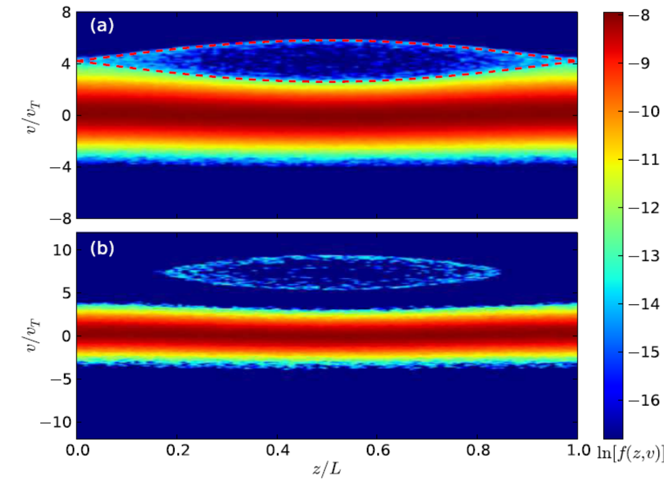
Whitham, *Linear and Nonlinear Waves*

- BGK-like waves are ubiquitous
  - Laser-plasma interactions
  - Energetic particle modes in tokamaks
  - Magnetosphere
- Fundamental problems:
  - Dispersion, dynamics, stability
  - Basic physics is hard to capture

Adiabatic amplification of a BGK wave in compressing plasma

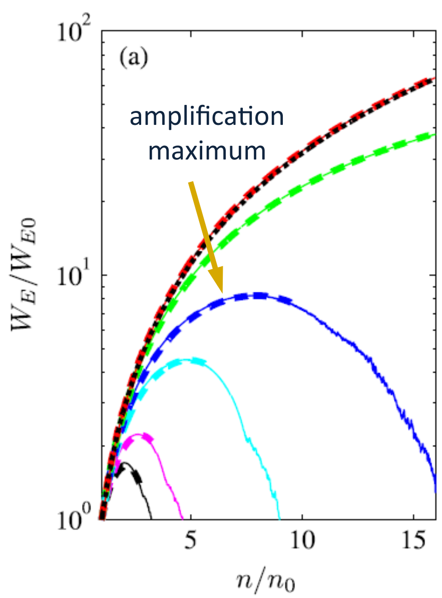
Plasma compression perpendicular to **k**

- Note that $\omega \sim \omega_p \sim n^{1/2}$ grows, but k is fixed
- Trapped electrons are accelerated as ω/k grows



How does the wave evolve during compression?

- *A simple Lagrangian model gives the answer*



$$\mathfrak{L} \approx \epsilon(\omega, k)\,\frac{E_{\rm m}^2}{16\pi} - n_{\rm t}\,\langle \mathcal{E}_{\rm t}(k, E_{\rm m})\rangle + \frac{m n_{\rm t}}{2}\left(\frac{\omega}{k}\right)^2$$

$$\boxed{\text{const} = \int \mathfrak{L}_\omega\, d^3x \sim \left[(\partial_\omega \epsilon) E_{\rm m}^2/(16\pi) + m n_{\rm t} v_{\rm ph}^2\right]/n}$$

Wave action contains an unusual nonlinear term:

Energy eventually goes to particle acceleration $\rightarrow$ damping

Schmit, Dodin, Rocks, and NJF, PRL (2013)

What can compressed waves usefully do?

