Drift-Energy Replacement Effect in Multi-ion Magnetized Plasma

M. E. Mlodik, E. J. Kolmes, and N. J. Fisch
Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544, USA

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Fast rotation can improve the stability and confinement of fusion plasmas. However, to maintain a rapidly rotating fusion plasma in steady state, significant energy must be invested in spinning up each incoming fuel ion. We show here that, under the right circumstances, collisional cross-field radial fueling can directly transfer drift energy between outgoing and incoming ions without the need for external power recirculation, thereby reducing the energy costs of maintaining the rotation.

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Introduction—Fast rotation can be desirable in plasma traps for nuclear fusion. Rotation often suppresses instabilities [1–8]. In centrifugal mirror fusion, rotation provides axial confinement [9–16]. For significant axial confinement, the rotation typically must be supersonic, often with a Mach number much greater than 1.

However, high Mach numbers present a very serious problem for centrifugal fusion devices. In steady-state operation, new fuel ions replenish the ions that are burned through fusion. For plasma spinning at a high Mach number, the energetic "spin-up cost" for each new ion can be large, possibly a large fraction of the energy derived from fusion. The ratio of spin-up cost to fusion energy production depends on plasma temperature, Mach number, and fusion energy output. Hence, it is sensitive to the choice of fuel in the fusing plasma. Aneutronic fuels such as p-B11, while being attractive for technological and regulatory reasons, require high-temperature ($\gtrsim 100$ keV) plasma to access sufficient fusion reactivity, exacerbating an already thin power balance margin [17–22]. Therefore, understanding the properties of the rotation energy is crucial to determine the viability of sonic or supersonic rotation in a plasma fusion reactor if aneutronic fuel is used. However, the same considerations apply to any steady-state, supersonically rotating fusion plasma at sufficiently high Mach number.

Of course, fast plasma rotation also means that the fusion byproducts are then born with substantial rotation energy. When the fusion by-products are extracted, their rotation energy might then be captured and converted into electricity, which could offset the energy cost of spinning up the fuel. However, converting and reinjecting energy in this way involves substantial inefficiencies.

However, remarkably, we find that, under certain circumstances, fuel ions may be collisionally exchanged for the fusion ash (fusion by-products), while the fuel ion rotation energy is retained in the plasma. This surprising result flows from the constraint that collisions in magnetized plasma do not move charge across field lines. In fusion reactions, were the ash to include neutrons, rotation energy must be lost. However, for aneutronic fusion reactions, where both reactants and by-products are charged and magnetized, the reactants can pick up the by-product rotation energy. This very fortuitous "drift-energy replacement effect" is illustrated in Fig. 1, where ions in the lower slab have drift velocities larger than those in the upper slab. If a helium ion from the upper slab is exchanged with two deuterium ions from the lower slab, which conserves charge, then the helium ion must gain in drift energy, but that energy can be provided exactly by the drift energy of the exchanged deuterium ions.

Stating the cost issue—Before uncovering the conditions under which and the extent to which the drift-energy replacement effect occurs, let us state precisely the cost issue. The energy gained per fusion event is $\Delta\epsilon = \alpha(\epsilon_{\text{fus}} + \epsilon_{\text{kin}}) - \beta\epsilon_{\text{kin}}$, where ϵ_{fus} is the fusion energy yield, ϵ_{kin} is total thermal energy residing in particles participating in a fusion event, $\alpha \le 1$ is the fusion energy conversion efficiency, and $\beta \ge 1$ is the energy cost of fueling a particle and heating it up to the temperature of

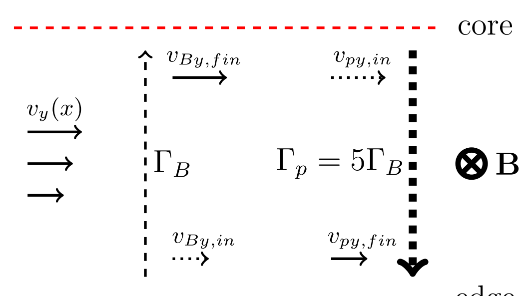

FIG. 1. Cross-field boron fueling while replacing protons. Here Γ_B and Γ_p are particle fluxes of boron and protons, respectively, while v_{sy} is y-directed component of flow velocity of species s with the last part of the index signifying location. Combination of pairs of species participating in a given exchange can be arranged to produce desired outcome.

