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LETTERS

FIELD REVERSAL BY ROTATING WAVES

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ABSTRACT. Rotating waves have been suggested as a means of generating currents that, in turn, can produce a compact-torus configuration. The practicality of this scheme is assessed with regard to stability and power requirements. The method of generating current is compared to other methods of non-Ohmic current drive.

1. INTRODUCTION

The 'Rotamak' [1] is a conceptual device that employs rotating magnetic fields to produce a compact-torus configuration in a mirror device. The fields rotate in the plane perpendicular to the mirror axis in the so-called toroidal or θ-direction. The expectation is that the fields cause electrons to rotate synchronously to drive a toroidal current. The toroidal current, in turn, creates an axial magnetic field which can cancel the original magnetic field, leading to field reversal and the compact-torus configuration. Recently, an experiment [2] recorded very substantial production of current in this manner and there were indications that the desired compact-torus configuration had been achieved.

The idea of producing electric current by rotating waves originated with Blevin and Thonemann [3] and has been expanded upon in several other papers [4–7]. The fundamental principle guiding all relevant theoretical and experimental work is that, in the presence of a rotating field, electrons may be tied to the magnetic field lines when ions are not, so that the field causes a synchronous rotation of electrons rather than ions. It is supposed that this will occur provided $\omega_{ci} \ll \omega \ll \omega_{ce}$, where ω is the frequency of the rotating field and ω_{ci} and ω_{ce} are, respectively, the ion and electron gyro-frequencies in this field. To be specific, it is commonly assumed in the above-referenced literature that, in the presence of an imposed magnetic field,

$$\vec{B} = (B\cos\omega t,\ B\sin\omega t,\ B_0) \tag{1a}$$

and the consistent electric field

$$\vec{E} = (0,\ 0,\ \omega B x\cos\omega t + \omega B y\sin\omega t) \tag{1b}$$

where the vector components are given, respectively, in the rectangular co-ordinates x, y and z (z is the axial co-ordinate), electrons will follow the rotating field (if collisions are infrequent enough) so that a toroidal current

$$J_\theta = -ner\omega \tag{2}$$

will be generated, where n is the electron density, e the electron charge, and cylindrical co-ordinates ($r^2 = x^2 + y^2$, $\theta = \cos^{-1} x/r$) are useful to describe the current. The evaluation of this claim, expressed by Eq.(2) and fundamental to the Rotamak concept, is the main aim of this paper.

Our approach is to follow the exact particle motion in the given fields. It resembles other analyses [8–12] of the rotating-field problem which employ slightly different boundary conditions and concentrate on the localization of particles rather than, as we shall do, on the current generation and power dissipation. In particular, we wish to define the role played by the axial magnetic field in inhibiting particle motion transverse to it and to place this mechanism of current generation in the general context of non-Ohmic current generation methods, for example, in terms of its efficiency.

Solving exactly the equations of motion in the applied fields given by Eqs (1) and (2) will allow us to describe accurately the plasma, including current generation and power dissipation. This description will be exact in the limit of vanishing plasma density, where the additional fields consistent with the plasma motion may be neglected. An actual Rotamak is, of course, far more complicated because of the fields associated with the motion of dense plasma; so the relevance of our findings must be qualified.

Having presented this caveat, we observe that the principle behind the Rotamak – that Eqs (1) and (2) alone imply Eq.(3) – is incorporated in the present study. It is, therefore, if not conclusive, then at least more than likely that some important qualitative features of the Rotamak, including the law governing power dissipation, may be verified by thoroughly examining the limit in question.

The paper is organized as follows: In Section 2, we shall derive the general solution to particle motion in the fields described by Eqs (1). In Section 3 the important parameter regimes are identified and appropriate limiting cases of the general solution are considered to find useful expressions for the generated current. In Section 4, a resistivity law for this mechanism of current generation is derived. In Section 5, we state our conclusions and evaluate our findings with reference to the Rotamak concept.

2. GENERAL SOLUTION

We adopt the following normalizations in solving for the single-particle motion: $\tau = \omega t$, $X = x/a$, $Y = y/a$, $Z = z/a$, $w = qB/m\omega$ and $p = qB_0/m\omega$, where a is the mirror radius and q/m is the charged-particle charge-to-mass ratio. The Lorentz force law then becomes

$$\ddot{X} = -w\dot{Z} \sin\tau + p\dot{Y} \tag{3a}$$

$$\ddot{Y} = w\dot{Z} \cos\tau - p\dot{X} \tag{3b}$$

$$\dot{Z} = w(X \sin\tau - Y \cos\tau) + c_z \tag{3c}$$

where dots indicate differentiation with respect to τ and the force law has been integrated once in the z-direction, defining c_z as a constant of motion.