1. Drive current

2. Reduce heat conduction

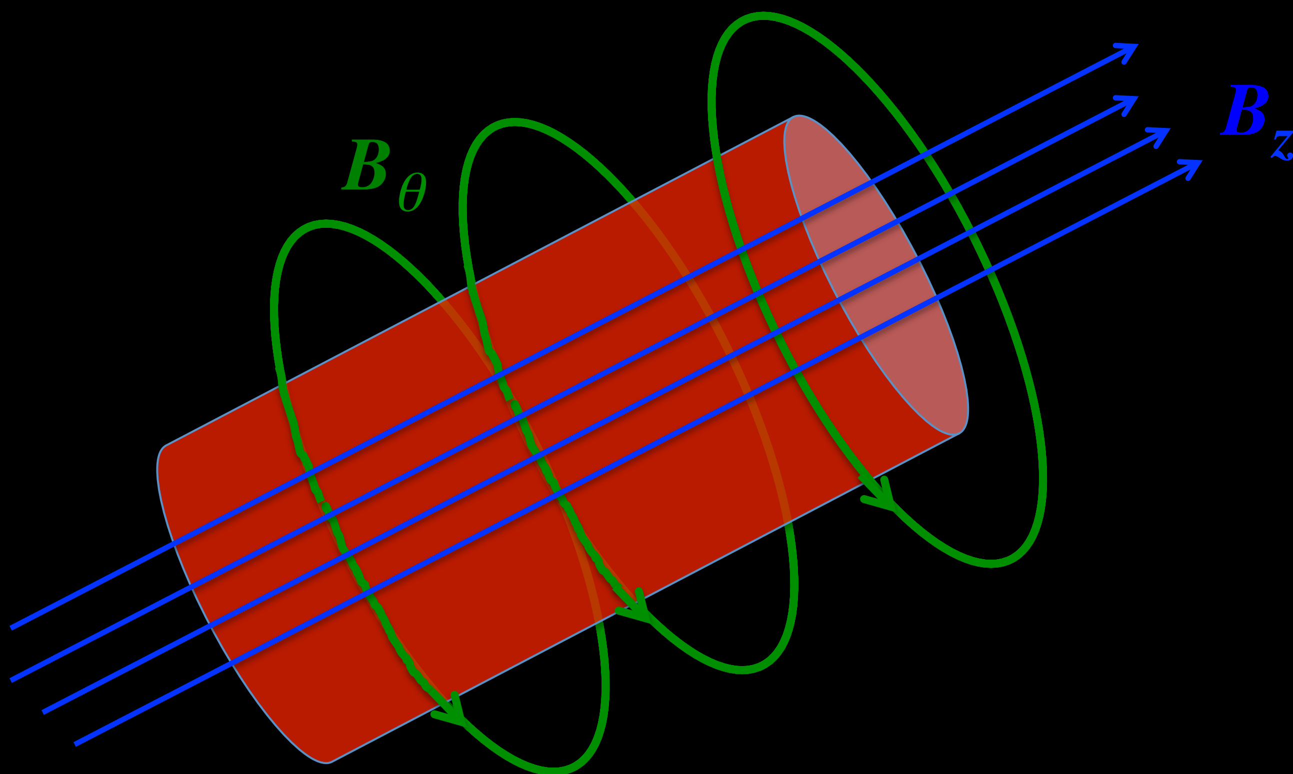
3. Heat ions preferentially

4. Slow down alpha particles

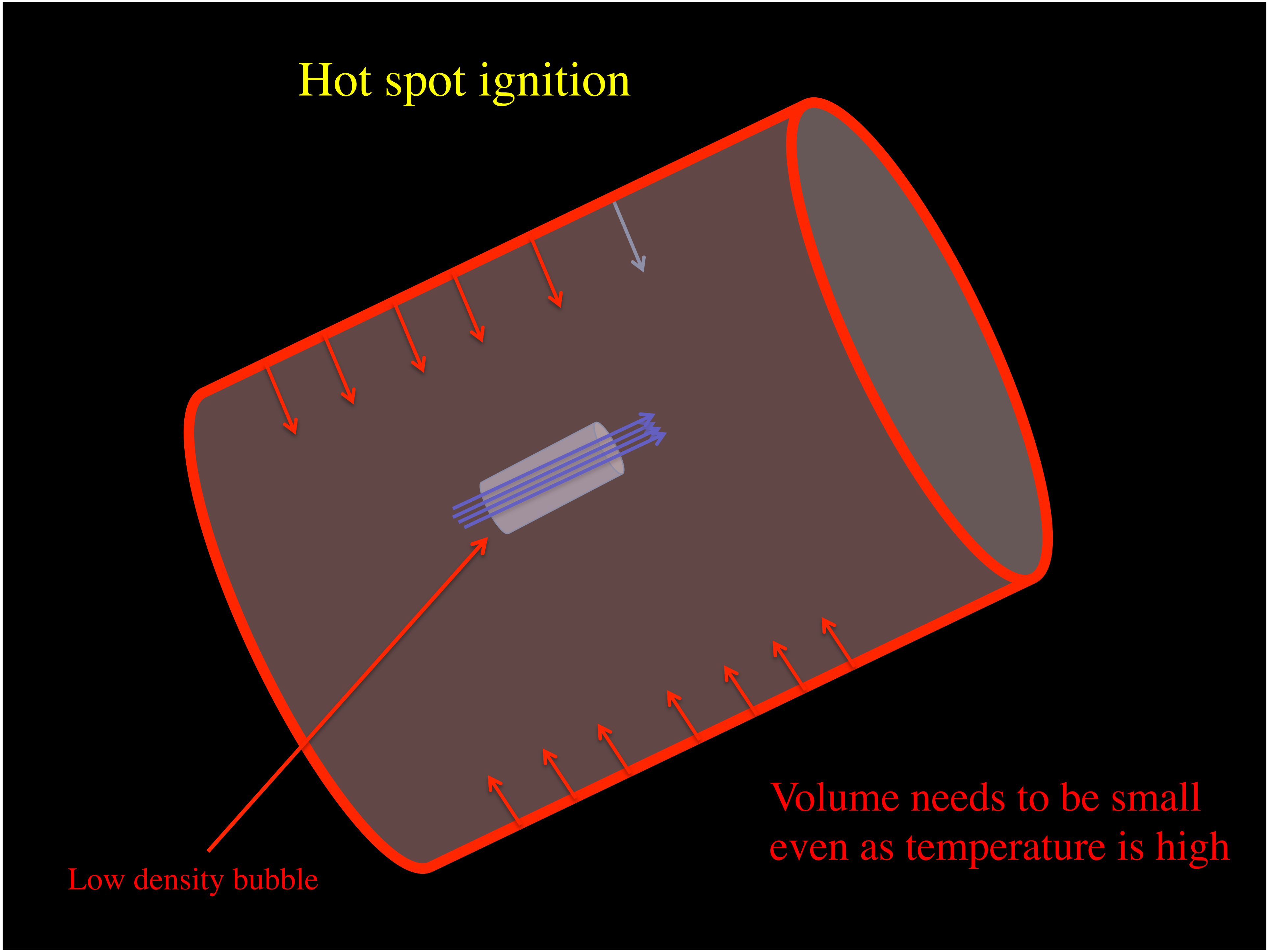
5. Reduce plasma pressure (increase compressibility)

6. All of the above, but with precise control in time

Z-pinch



Plasma and axial B trapped under compression


Hot spot ignition
Low density bubble
Volume needs to be small
even as temperature is high

Speculative Goal: Hot spot ignition with hot ion mode

$$\nu_{\perp}^{e} \gg \nu_{\perp}^{i} \gg \nu_{T}^{ie}$$

$$\frac{1}{\nu_{\perp}^{e}} \cong 1\mathrm{ps}\left[\frac{T_{10}^{3/2}}{n_{22}}\right]$$

Thus, if main loss through electrons,
get $T_i > T_e$ if

$$\boxed{\nu_{\perp}^{e} \gg \nu_{\perp}^{i} \gg \nu_{comp} \gg \nu_{T}^{ie}}$$

$$\nu_{\perp}^{e} \sim \sqrt{\frac{m_i}{m_e}}\nu_{\perp}^{i} \sim \frac{m_i}{m_e}\nu_{T}^{ie}$$

$$\nu_{\perp}^{i} \gg \nu_{\perp}^{i(hot)}$$

Get superthermal ion tail, if

$$\nu_{\perp}^{e} \gg \nu_{\perp}^{i} \gg \nu_{comp} \gg \nu_{\perp}^{i(hot)} \gg \nu_{T}^{ie}$$

Thus, with nanosecond or smaller compression time, create hot ion mode conditions

Hot ion mode facilitated since cold electrons do not conduct heat, magnetized ions and electrons minimize heat flow, and low parallel velocity ions stay in region even if very hot.

Summary

Some uses of a Plasma Wave

1. High-gradient accelerators
2. Toroidal current in tokamaks
3. Mediate resonant Raman compression of optical lasers in plasma
4. Compression of x-rays, short wavelength optical
5. Switch-like mechanisms in compression of plasma waves

Goal: Achieve next generation of light intensities

Goal: Realize new (timely) effects in new facilities for
highly compressing plasma

Goal: Facilitate economical fusion power