Ponderomotive Forces *on* Waves in Modulated Media

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Nonlinear interactions of waves via instantaneous cross-phase modulation can be cast in the same way as ponderomotive wave-particle interactions in high-frequency fields. The ponderomotive effect arises when rays of a probe wave scatter off perturbations of the underlying medium produced by a second, modulation wave, much like charged particles scatter off an oscillating electromagnetic field. Parallels with the point-particle dynamics, which itself is subsumed under this theory, lead to new methods of wave manipulation, including asymmetric barriers for light.

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Introduction.—One of the curious effects in wave-particle interactions is that a rapidly oscillating electromagnetic (EM) field can produce a time-averaged force, known as the ponderomotive force, on any particle that is charged or, more generally, has a nonzero polarizability [1,2]. This effect, which can be attractive or repulsive depending on a specific interaction, is widely used in various applications ranging from atomic cooling to plasma confinement [3,4]. Moreover, it was shown recently that ponderomotive forces can cause nonreciprocal dynamics, such as one-way-wall effects [4–7], and perform other nonintuitive transformations of the particle phase space [8]. As it turns out, and as we argue in this Letter, the same effects can be practiced also *on waves*, if the parameters of the medium are modulated in time or space.

Specifically, what we show here is that wave interactions in Kerr media via cross-phase modulation (XPM) can be cast in the same way as ponderomotive wave-particle interactions. The ponderomotive effect arises when rays of a geometrical-optics (GO) probe wave (PW) scatter off medium perturbations produced by a second, modulation wave (MW), much like particles scatter off EM waves. In contrast to the PW refraction caused by gradual changes of the medium average parameters ("slow" nonlinearity), the ponderomotive effect on rays is instantaneous and can be inferred from the PW linear dispersion alone, irrespective of the medium equations.

The practical utility of these findings is twofold: (i) Based on parallels with the wave-particle dynamics, we predict new qualitative effects, including ponderomotive reflection (which must not be confused with resonant, Bragg reflection) and asymmetric barriers for light; (ii) the XPM via instantaneous nonlinearities can now be described, both generally and quantitatively, beyond the special cases studied in literature [9–11]. In particular, we derive equations for the PW continuous ponderomotive dynamics that remain manifestly conservative even when the medium average parameters slowly evolve in time or space. We also show that the traditional theory of ponderomotive forces on point particles is subsumed under this general wave theory if particles are treated quantum-mechanically, as wave envelopes.

Basic equations.—Consider a linear PW propagating in a general dissipationless medium such that the GO approximation is justified. This implies, in particular, that the wave resides on a single branch of the dispersion relation, even though parameters of the medium may vary with time t and coordinates $\mathbf{x} \equiv (x^1, \ldots, x^D)$. (The number of spatial dimensions, D, can be arbitrary.) Assuming, for simplicity, that the wave is of the scalar type (which includes vector waves with fixed polarization too), it can be assigned a canonical phase $\theta(t, \mathbf{x})$, a scalar action density $\mathcal{I}(t, \mathbf{x})$, and the Lagrangian density [12,13]

$$\mathfrak{L} = -\mathcal{I}[\partial_t \theta + \omega(t, \mathbf{x}, \nabla\theta)]. \tag{1}$$

Both $\theta(t, \mathbf{x})$ and $\mathcal{I}(t, \mathbf{x})$ are independent functions here, so Eq. (1) generates two Euler-Lagrange equations,

$$\partial_t \theta + \omega = 0, \tag{2}$$

$$\partial_t \mathcal{I} + \nabla \cdot (\mathcal{I}\mathbf{v}_g) = 0, \tag{3}$$

where $\mathbf{v}_g(t, \mathbf{x}) \doteq \partial_{\mathbf{k}} \omega(t, \mathbf{x}, \mathbf{k}(t, \mathbf{x}))$ is the group velocity, and $\mathbf{k} \doteq \nabla\theta$ is the local wave number, $\mathbf{k} \equiv (k_1, \ldots, k_D)$. (We use the symbol $\doteq$ to denote definitions. We also allow for an arbitrary spatial metric g_{ij} and thus generally distinguish upper and lower indexes in the usual manner.) Equation (2) is of the Hamilton-Jacobi type and serves as the dispersion relation, $\omega = \omega(t, \mathbf{x}, \mathbf{k})$, since $-\partial_t \theta$ is, by definition, the wave local frequency. Equation (3) has the form of a continuity equation and represents the action conservation theorem. To close this set of equations, the so-called consistency relations are used,