It turns out that the set of Eqs (3) is tractable in the rotating co-ordinates u, v, where

$$u = X \cos\tau + Y \sin\tau \tag{4a}$$

$$v = Y \cos\tau - X \sin\tau \tag{4b}$$

In these co-ordinates, the equations of motion reduce to

$$\ddot{u} - (p+1)u = (2+p)\dot{v} \tag{5a}$$

$$\ddot{v} + (w^2-p-1)v = -(2+p)\dot{u} + wc_z \tag{5b}$$

i.e. a set of linear differential equations with constant coefficients. Accordingly, a general solution may be written with

$$u = A_1 \cos\omega_1\tau + A_2 \cos\omega_2\tau$$
$$+ A_3 \sin\omega_1\tau + A_4 \sin\omega_2\tau \tag{6a}$$

$$v = D_1 \sin\omega_1\tau + D_2 \sin\omega_2\tau$$
$$+ D_3 \cos\omega_1\tau + D_4 \cos\omega_2\tau \tag{6b}$$

where the characteristic frequencies are found from the quadratic

$$\omega^2_{1,2} = \frac{(p+1)^2+w^2+1 \pm \sqrt{[(p+1)^2+w^2+1]^2+4(p+1)(w^2-p-}}{2} \tag{7}$$

The first thing to note from Eq.(7) is that particle motion is governed by a stability criterion

$$(p+1)(w^2-p-1)<0 \tag{8}$$

which is both necessary and (as is easy to show) sufficient for stability. When inequality (8) is not satisfied, particle orbits become unbounded in time. For ions, we expect w^2, $p^2 \ll 1$, so that motion is always bounded. For electrons, however, we expect w^2, $p^2 \gg 1$, so that we may say that, for stability, we need either of the two sufficient criteria:

$$w^2 < |p+1| \tag{9a}$$

or

$$p < -1 \tag{9b}$$

The general, detailed solution to Eqs (5) is hardly needed; we shall, however, provide the solutions to the initial conditions $Y = Z = \dot{X} = \dot{Z} = 0$, $X = X_0$ and $\dot{Y} = V_0$, which are sufficient for our purposes. The solution is given by Eqs (6) with $A_3 = A_4 = D_3 = D_4 = 0$ and

$$A_1 = \frac{(\omega_2^2 - 1)X_0 + (2+p)V_0}{\omega_2^2 - \omega_1^2} \quad (10a)$$

$$A_2 = \frac{(\omega_1^2 - 1)X_0 + (2+p)V_0}{\omega_1^2 - \omega_2^2} \quad (10b)$$

$$D_1 = -\frac{(\omega_1^2 + p + 1)}{\omega_1(2+p)} A_1 \quad (10c)$$

$$D_2 = -\frac{(\omega_2^2 + p + 1)}{\omega_2(2+p)} A_2 \quad (10d)$$

The solution is linear in the initial conditions so, of course, even more complicated but less revealing dynamics could be described. In the next section, we shall take the relevant limits to extract useful information from this solution.

3. LIMITING CASES

To concentrate on the relevant parameter regime, we note that, for practical densities and temperatures in fusion applications, we can expect, if the current generated is, in fact, given by Eq.(2), that the synchronous electron velocity, $v_\theta = a\omega$, should be much smaller than the electron thermal velocity, v_{Te}. Note also that, by assumption, ions will be confined radially by the steady axial magnetic field only. Thus, we can also expect that the ion Larmor radius in the steady field should be less than the radius of the mirror device, a. It follows then that $a_e/a \ll 1$, where a_e is the electron Larmor radius in the steady field. From the two inequalities given above regarding electrons, we can deduce that

$$|p| \equiv \left(\frac{a}{a_e}\right)\left(\frac{v_{Te}}{a\omega}\right) >> 1 \quad (11)$$

Even if field reversal is achieved, the above inequality should be true everywhere except in a very narrow region near the point of reversal.

In view of inequality (11), let us first consider the limit of weak rotating fields, i.e. $|p| \gg w^2$. Note that orbits are always stable in this regime since condition (9a) is satisfied. In this limit, we find after some straightforward but tedious algebra that

$$R^2 \equiv X^2 + Y^2 = u^2 + v^2 \simeq X_0^2 \quad (12)$$

and the normalized toroidal velocity is found to be

$$V_\theta \equiv R\dot{\theta} = \frac{X\dot{Y} - Y\dot{X}}{R} \simeq \frac{X_0 w^2}{p} \sin^2\tau \quad (13)$$

Note that the time-averaged V_θ is only $X_0 w^2/2p$, which implies that $\dot{\theta} \ll 1$, since $w^2/p \ll 1$ by assumption. This means that the electrons, while moving in the direction of field rotation, do considerably lag behind when the axial field is strong. This result is satisfying because of the intuitive expectation that a strong axial magnetic field should inhibit particle motion transverse to it. Note also that the electron current is considerably less in this limit than the expectations in the literature.

A more generous electron current is, however, obtained in the opposite limit, $w^2 \gg |p| \gg 1$, with stability achieved now only when inequality (9b) is satisfied. In this limit, to leading order, we find

$$u \simeq X_0 \cos\Omega\tau \quad (14a)$$

$$v \simeq \frac{V_0 - X_0}{w} \sin w\tau - \frac{\Omega^3}{w^2} X_0 \sin\Omega\tau \quad (14b)$$

where we employed the notational convenience $\Omega^2 = -p$. Thus, we can calculate, also to leading order in $1/w$,

$$R \simeq X_0 |\cos\Omega\tau| \quad (15a)$$

$$V_\theta \simeq X_0 |\cos\Omega\tau| + (V_0 - X_0)\cos w\tau \frac{\cos\Omega\tau}{|\cos\Omega\tau|} \quad (15b)$$

where the second term on the right-hand side of Eq.(15b) will vanish upon time-averaging. It follows then that in this limit, on average, $\dot{\theta} = 1$, which means that the electrons do, indeed, synchronously follow the rotating magnetic field, and the current is, indeed, correctly given by Eq.(2). This is the current-drive scheme envisaged for the Rotamak concept; in the next two sections we shall, however, raise some questions regarding its practicality.