the fusing plasma. For viable fusion, $\Delta\epsilon \geq 0$, or

$$\frac{\beta}{\alpha} - 1 \leq \frac{\epsilon_{\mathrm{fus}}}{\epsilon_{\mathrm{kin}}}. \tag{1}$$

Now suppose that plasma in the box is drifting. Then the drift energy must be taken into account. As it turns out, how to do that depends on how the plasma is fueled.

Suppose the plasma is fueled via the injection of (stationary) neutral pellets. Upon ionization, a neutral picks up $m\mathbf{v}_d^2/2$ drift energy and the same energy in Larmor gyration (perpendicular heating), based on instantaneous momentum conservation. Both drift energy and heating are at the expense of the electric field energy, which is lowered through the induced charge separation.

For Mach number Ma of the plasma flow, the drift kinetic energy is $\epsilon_{\mathrm{dr}} = \mathrm{Ma}^2\epsilon_{\mathrm{kin}}$, so that

$$\Delta\epsilon = \alpha(\epsilon_{\mathrm{fus}} + \epsilon_{\mathrm{kin}} + \epsilon_{\mathrm{dr}}) - \beta(\epsilon_{\mathrm{kin}} + \epsilon_{\mathrm{dr}}), \tag{2}$$

producing the more stringent requirement on α and β

$$\frac{\beta}{\alpha} - 1 \leq \frac{\epsilon_{\mathrm{fus}}}{\epsilon_{\mathrm{kin}}} \frac{1}{1 + \mathrm{Ma}^2}. \tag{3}$$

The drift energy must then be externally recirculated (removed from fusion products, converted to electricity, and then used to sustain the electric field).

Similarly, if the pellet is instead injected at high velocity, the conclusions are largely the same, due to the launch energy cost.

It is thus both surprising and fortuitous that collisional cross-field fueling can facilitate a direct transfer of the rotational kinetic energy from outgoing to incoming ions. If the drift-energy exchange is one to one, the energy balance that leads to Eq. (1) is unaffected.

The drift-energy replacement effect—Consider a plasma slab immersed in a uniform magnetic field $\mathbf{B} = B\hat{z}$, in Cartesian (x, y, z) coordinates with unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$. Suppose the plasma is homogeneous in the $\hat{y}$ direction, with plasma parameters such as the density n varying only in the $\hat{x}$ direction. Note that drift-energy partition between species is affected by momentum-changing collisions. Therefore, ions change their drift energy more via interaction with other ions, not electrons, because of the mass disparity between species. Moreover, to the extent that energy slowing down of ions on electrons is important, that slowing down occurs in the rotating frame, so only the thermal energy is lost to electrons, leaving the rotation energy intact and important to recover. Consider two particles interacting at point $\mathbf{r}$, where

$$\mathbf{r} = \mathbf{R}_j + \boldsymbol{\rho}_j. \tag{4}$$

Here $\mathbf{R}_j$ is position of the gyrocenter of particle j, $\boldsymbol{\rho}_j$ is the vector between gyrocenter and actual particle position, and we have taken the Larmor radii to be large compared with any distance between the interacting particles. The velocity of either particle can be expressed as

$$\mathbf{v}_j = \mathbf{v}_{dj}(\mathbf{R}_j) + \mathbf{w}_j(\boldsymbol{\rho}_j) + v_{\parallel j}\hat{\mathbf{b}}. \tag{5}$$