$$\partial_t \mathbf{k} + \nabla\omega = 0, \qquad \nabla \times \mathbf{k} = 0, \tag{4}$$

which flow from the definitions of ω and $\mathbf{k}$. Equations (2)–(4) are known as the Whitham equations [14], and they also subsume, as their characteristics, the familiar ray equations for the components of $\mathbf{x}$ and $\mathbf{k}$ [15,16],

$$\dot{x}^\ell = \partial\omega(t,\mathbf{x},\mathbf{k})/\partial k_\ell, \qquad \dot{k}_\ell = -\partial\omega(t,\mathbf{x},\mathbf{k})/\partial x^\ell. \quad (5)$$

Reduced equations.—Suppose now that $\omega = \bar{\omega} + \tilde{\omega}$, where $\tilde{\omega}(t,\mathbf{x},\mathbf{k}) = \mathrm{Re}[\tilde{\omega}_c(t,\mathbf{x},\mathbf{k})e^{i\Theta(t,\mathbf{x})}]$ is a small perturbation. We term the latter a MW and introduce its frequency $\Omega \doteq -\partial_t\Theta$ and wave vector $\mathbf{K} \doteq \nabla\Theta$. Suppose also that Θ evolves slowly enough, so that the GO approximation for the PW holds (and, in particular, resonant effects like Bragg scattering do not occur). On the other hand, we will assume that Θ evolves fast compared to the rate at which $\bar{\omega}$ and the MW parameters (Ω, $\mathbf{K}$, and the amplitude) change in time and space. Hence we can unambiguously introduce the slow, Θ-independent, or adiabatic dynamics, which is done as follows.

Let us express the PW phase as $\theta = \bar{\theta} + \tilde{\theta}$ and the PW action density as $\mathcal{I} = \bar{\mathcal{I}} + \tilde{\mathcal{I}}$, where $\tilde{\theta}$ and $\tilde{\mathcal{I}}$ are oscillating functions of the order of $\tilde{\omega}_c$; also, $\bar{\theta} \doteq \langle\theta\rangle$, and $\bar{\mathcal{I}} \doteq \langle\mathcal{I}\rangle$, where the angular brackets denote local averaging over Θ. As usual [14,17], the Lagrangian density of slow, adiabatic dynamics can then be calculated as $\bar{\mathfrak{L}} = \langle\mathfrak{L}\rangle$. After neglecting terms of order $|\tilde{\omega}_c|^r$ with $r > 2$, one gets

$$\bar{\mathfrak{L}} = -\bar{\mathcal{I}}[\partial_t\bar{\theta} + w(t,\mathbf{x},\nabla\bar{\theta})], \quad (6)$$

where $w(t,\mathbf{x},\bar{\mathbf{k}}) = \bar{\omega} + \langle\nabla\tilde{\theta}\cdot\bar{\omega}_{\bar{\mathbf{k}}\bar{\mathbf{k}}}\cdot\nabla\tilde{\theta}\rangle/2 + \langle\tilde{\omega}_{\bar{\mathbf{k}}}\cdot\nabla\tilde{\theta}\rangle + h$, and $h \doteq \langle(\partial_t\tilde{\theta} + \bar{\omega}_{\bar{\mathbf{k}}}\cdot\nabla\tilde{\theta} + \tilde{\omega})\tilde{\mathcal{I}}\rangle$. (Note that $\nabla\tilde{\theta} \sim K\tilde{\theta}$, and K cannot be too large in the GO limit.) Both $\bar{\omega}$ and $\tilde{\omega}$ are evaluated here at $(t,\mathbf{x},\bar{\mathbf{k}})$, and the index $\bar{\mathbf{k}}$ denotes the corresponding partial derivative. The quiver phase, $\tilde{\theta}$, satisfies the linearized equation $\partial_t\tilde{\theta} + \bar{\omega}_{\bar{\mathbf{k}}}\cdot\nabla\tilde{\theta} + \tilde{\omega} = 0$, with $\bar{\mathbf{k}} = \nabla\bar{\theta}$. This leads to $h = 0$ and also $\tilde{\theta} = -i\tilde{\omega}/(\Omega - \mathbf{K}\cdot\bar{\omega}_{\bar{\mathbf{k}}})$. A straightforward calculation then yields