4. RESISTIVITY LAW

Non-Ohmic current-drive schemes have been found [12] to obey the resistivity law where the power dissipated is proportional to the current rather than to the current squared, as is the case in Ohmic current drive. The proportionality constant then serves as a measure of efficiency in comparing various schemes of driving current.

To find the power dissipated in driving current by means of the present scheme, we must first find the extent of coherent oscillation in energy between the electromagnetic fields and the electrons. The randomization of this energy through collisions results in electron heating. The power dissipated is then given as the product of the oscillating energy and the collision frequency.

From Eqs (3) and (4), we can write

$$\frac{dW}{d\tau} \equiv \frac{d}{d\tau}\,\frac{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2}{2} = -w^2 uv \qquad (16)$$

where W represents the normalized particle kinetic energy. In the important limit $w^2 \gg |p| \gg 1$, where the current generation is effective, we can use Eqs (14) and (16) to find that the oscillating part of the kinetic energy is given, to highest order in $1/w$, by

$$\Delta W = |X_0 V_0| + |pX_0{}^2| \qquad (17)$$

In the event of collisions, this sloshing energy becomes randomized every collision time so that the dimensional power dissipated may be written as

$$P \simeq \nu_0 nm(a\omega)^2 \Delta W \qquad (18)$$

where ν_0 is the collision frequency of thermal electrons. (In view of the approximate nature of the consideration here, we shall identify ν_0 with the electron-ion collision frequency. This means that we shall somewhat underestimate the power dissipation since electron-electron collisions are actually effective in the randomizing process, too. For our purposes here, this degree of accuracy is sufficient.) Using now Eqs (2), (17) and (18), and noting that we can expect $X_0 \sim 1$, and $V_0 \sim v_{Te}/a\omega$, we find the important dimensionless quantity

$$\frac{J}{P_d} \simeq \frac{1}{1 + a/a_e} \ll 1 \qquad (19)$$

where the normalized current and power dissipated are given, respectively, by

$$J = J_\theta / env_{Te} \qquad (20a)$$

$$P_d = P/\nu_0 nmv_{Te}^2 \qquad (20b)$$

The inequality in Eq.(19) was written on the basis of the expectations discussed in Section 3.

Note from Eq.(19) that the power dissipated is indeed proportional to the current generated, not to the square of the current generated. Hence, discussions [2] of Ohmic dissipation in connection with this scheme of current generation are misleading.

It is now possible to compare the efficiency of this method of current-drive with other non-Ohmic methods. There are several other mechanisms that can achieve J/P_d in the range of 10 to 50. Thus, from Eq.(19), we see that the present scheme is by orders of magnitude less efficient than other schemes. This comparison is not, however, entirely fair because the present scheme generates current transverse to an axial magnetic field and the other schemes generate only parallel currents. The only measure of importance is, of course, whether sufficient current can be generated with tolerable power dissipation for a given application, which here is to generate the reversed-field configuration. For fusion reactor application, this measure would be P/P_f, the ratio of wave power dissipated to fusion power generated. The calculation of this quantity, which was identified [13] in tokamak reactor applications in connection with other forms of current-drive, is beyond the scope of this paper. Nonetheless, we should think, on the basis of the relatively low values of J/P_d given by Eq.(19), that the level of power dissipation is likely to be a problem for 'Rotamak'-type reactors.

5. CONCLUSIONS

The salient findings from the single-particle analysis presented here include the verification of our intuition that an axial magnetic field does indeed inhibit motion transverse to it. Yet regimes where current may be generated through synchronous and stable electron motion were also identified. Although the identification of these regimes is consistent with the feasibility of the Rotamak scheme, the apparently large power dissipation associated with the current generation calls the economic practicality of such schemes in question.

In addition to the problem of power dissipation in attaining the reversed-field configuration, it should not be overlooked that the stability of particle motion is threatened just when such a configuration is attained. The stability condition (9b) implies a sense to the generated axial magnetic field relative to the background axial magnetic field. For electron motion, the case under consideration, these fields are in opposite directions. Thus, a decrease in the axial field strength is consistent with stable orbits. However, when the decrease is close to reversing the field direction, orbits are already de-stabilized.

The above reasoning may not apply to practical situations where inhomogeneous axial magnetic fields occur, representing more complexity than is allowed for in the problem that has been solved. What may be concluded, however, is that the picture of 'Ohmic' current generation by tying electrons to rotating field lines is an oversimplification, and that both orbit stability and power dissipation may be more troublesome to the Rotamak scheme than was thought previously.

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