Here $\mathbf{v}_{dj}$ is the drift velocity, $\mathbf{w}_j$ is the velocity associated with gyromotion, $v_{\parallel j}\hat{z}$ is the motion parallel to the magnetic field, and $\hat{\mathbf{b}} \doteq \mathbf{B}/B = \hat{z}$. The gyromotion velocity can be expressed as

$$\mathbf{w}_j = \Omega_j \boldsymbol{\rho}_j \times \hat{\mathbf{b}}. \tag{6}$$

Here $\Omega_j = q_j B/m_j$ is the gyrofrequency of particle j, where q_j and m_j are the charge and the mass of the particle, respectively. Conversely,

$$\boldsymbol{\rho}_j = -\frac{\mathbf{w}_j \times \hat{\mathbf{b}}}{\Omega_j}. \tag{7}$$

Note that collisions between particles are instantaneous compared with the Larmor gyration time of each particle. Therefore, the electromagnetic part of the momentum is not changed during the collision itself, and the total kinetic momentum of two particles is conserved. Note that a particle assumes drift velocity and random velocity commensurate with its new position after collision if macroscopic changes in plasma are slow compared to inverse gyrofrequency, e.g., $|\partial n/\partial t|/n, |\partial T/\partial t|/T \ll \Omega$.

Let $\delta\mathbf{p}$ be the kinetic momentum transferred from the second to the first particle during the collision and $\delta\mathbf{p}_j$ be the change of kinetic momentum of particle j (i.e., $\delta\mathbf{p}_1 = \delta\mathbf{p}$, $\delta\mathbf{p}_2 = -\delta\mathbf{p}$). Then

$$\delta\mathbf{v}_{dj} + \delta\mathbf{w}_j = \frac{\delta\mathbf{p}_j}{m_j}; \tag{8}$$

$$\delta\mathbf{R}_j + \delta\boldsymbol{\rho}_j = 0. \tag{9}$$

Using Eq. (6), we have $\delta\mathbf{w}_j = \Omega_j \delta\boldsymbol{\rho}_j \times \hat{\mathbf{b}}$, so that

$$\delta\mathbf{v}_{dj} - \Omega_j \delta\mathbf{R}_j \times \hat{\mathbf{b}} = \frac{\delta\mathbf{p}_j}{m_j}. \tag{10}$$

Let us write $\delta\mathbf{v}_{dj} = \omega_j \cdot \delta\mathbf{R}_j$. We will assume constant ω_j, which is exact if the shear is constant and it holds to leading order in the ion gyroradius compared to the shear length scale. (If the flow is incompressible, ω_j is related to the vorticity.) In the geometry here, $\omega_j = |\omega_j|\hat{y}\,\hat{x}$. The equations of motion in the perpendicular (x-y) plane can then be solved separately from those in z direction, with the $\hat{x}$ and $\hat{y}$ components of $\delta\mathbf{R}_j$ satisfying

$$\begin{pmatrix} 0 & -\Omega_j \\ \Omega_j + |\omega_j| & 0 \end{pmatrix} \delta \mathbf{R}_j \quad = \frac{\delta \mathbf{p}_j}{m_j}. \tag{11}$$

The solution for $\delta \mathbf{R}_j$ is

$$\delta \mathbf{R}_j = \begin{pmatrix} 0 & \frac{1}{\Omega_j + |\omega_j|} \\ -\frac{1}{\Omega_j} & 0 \end{pmatrix} \frac{\delta \mathbf{p}_j}{m_j}. \tag{12}$$

The structure of this correlation reflects the essential physics underlying the drift replacement effect, in which changes in energy are coupled to changes in position, much in the same way that the α channeling effect coupled diffusion in energy to diffusion in space [23]. Using this, we can show that the change in drift velocities is

$$\delta \mathbf{v}_{dj} = \hat{y} \frac{|\omega_j|}{\Omega_j + |\omega_j|} \frac{\delta p_{jy}}{m_j}. \tag{13}$$

The change of drift velocity energy due to a collision can be written as

$$\delta W_{\mathrm{drift}} = \sum_{j=1,2} \mathbf{v}_{dj} \cdot m_j \delta \mathbf{v}_{dj} + \frac{1}{2} \sum_{j=1,2} m_j (\delta \mathbf{v}_{dj})^2, \tag{14}$$

or, alternatively,

$$\delta W_{\mathrm{drift}} = \delta p_y \left[v_{d1y} \frac{|\omega_1|}{\Omega_1 + |\omega_1|} - v_{d2y} \frac{|\omega_2|}{\Omega_2 + |\omega_2|} \right] + \frac{1}{2} \left[m_1 (\delta \mathbf{v}_{d1})^2 + m_2 (\delta \mathbf{v}_{d2})^2 \right]. \tag{15}$$