$$w(t,\mathbf{x},\bar{\mathbf{k}}) = \bar{\omega} + \frac{\mathbf{K}}{4}\cdot\frac{\partial}{\partial\bar{\mathbf{k}}}\left(\frac{|\tilde{\omega}_c|^2}{\Omega - \mathbf{K}\cdot\bar{\omega}_{\bar{\mathbf{k}}}}\right), \quad (7)$$

where, within the adopted accuracy, $\bar{\omega}_{\bar{\mathbf{k}}}$ can be replaced with $\bar{\mathbf{v}}_g \doteq \partial_{\bar{\mathbf{k}}}\omega_0$, and ω_0 is the unperturbed PW frequency evaluated at $\bar{\mathbf{k}}$. (The possible difference between ω_0 and $\bar{\omega}$ will become clear from examples below.) Equations (6) and (7) also lead to dynamic equations akin to the original Whitham equations (2)–(4):

$$\partial_t\bar{\theta} + w = 0, \quad (8)$$

$$\partial_t\bar{\mathcal{I}} + \nabla\cdot(\bar{\mathcal{I}}\mathbf{u}_g) = 0, \quad (9)$$

$$\partial_t\bar{\mathbf{k}} + \nabla\bar{\omega} = 0, \qquad \nabla\times\bar{\mathbf{k}} = 0, \quad (10)$$

where $\mathbf{u}_g \doteq \partial_{\bar{\mathbf{k}}}w$ serves as a new, effective group velocity modified by the presence of the MW. Then the corresponding "oscillation-center" (OC) ray equations, which can be considered as time-averaged Eqs. (5), are

$$\dot{\bar{x}}^\ell = \partial w(t,\bar{\mathbf{x}},\bar{\mathbf{k}})/\partial\bar{k}_\ell, \qquad \dot{\bar{k}}_\ell = -\partial w(t,\bar{\mathbf{x}},\bar{\mathbf{k}})/\partial\bar{x}^\ell. \quad (11)$$

Here w acts as the OC Hamiltonian of PW rays, or their ponderomotive Hamiltonian, so one may recognize Eq. (7) as an extension to continuous waves of what is a known theorem in classical mechanics of discrete systems [18,19]. [The cause of this parallel is that Eq. (2), which describes the dispersion relation of a continuous wave, is identical to the Hamilton-Jacobi equation for a ray as a discrete quasiparticle governed by Eqs. (5).] From the particle analogy (cf. e.g., Ref. [5]), one also obtains the adiabaticity condition underlying Eqs. (6)–(11); namely, in addition to the smallness of $\tilde{\omega}$, one must have

$$\dot{\tau} \ll 1, \qquad \tau \doteq |\Omega - \mathbf{K}\cdot\bar{\mathbf{v}}_g|^{-1}, \quad (12)$$

where τ is the modulation time scale in the ray reference frame, and the time derivative is taken along rays.

Equations (6)–(12) are the main analytical results of our Letter. They provide a new, *general* description of the MW effect on the GO propagation of a nondissipative continuous PW in any medium with a Kerr-type, cubic nonlinearity [20]. (XPM via second-order nonlinearities does not appear in our picture because the Pockels effect, such as in Ref. [21], requires $\dot{\tau} \gtrsim 1$ and otherwise averages to zero.) Slow nonlinearities enter here through the dependence of w on Θ-averaged parameters of the medium. To assess this effect quantitatively, one merely needs to add the OC Lagrangian density of the medium to $\bar{\mathfrak{L}}$ [15,22,23] and calculate the medium evolution in response to the ponderomotive force that a MW produces *on matter*. However, below we will focus instead on the general nonlinearity that is independent of the medium inertia. It can be viewed as an instantaneous ponderomotive effect that the MW produces *directly on PW rays* and hence is termed "ponderomotive refraction."

Examples of adiabatic dynamics.—Even at small $\tilde{\omega}$, ponderomotive refraction can be a significant factor in the PW evolution, especially when the underlying medium is homogeneous and stationary. The effect can be particularly strong near the group-velocity resonance (GVR), $\Omega \simeq \mathbf{K}\cdot\bar{\mathbf{v}}_g$. This is naturally understood for broad-spectrum PW pulses, as then the GVR can be (at least loosely) interpreted as the Cherenkov resonance between PW "quanta" and the MW [10,11,24]. However, as seen from our theory, the GVR remains a peculiar regime even for homogeneous waves, in which case $\bar{\mathbf{v}}_g$ does not have the transparent meaning of the envelope velocity. What is also remarkable is that Eq. (7) describes ponderomotive refraction solely from the PW linear dispersion, irrespective of equations that describe the medium nonlinear dynamics, in contrast to traditional theories [25]. Here are some examples.

(i) First, suppose a sound-like wave, $\omega(t,\mathbf{x},\mathbf{k}) = kC(t,\mathbf{x})$, where $C(t,\mathbf{x}) = C_0(t,\mathbf{x}) + \mathrm{Re}[\tilde{C}(t,\mathbf{x})e^{i\Theta(t,\mathbf{x})}]$, such that C_0 and $\tilde{C}$ are slow functions. Then $\bar{\omega} = \omega_0 = \bar{k}C_0$, and $\tilde{\omega} = \bar{k}\tilde{C}$, so Eq. (7) yields

$$w = \omega_0 \left[1 + \frac{\epsilon^2 \cos\chi}{U - \cos\chi} + \frac{\epsilon^2 \sin^2\chi}{2(U - \cos\chi)^2} \right], \quad (13)$$

where $U \doteq \Omega/(KC_0)$, $\epsilon^2 \doteq |\tilde{C}|^2/(2C_0^2)$, and χ is the angle between $\bar{\mathbf{k}}$ and $\mathbf{K}$. Suppose, for simplicity, that $U \sim 1$ and that any spatial gradients are along $\mathbf{K}$, so the transverse wave vector, $\bar{\mathbf{k}}_\perp$, is conserved. At quasiparallel propagation ($\bar{k}_\parallel \gg \bar{k}_\perp$, where $\bar{k}_\parallel$ is the parallel component of the wave vector), one then gets $w \simeq \bar{k}_\parallel C_{\rm eff}$, where $C_{\rm eff} \doteq C_0[1 + \epsilon^2/(U-1)]$; i.e., the ponderomotive effect changes the sound speed from C_0 to $C_{\rm eff}$. At quasitransverse propagation ($\bar{k}_\parallel \ll \bar{k}_\perp$), one gets $w/(\bar{k}_x C_0) \simeq 1 + (p-\alpha)^2/2 + \phi$, where $p \doteq \bar{k}_\parallel/\bar{k}_\perp$, $\alpha \doteq -\epsilon^2(1+U^2)/U^3$, and $\phi \doteq \epsilon^2/(2U^2)$. If C_0 is constant, and α is independent of time, then α is removable by gauge transformation as an effective vector potential; hence the ponderomotive effect consists of ray repulsion by the scalar potential ϕ. Notably, the linear theory of mode conversion [26] does not capture these effects.

(ii) As another example, consider an EM wave in plasma with electron-density relative perturbation $\tilde{N}(t, \mathbf{x})$. Then $\omega(t, \mathbf{x}, \mathbf{k}) = [\omega_p^2(t, \mathbf{x}) + k^2c^2]^{1/2}$, where $\omega_p = \omega_{p0}(1 + \tilde{N})^{1/2}$ is the plasma frequency, ω_{p0} is its unperturbed value, and c is the speed of light. (Relativistic effects [27] are neglected in this model.) Hence $\tilde{\omega} \simeq \tilde{N}\omega_{p0}^2/(2\omega_0)$, $\bar{\omega} \simeq (1-\epsilon^2)\omega_0$, and $\bar{\mathbf{v}}_g = c^2\bar{\mathbf{k}}/\omega_0$, where $\omega_0 = (\omega_{p0}^2 + \bar{k}^2c^2)^{1/2}$, $\epsilon \doteq \tilde{N}_m\omega_{p0}^2/(4\omega_0^2)$, and $\tilde{N}_m$ is the amplitude of $\tilde{N}$. One then gets

$$w = \omega_0[1 - \epsilon^2(\Omega^2 - K^2c^2)/(\Omega - \mathbf{K}\cdot\bar{\mathbf{v}}_g)^2]. \quad (14)$$