In the case of small shear (i.e., $|\omega_1| \ll \Omega_1$, $|\omega_2| \ll \Omega_2$), the change in drift energy is

$$\delta W_{\mathrm{drift}} = \delta p_y \left(\frac{v_{d1y} |\omega_1|}{\Omega_1} - \frac{v_{d2y} |\omega_2|}{\Omega_2} \right). \tag{16}$$

The change of drift energy associated with particle transport of particles of species 1 represents the correlation between δR_{1x} (jump in gyrocenter position of particle 1 due to collision) and $\delta W_{\mathrm{drift}}$. Using the expression for δR_{1x} from Eq. (12), and taking the drift velocities $\mathbf{v}_{dj}(\mathbf{r})$ to be the same for both species (as is the case for the $\mathbf{E} \times \mathbf{B}$ drift), this correlation can be put as

$$\frac{\delta W_{\mathrm{drift}}}{\delta R_{1x}} = m_1 v_{dy} |\omega| \left(1 - \frac{\Omega_1}{\Omega_2} \right) \left[1 + \mathcal{O}\left(\frac{|\omega|}{\Omega} \right) \right]. \tag{17}$$

Equations (16) and (17) represent the drift-energy replacement effect, the major result of this Letter. These equations demonstrate that collisions between species 1 and 2 drive in correlation both the cross-field transport and the transfer of drift energy between the two species. Equation (17) shows that, for small enough shear in drift velocity, regardless of the details of collision operator, the drift velocity profiles, and the extent of collisional transport, the change in total rotation energy is $(1 - \Omega_1/\Omega_2)$ times the rotation energy acquired by the particle of species 1. Further explanation and expanded derivation can be found in the End Matter part of the Letter. Moreover, we conducted a numerical simulation using the code MITNS [24], which shows drift-energy replacement effect. See Supplemental Material [25] for the details on simulation setup.

To understand Eq. (17), consider some limiting cases. If $|\omega| = 0$, which is equivalent to $\mathbf{v}_{dj} = \mathrm{const}$ (uniform drift), then $\delta W_{\mathrm{drift}} = 0$. In other words, if plasma moves as a whole without shear, there is no change in drift energy due to collisional transport. If $\Omega_1 = \Omega_2$ (both types of particles have the same gyrofrequency), then $\delta W_{\mathrm{drift}} = 0$. In other words, the drift energy gained by incoming ions is matched by the drift energy given up by the outgoing ions, and no interaction with other energy reservoirs, such as potential energy, is required.

This can substantially reconfigure the flow of energy as particles move in and out of the system. For example, take boron as species 1 and protons as species 2, as shown in Fig. 1. Then 5/11 of the boron rotational energy is provided by the protons, which are deconfined, leaving only 6/11 to be taken from the potential energy. Notably, it is also possible to extract drift energy. If $\Omega_1 > \Omega_2$, then $\delta W_{\mathrm{drift}}$ is negative, so outgoing ions have smaller gyrofrequency than incoming ions. Then outgoing ions not only give their rotational energy to the incoming ions, they also recharge the potential energy of the field.

Multiple ion drift energies—The discussion thus far considered only two species of interacting ions. For more than two species, high-efficiency energy transfers can be accomplished even without precise matching of the gyrofrequencies. For an arbitrary number of interacting ion species, the change in drift energy can be written as

$$\delta W_{\mathrm{drift}} = \sum_j m_j \mathbf{v}_{dj} \cdot \delta \mathbf{v}_{dj} \left[1 + \mathcal{O}\left(\frac{|\delta \mathbf{v}_{dj}|}{|\mathbf{v}_{dj}|} \right) \right] \tag{18}$$

$$= \sum_j v_{dj} |\omega_j| m_j \delta R_{jx} \left[1 + \mathcal{O}\left(\frac{|\omega_j|}{\Omega_j} \right) + \mathcal{O}\left(\frac{|\delta \mathbf{v}_{dj}|}{|\mathbf{v}_{dj}|} \right) \right]. \tag{19}$$

In the limit where all species have the same drift velocity profile, this means that (to leading order) $\delta W_{\mathrm{drift}}$ vanishes any time the in- and outward mass flux is equal. This is equivalent to matching gyrofrequencies in the two-species case because, in an interaction between two species, $Z_1 \delta R_{j1} + Z_2 \delta R_{j2} = 0$. Thus, for example, in p-B11 centrifugal fusion devices, utilizing $E \times B$ drifts, there would be complete drift-energy replacement for both protons and boron replacing α particle ash.

Drift-energy release in a fusion event—Interestingly, and closely related to the drift-energy replacement effect, depending on the physical mechanism of the drift and the species involved, drift energy can be released in a fusion event in magnetized plasma. Consider a force $\mathbf{F}_s = -\nabla\Phi_s$. The gyroradius vector of a particle s is

$$\boldsymbol{\rho}_s = -\frac{m_s \mathbf{w}_s \times \hat{\mathbf{b}}}{q_s B}. \tag{20}$$

The potential energy of particle s relative to the location of the fusion event is

$$W_s = \mathbf{F}_s \cdot \boldsymbol{\rho}_s. \tag{21}$$

Given that $\mathbf{w}_s = \mathbf{v}_s - \mathbf{v}_{ds}$,

$$W = \sum_s W_s = -\sum_s \frac{m_s(\mathbf{v}_s - \mathbf{v}_{ds}) \times \hat{\mathbf{b}} \cdot \mathbf{F}_s}{q_s B}. \tag{22}$$

Alternatively, if $\mathbf{p}_s = m_s \mathbf{v}_s$, then

$$W = \sum_s \frac{m_s \mathbf{v}_{ds} \times \hat{\mathbf{b}} \cdot \mathbf{F}_s}{q_s B} - \sum_s \frac{\mathbf{p}_s \times \hat{\mathbf{b}} \cdot \mathbf{F}_s}{q_s B}. \tag{23}$$

If $\mathbf{v}_{ds}$ is caused by total force $\mathbf{F}_{\mathrm{tot},s}$, then

$$W = -\sum_s \frac{\mathbf{F}_s \cdot \mathbf{F}_{\mathrm{tot},s}}{m_s \Omega_s^2} - \sum_s \frac{\mathbf{F}_s \times \mathbf{p}_s}{q_s B} \cdot \hat{\mathbf{b}}. \tag{24}$$

If the $\mathbf{E} \times \mathbf{B}$ drift is the only drift, then the second term does not change in a fusion event due to momentum conservation. The first term becomes $\sum_s m_s E^2/B^2$ and is constant (to the extent that the fusion event conserves total mass). Therefore, there is no change in electric potential in a fusion event in an $\mathbf{E} \times \mathbf{B}$ drifting plasma.

The situation is very different if the force is proportional to mass, for example, if $\mathbf{F}_s = m_s \mathbf{g}$, then

$$W = -\sum_s \frac{m_s^3}{q_s^2} \frac{g^2}{B^2} + \sum_s \frac{\mathbf{p}_s}{\Omega_s} \cdot (\mathbf{g} \times \hat{\mathbf{b}}). \tag{25}$$

Now, in general, the potential energy W changes in a fusion event. For example, in the p-B11 reaction, the change of the first term is $\Delta W_1 = +6.24(m_p^3/q_p^2)(g^2/B^2)$, while the change of the second term is

$$\Delta W_2 = \left[\mathbf{p}_p\left(\frac{1}{\Omega_\alpha} - \frac{1}{\Omega_p}\right) + \mathbf{p}_B\left(\frac{1}{\Omega_\alpha} - \frac{1}{\Omega_B}\right)\right] \cdot (\mathbf{g} \times \hat{\mathbf{b}}). \tag{26}$$

Alternatively,

$$\Delta W_2 = -\left(\frac{1}{2}\mathbf{p}_p - \frac{1}{22}\mathbf{p}_B\right) \cdot \frac{(\mathbf{g} \times \hat{\mathbf{b}})}{\Omega_p}. \tag{27}$$

Thus, there is a release of potential energy as a result of the fusion event, unless $\Delta W_1 + \Delta W_2 = 0$.