Note that EM wave propagation in static modulated media, like photonic crystals [28], are described by Eq. (14) as a special case corresponding to $\Omega = 0$ (cf. e.g., Ref. [29]); then $w = \omega_0[1 + (\epsilon/\mathsf{n}_\parallel)^2]$, where $\mathsf{n}_\parallel \equiv \bar{k}_\parallel c/\omega_0$ is the PW refraction index along $\mathbf{K}$. (However, this result applies only at large enough $\mathsf{n}_\parallel$, such that $\mathbf{u}_g$ does not deviate much from $\bar{\mathbf{v}}_g$.) Also let us consider the opposite limit, $\Omega \gg \mathbf{K}\cdot\bar{\mathbf{v}}_g$. Assuming, for simplicity, that $\bar{k}$ is small, in this case one gets $w/\omega_{p0} \simeq 1 + c^2\bar{k}^2/(2\omega_{p0}^2) + \phi$, where $\phi = \epsilon^2(\mathsf{N}^2 - 1)$ acts as an effective potential. Its sign is determined by the MW refraction index, $\mathsf{N} \doteq Kc/\Omega$, which, in principle, can have any value, especially if the MW is produced by driven fields. Such ϕ thereby attracts PW rays if $\mathsf{N} < 1$ and repels them [30] if $\mathsf{N} > 1$ (Fig. 1). (In particular, the latter case is realized when the MW is one of the natural plasma waves, e.g., an ion acoustic or Langmuir wave.) Such an effective potential is similar to the adiabatic ponderomotive potential seen by point charges in a high-frequency EM field [1].

Nonadiabatic dynamics.—The wave-particle analogy is also naturally extrapolated to the Pockels regime ($\dot{\tau} \gtrsim 1$), where the interaction is nonadiabatic [4,8]. Based on what is known about the particle dynamics in nonadiabatic ponderomotive barriers [4,8,31], one readily anticipates that regions of strong MW can be arranged in this regime to scatter PW rays probabilistically and, when Ω is nonzero, also *asymmetrically*. This is confirmed in simulations already for simple MW envelopes (Fig. 2), and asymmetry can be made even stronger if the MW shape is specially adjusted (Fig. 3). (Notably, these manipulations are somewhat akin to those produced by effective gauge fields on PWs in externally driven lattices of multimode resonators [32]. The difference is, however, that our ponderomotive forces can be applied to single-mode pulses and in simpler, continuous media.)

Such "one-way walls" can be used to direct rays in a ratchet manner, as suggested in Refs. [5,6] for charged particles, or even to concentrate them, as proposed and

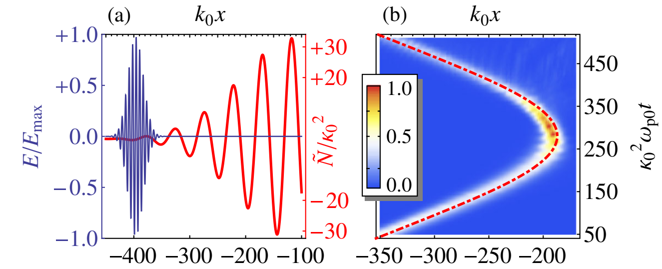

FIG. 1 (color online). Results of one-dimensional full-wave simulations of the EM pulse scattering in plasma by a density wave with stationary envelope, $\Omega = \omega_{p0}\kappa_0^2$, and $K = -0.12k_0$; here $\kappa_0 \doteq ck_0/\omega_{p0} \lll 1$, and k_0 is the initial wave number. Slow nonlinearities are ignored, and oscillations at the constant carrier frequency ω_{p0} are mapped out. The electric field envelope E is given in units of its maximum amplitude, $E_{\rm max}$; $\tilde{N}$ is given in units κ_0^2; t is given in units $\kappa_0^{-2}\omega_{p0}^{-1}$; x is given in units k_0^{-1}. (a) Initial setup ($t = 0$); shown are E (blue, narrow envelopes) and $\tilde{N}$ (red, wide envelope). (b) $|E(t,x)|^2$ and the ray trajectory found by numerical integration of Eqs. (11).

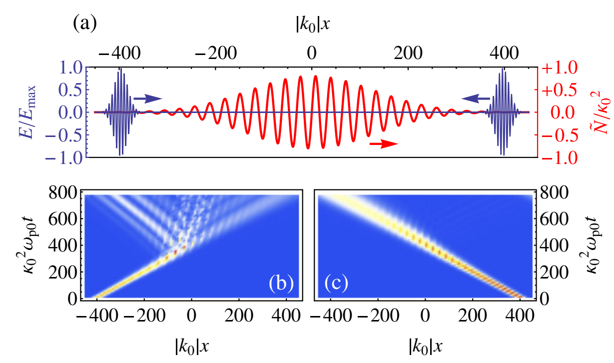

FIG. 2 (color online). Same as in Fig. 1 but for $\Omega = 0.03\omega_{p0}\kappa_0^2$ and $K = 0.2|k_0|$. Two cases are considered, $k_0 > 0$ and $k_0 < 0$, with identical $|k_0|$. (a) Initial setup ($t = 0$); the arrows denote the directions of propagation. (b) $|E(t,x)|$ for $k_0 > 0$. The pulse hits the GVR at about $k_0x \simeq -100$; then it is partially transmitted and partially reflected, much like it is described for particles in Ref. [31]. (c) $|E(t,x)|$ for $k_0 < 0$. As the signs of k_0 and K are different, the pulse never hits the GVR and is fully transmitted.