Of practical interest in fusion devices is the centrifugal force, rather than the gravitational force, which is also proportional to mass. In a rapidly rotating system where mass-dependent inertial drifts are important, the redistribution of the gyrocenters after a fusion event changes the total potential energy of the participating particles. The mechanism underlying this is much the same as the one behind the drift-energy replacement effect: interactions between particles at a given point can shift their gyrocenters up or down a potential gradient.

In the case of the p-B11 reaction undergoing fusion in a centrifugal mirror machine, the mass-weighted centrifugal force on the fusion reactants is greater than the mass-weighted force on the fusion by-products, so the rotational energy of the fusion reactants is greater than that of the fusion by-products. Thus, drift energy is released in the p-B11 fusion reaction. In fact, this energy release is greatest when the centrifugal fusion reactor is charged positive (the usual case), for then the centrifugal forces are in the same direction as the electric forces, thus enhancing the drift-energy difference between the reactants and the by-products. The energy release settles in part the account of drift energy to be replaced. It is supplementary in fact to the collisional drift-energy replacement effect. To the extent that the fusion by-products do not have quite the drift energy of the fusion reactants, then the collisional drift-energy replacement effect cannot replace the full reactant drift energy. However, the sum of the two effects does replace the full reactant drift energy.

Summary and discussion—The power cost of refueling ions can render uneconomical fusion devices requiring high plasma drift velocities. Although fusion products might be recovered with that drift energy, the recovery and reinjection of the energy (as new fuel ions need to be "spun up") can be a source of significant inefficiency.

This Letter identifies the drift-energy replacement effect, a new and fundamental transport effect that can obviate the need to recover the drift energy of fusion ash. Surprisingly, when ions of different species move via collisions across magnetic field lines in opposite directions, incoming ions can automatically take all or part of the drift energy directly out of the outgoing ions.

Near equilibrium, this process represents a particular and fortuitous case of multiple-ion-species redistributions [24,26–39]. For plasma near collisional equilibrium, irreversible heating associated with particle transport can take place, but is small if the transport is not fast. Since the ion rearrangement itself does not produce heating, it is reversible; with reversed boundary conditions or forces, the net result of ion exchanges can be reversed. The interaction between incoming and outgoing ions is not specific to collisions, so the calculations in this Letter are applicable also to wave-mediated transport, so long as the wave does not impart net momentum to ions and the momentum transfer can be regarded as instantaneous.

The energy transfer is possible at near-perfect efficiencies and is reversible, but only if the conditions outlined here are met. In cases where the transfer does not meet the drift energy needed, the difference between the drift energies of incoming and outgoing ions would be provided by the potential energy driving the drift (the electrostatic potential, in the case of the $\mathbf{E} \times \mathbf{B}$ drift).

This difference can also be negative if more drift energy is removed than replaced; in that case, power can be drawn from the circuit providing for the rotation. Also, note that the drift-energy replacement cannot be perfect in neutronic reactions, as the balance of incoming and outgoing particles depends only on those species that participate in the collisional transport; neutrons leave promptly, so any drift energy they carry would not be recoverable within the plasma.