implemented in Ref. [7] for atoms. For example, suppose a barrier shown in Fig. 3 and the concentrator scheme as in the inset. With the aid of an additional mirror, this barrier can confine photons on its left side. That applies, of course, only for photons with energies below a certain threshold, whereas those transmitted from the right have energies above that threshold and thus can escape. (This is because, in the adiabatic domain, the motion is reversible, so any photon that once was at the top of the barrier can return there in the future.) However, like in the case of charged particles [5,6] and atoms [7], the barrier can serve as a one-way wall if dissipation is added. Suppose that a transmitted photon, with some frequency ω_1, undergoes Raman decay into some natural oscillations with frequency ω_R and another photon with frequency $\omega_2 < \omega_1$. The former will dissipate, but, assuming $\omega_R \ll \omega_1$, this energy loss can be negligible. The second photon, however, now has a smaller (ideally, zero) probability to escape due to its lower energy and thus is stuck between the one-way wall and the mirror until it decays through the Raman cascade. Hence the photon density in that region will be higher than outside.

Classical particles as PWs.—Although the above discussion appeals to understanding waves as particles, the particle dynamics itself can be viewed merely as a special case of the ponderomotive-refraction theory. To show this, we approach it quantum-mechanically as follows. Consider the Lagrangian density of a nonrelativistic quantum particle, $\mathfrak{L} = (i\hbar/2)(\psi^*\partial_t\psi - \psi\partial_t\psi^*) - \psi^* H(t, \mathbf{x}, -i\hbar\nabla)\psi$, where H is a Hamiltonian, and $\psi(t, \mathbf{x})$ is the wave function in the spatial representation [13]. (Relativistic particles can be described similarly, but they generally must be treated as vector waves.) Let us represent this function through its (real) phase $S/\hbar$ and amplitude $\sqrt{\mathcal{I}}$. Specifically, we write

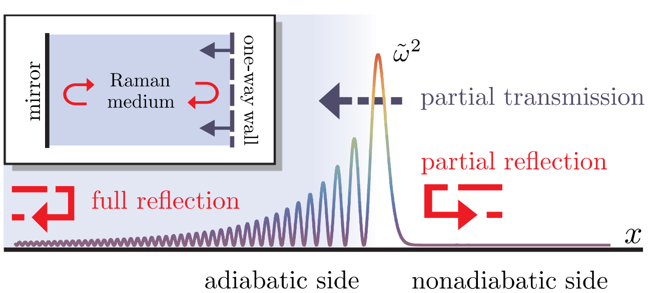

FIG. 3 (color online). Main figure: schematic of a one-dimensional asymmetric ponderomotive wall for PW rays. Rainbow-colored and oscillating is $\tilde{\omega}^2$, which determines w [Eq. (7)]. On the left, adiabatic side (shaded), the ponderomotive force reflects all rays below a certain frequency before they even reach the nonadiabatic region. On the right side, rays enter the nonadiabatic region first; then, depending on the initial phase, some of them are reflected, but others are transmitted, much like particles in asymmetric barriers discussed in Ref. [6]. Inset: possible scheme of a light concentrator (see the main text).