The drift-energy replacement effect shows that, in fusion plasmas with large drift flows, the fueling mechanism can significantly impact the power flow, especially if advanced fuels are considered. Although we focus on nuclear fusion, the replacement effect uncovered here applies also to other technologies involving rapid rotation, such as rotating plasma mass filters [9,27,28,40–42]. While the plasma needs to span at least a few Larmor radii across the magnetic field for drift-energy replacement effect to be applicable, this condition is met by many plasmas. Indeed, in a deuterium-tritium fusion plasma with 10 keV ions and 3.5 MeV α particles in 10 T magnetic field, the ions have ion Larmor radii, $\rho_D = 0.14$ and $\rho_\alpha = 1.9$ cm, respectively. Thus, any device with plasma extent of tens of centimeters or more in the perpendicular direction would satisfy this condition. In the case of p-B11 plasma at 200 keV ion temperature in 10 T magnetic field, the proton Larmor radius is $\rho_p = 0.45$ cm. The drift-energy replacement effect does not require any particular restrictions on the ion Hall parameter. Moreover, while the slab case is considered in this Letter for simplicity, similar effects will occur in other magnetic field geometries. Also, note that even if the density of fusion products is small due to low burn-up fraction of fuel, the number of fuel ions with which by-product ions interact is sufficient to replace burned fuel in the core of the fusion device.

Apart from its inherent academic interest, the drift-energy replacement effect identified here has very practical interest; it represents the very rare case in which collisional effects, in particular, render economic nuclear fusion to be easier rather than harder. The cost of spinning up new fuel ions could very well have proved prohibitive for any steady-state fusion reactor based on the centrifugal confinement of fuels for aneutronic fusion, since, for the needed sonic or supersonic rotation, these very hot plasmas would feature very large invested rotation energy. At the same time, reactor designs based on aneutronic reactions tend to have very small margins of net energy produced compared to invested energy. Even for more conventional lower-temperature fuels, the spin-up cost could be significant. But if this spinning-up can be accomplished “for free,” with a direct transfer of rotation energy from the outgoing fusion products, then this cost will not be as formidable as might have been expected.

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End Matter

Expanded derivation of drift-energy replacement effect—Here, we expand upon the derivation of drift-energy replacement effect and show that this effect readily follows from fluid equations and fundamental conservation laws.

Momentum equation for species s is [43]

$$\frac{\partial(m_s n_s \mathbf{u}_s)}{\partial t} + \nabla\cdot(m_s n_s \mathbf{u}_s \mathbf{u}_s) = q_s n_s(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) - \nabla p_s - \nabla\cdot\pi_s + \sum_{s'} \mathbf{R}_{ss'} + \mathbf{F}_s. \tag{A1}$$

Here index s refers to species s, m_s is the mass, q_s is the charge, n_s is density, $\mathbf{u}_s$ is flow velocity, p_s is pressure, π_s is the traceless component of pressure tensor, $\mathbf{R}_{ss'} = \mathbf{R}^u_{ss'} + \mathbf{R}^T_{ss'}$ is the friction force between species s and s', including both regular friction $\mathbf{R}^u_{ss'}$ and thermal friction $\mathbf{R}^T_{ss'}$ (Nernst effect), and $\mathbf{F}_s$ is an external force. Summing Eq. (A1) over all species s and using quasineutrality $\sum_s q_s n_s = 0$ and reciprocity $\mathbf{R}_{ss'} + \mathbf{R}_{s's} = 0$ yields

$$\frac{\partial(\sum_s m_s n_s \mathbf{u}_s)}{\partial t} + \nabla\cdot\Big(\sum_s m_s n_s \mathbf{u}_s \mathbf{u}_s\Big) = \sum_s q_s n_s \mathbf{u}_s \times \mathbf{B} - \sum_s \nabla p_s - \sum_s \nabla\cdot\pi_s + \sum_s \mathbf{F}_s. \tag{A2}$$

On the other hand, flow velocity in perpendicular direction to magnetic field can be found from Eq. (A1) to be

$$\mathbf{u}_s = \frac{\mathbf{E}\times\mathbf{B}}{B^2} - \frac{\nabla p_s \times \hat{b}}{q_s n_s B} - \frac{\nabla\cdot\pi_s \times \hat{b}}{q_s n_s B} - \frac{m_s d_s \mathbf{u}_s/dt \times \hat{b}}{q_s B} + \frac{\sum_{s'} \mathbf{R}_{ss'} \times \hat{b}}{q_s n_s B} + \frac{\mathbf{F}_s \times \hat{b}}{q_s n_s B}. \tag{A3}$$

Here $B = |\mathbf{B}|$, $\hat{b} = \mathbf{B}/B$, and $d_s/dt = \partial/\partial t + \mathbf{u}_s\cdot\nabla$. Suppose that plasma has geometry with a direction of symmetry (slab or cylinder geometry) and that all gradients are in one direction. In the subsequent discussion, cylindrical geometry is considered and the terms azimuthal and radial directions are used. However, they can be substituted by "direction of drifts" and "direction of gradients," and the analysis will apply to slab geometry.