$\psi = e^{iS(t,\mathbf{x})/\hbar}\sqrt{\mathcal{I}(t,\mathbf{x})}$, where $\mathcal{I}$ is now chosen to have units of number density rather than of action density, as before. Assuming that $\psi(t, \mathbf{x})$ is quasiclassical, we have $H(t, \mathbf{x}, -i\hbar\nabla)\psi \simeq H(t, \mathbf{x}, \nabla S)\psi$, so $\mathfrak{L} = -\mathcal{I}[\partial_t S + H(t, \mathbf{x}, \nabla S)]$. This expression does not contain $\hbar$ and has the same form as Eq. (1). [Notably, it also reproduces the well-known Lagrangian density of cold classical fluid [33] as a special case.] Therefore, if the particle Hamiltonian consists of slow and rapidly oscillating parts, $H = \bar{H} + \tilde{H}$, we can introduce a ponderomotive Lagrangian $\bar{\mathfrak{L}} = -\bar{\mathcal{I}}[\partial_t \bar{S} + \mathcal{H}(t, \mathbf{x}, \nabla\bar{S})]$ that describes the particle dynamics averaged over the oscillations of $\tilde{H}$. Here,

$$\mathcal{H}(t, \mathbf{x}, \mathbf{P}) = \bar{H} + \frac{\mathbf{K}}{4}\cdot\frac{\partial}{\partial\mathbf{P}}\left(\frac{|\tilde{H}|^2}{\Omega - \mathbf{K}\cdot\mathbf{V}}\right), \tag{15}$$

$\mathbf{V} \equiv \bar{\mathbf{v}}_g$ is the OC velocity, and the remaining notation is self-explanatory. The model of *point* particles corresponds to $\mathcal{I}(t, \mathbf{x}) = \delta(\mathbf{x} - \mathbf{X}(t))|\det g_{ij}|^{-1/2}$. The corresponding *total* Lagrangian $\mathcal{L}$ of the OC is obtained by integrating $\bar{\mathfrak{L}}$ over the volume. That yields $\mathcal{L} = \mathbf{P}\cdot\dot{\mathbf{X}} - \mathcal{H}(t, \mathbf{X}, \mathbf{P})$ with $\mathbf{P} \doteq \nabla\bar{S}$, so $\mathcal{H}$ serves as the Hamiltonian for the canonical pair $(\mathbf{X}, \mathbf{P})$.

In particular, for an elementary particle with mass m and charge e, one has $H(t, \mathbf{x}, \mathbf{P}) = \{m^2c^4 + [\mathbf{P} - e\mathbf{A}(t, \mathbf{x})/c]^2\}^{1/2} + e\varphi(t, \mathbf{x})$. Here $\mathbf{A} = \bar{\mathbf{A}} + \tilde{\mathbf{A}}$ and $\varphi = \bar{\varphi} + \tilde{\varphi}$ are the vector and scalar potentials; $\bar{\mathbf{A}}$ and $\bar{\varphi}$ describe quasistatic fields, if any; $\tilde{\mathbf{A}} = \mathrm{Re}[\tilde{\mathbf{A}}_c e^{i\Theta(t,\mathbf{x})}]$ and $\tilde{\varphi} = \mathrm{Re}[\tilde{\varphi}_c e^{i\Theta(t,\mathbf{x})}]$ describe a MW, which is now comprised of oscillations of the electric field with complex amplitude $\tilde{\mathbf{E}}_c = i\Omega\tilde{\mathbf{A}}_c/c - i\mathbf{K}\tilde{\varphi}_c$ and magnetic field with complex amplitude $\tilde{\mathbf{B}}_c = i\mathbf{K}\times\tilde{\mathbf{A}}_c$. This leads to

$$\bar{H} = H_0 + e^2|\tilde{\mathbf{A}}_c|^2/(4mc^2\bar{\gamma}^3), \tag{16}$$

$$\tilde{H} = -e(\mathbf{P} - e\bar{\mathbf{A}}/c)\cdot\tilde{\mathbf{A}}/(mc\bar{\gamma}) + e\tilde{\varphi}, \tag{17}$$

where $H_0 \doteq mc^2\bar{\gamma} + e\bar{\varphi}$, $\bar{\gamma} \doteq [1 + (\mathbf{P} - e\bar{\mathbf{A}}/c)^2/(mc)^2]^{1/2}$, and quiver terms scaling as second and higher powers of $\tilde{\mathbf{A}}$ and $\tilde{\varphi}$ are neglected. One can check then that Eq. (15) reproduces the OC Hamiltonians derived earlier, and $\Phi \doteq \mathcal{H} - H_0$ is the well-known ponderomotive potential [1].

Summary.—Nonlinear interactions of waves via instantaneous XPM can be cast in the same way as ponderomotive wave-particle interactions in a high-frequency EM field. The ponderomotive effect arises when rays of a PW scatter off medium perturbations produced by a MW, much like charged particles scatter off a quasiperiodic EM field. The striking parallels to the point-particle dynamics lead to new methods of wave manipulation, including asymmetric barriers for light.

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