Note that cross-field transport in one direction is governed by forces in the perpendicular direction ($\mathbf{F} \times \mathbf{B}$ drift). In cylindrical geometry, the pressure force and many other forces are in radial direction. They do not contribute to radial transport directly. They produce drift in azimuthal direction and, subsequently, friction force between species, which in turn produces radial transport. Note that friction force is present in Eq. (A3) but not in Eq. (A2). Also note from Eq. (A3) that particle flux due to a force is proportional to that force. Therefore, if friction force is larger than other azimuthal forces, then azimuthal momentum changes on a much slower timescale than ion density of individual ion species. Moreover, ion-ion friction force is larger than ion-electron friction force if ion density profiles are not in the equilibrium. Consequently, ion cross-field transport is accompanied by transfer of azimuthal momentum from one ion species to another.

Another feature of frictional drift [$\mathbf{R}_{ss'} \times \hat{b}$ term in Eq. (A3)] is that it is ambipolar, i.e., it does not move charge across magnetic field lines. Combined, these constraints on cross-field transport (azimuthal momentum and net charge) amount to the following about flow energy. Suppose that plasma has two ions species a and b, and that a chunk δn_a of ion species a ($\delta n_a \ll n_a$) is moved by cross-field transport from the place where it had flow velocity $\mathbf{u}_a$. Then chunk δn_b of ion species b moves in place of ion species a, and $\delta n_b = q_a \delta n_a / q_b$ so net charge density conservation is observed. If $m_a/q_a = m_b/q_b$, then momentum carried out by the chunk δn_a, $m_a \mathbf{u}_a \delta n_a$ is equal to momentum obtained by the chunk δn_b, $m_b \mathbf{u}_b \delta n_b$. Then momentum conservation is automatically observed in this exchange process, and flow velocities remain the same. Then drift energy, in turn, is exchanged between species a and species b in chunks of size $m_a \delta n_a \mathbf{u}_a^2/2$. The same argument can be generalized to plasma of multiple ion species if particle fluxes are matched in such a way that net cross-field in- and outflows of ion mass and charge are matched (e.g., one proton and one boron are exchanged with three α particles in p-B plasma in cross-field transport). If $m_a/q_a \neq m_b/q_b$, then the exchange of chunks is accompanied by the change in flow velocities, which shows up as the change in potential energy of the force driving plasma flow. However, analysis of energy sources and sinks shows once again that the drift energy of species a is given to the drift energy of species b, and potential energy of the force driving plasma flow provides the difference in these drift energies.

Additionally, note that there is an equivalence between $\delta \mathbf{R} \leftrightarrow \delta \mathbf{p}$ relationship in single-particle description and $\Gamma_s \leftrightarrow \mathbf{F}_s$ in relationship in fluid description, where $\Gamma_s = n_s \mathbf{u}_s$ is particle flux of species s. To see this, note that if in time period Δt there are $\nu n \Delta t$ momentum kicks, then net motion of ions is $\nu n \Delta t \delta \mathbf{R}$, so net motion of ions per unit time is $\nu n \delta \mathbf{R}$. During the same time period Δt, total amount of momentum imparted upon ions is $\nu n \Delta t \delta \mathbf{p}$ (number of momentum kicks times size of a kick). Hence, the size of the force that drives the cross-field transport is $\nu n \delta \mathbf{p}$, the rate of momentum change. Therefore, after appropriate averaging over the distribution of momentum kicks cross-field particle flux Γ is proportional to force $\mathbf{F}$, with the proportionality coefficient being the same matrix as in Eq. (12).