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Current generation in a relativistic plasma

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The relativistic dynamics of fast current carriers are rich in intriguing phenomena not appearing in the Newtonian limit. It is shown that because of these dynamics there is a bound to the efficiency of driving current in a tokamak either by waves or other means. Analytic techniques uncover the current-drive scheme which yields the maximum attainable efficiency.

Among the most promising schemes for sustaining continuous toroidal current in a tokamak are those that exploit the relative infrequency with which superthermal electrons suffer collisions. The higher the speed of the current carrier, the more effortlessly the current is carried, so that the power requirements for sustaining the current are diminished.[1] These fast electrons can be selectively induced to carry current by means of the resonant absorption of plasma waves which carry a component of toroidal momentum, such as lower hybrid waves,[1] or even by means of the absorption of waves with vanishingly small toroidal momentum, such as electron cyclotron waves,[2] which merely heat those electrons with the proper sign of toroidal momentum. The efficiency of current generation by means of the absorption of either wave increases as $p_{\|}^2$, the square of the toroidal momentum of the current carriers, where the measure of efficiency is J/P_d, the ratio of current generated to power dissipated.

The absence in the above scaling of a theoretical bound on the efficiency of current generation naturally promotes to prime importance the question of whether such a bound exists. Previous calculations of the efficiency, carried out for fast but nonrelativistic current carriers, are no longer valid as the current carriers approach the speed of light. It is this regime, which is pertinent to plasma under tokamak reactor conditions, that is to be the focus of the present study; we wish, however, to anticipate here a number of new and interesting effects that come into play. There arises, for example, the purely relativistic production of current, without any tampering with the parallel momentum of an electron, merely when the energy in its perpendicular motion is increased. (By parallel and perpendicular we refer to the direction with respect to a dc magnetic field, presumably largely in the toroidal direction in tokamaks.) The absorption of a perpendicularly traveling photon will accomplish this intriguing effect. Although the parallel momentum is unchanged, the parallel velocity is of necessity decreased since the electron is now heavier. Since the electron current is proportional to its velocity rather than its momentum, clearly current may be produced in this manner.

While what might be called relativistic mass enhancement is a completely collisionless effect that generates current in the parallel direction opposite to the current of the affected electron, it is always accompanied by a collisional effect,[2] arising from the decreased collisionality of the same electron, which serves to generate current in exactly the canceling direction. That even a vanishingly small number of collisions ensures the dominance of the latter over the former effect is revealed only by a thorough treatment of the relativistic dynamics of current generation which we now set out to perform. Motivated thusly by the unexpected phenomena that arise in this limit, we define our goal as the precise calculation of the efficiency measure J/P_d in full generality, i.e., in the case of acceleration of superthermal electrons in an arbitrary direction.

Consider the current produced when an electron in a plasma is pushed in momentum space to a nearby position. Not only is the electron current changed instantaneously by this push, but also the current carried at later times by this electron is changed as a result of the different electron collisionality if the push is to a position of different energy. Let $j_{\|}(t,\vec{p})$ be the current associated with one electron initially at momentum coordinate $\vec{p}$ and initially with energy $E(\vec{p})$. We imagine the continuous pushing of electrons in momentum space along the incremental vector $\vec{S}$ and with constant power p_d. The current thus produced evidently approaches the time-asymptotic value J such that

$$\frac{J}{P_d}=\frac{\vec{S}\cdot\vec{\nabla}\int_0^\infty j_{\parallel}(t,\vec{p})dt}{\vec{S}\cdot\vec{\nabla}E(\vec{p})}, \tag{1}$$

where $\vec{\nabla}$ is the gradient operator in momentum space. That the integral is convergent is only a statement of the negligibility of contributions to the present and steady current J arising as a result of pushes given to the electrons many collision times previously.

To evaluate Eq. (1), we first find $j_{\parallel}(t,\vec{p})$. Note that for superthermal electrons the diffusion in energy of test electrons is negligible compared to their slowing down in energy. Considering then only electrons with initial energy substantially superthermal, the relevant dynamics are contained in the slowing down equations of energy and parallel momentum which are obtained from a relativistic treatment of the Fokker-Planck equation.[3] These equations may be written as

$$d\lambda/\partial t=-\nu_E\lambda\ , \tag{2a}$$

$$d\zeta/\partial t=-\nu_{\parallel}\zeta\ , \tag{2b}$$

where we normalized momentum to mc, i.e., $\lambda=p/mc$ and $\zeta=p_{\parallel}/mc$, where m is the electron rest mass and c is the velocity of light. The slowing down frequencies are given by

$$\nu_E=\gamma^2\nu/2\lambda^3\ , \tag{3a}$$

$$\nu_{\parallel}=\gamma^2\nu/2\lambda^3+\gamma(1+Z_i)\nu/2\lambda^3\ , \tag{3b}$$

where $\gamma(\lambda)=(1+\lambda^2)^{1/2}$, Z_i is the ion charge state, and $\nu=\omega_p^4\ln\Lambda/2\pi n_0c^3$. Note that $\nu=\nu_0v_t^3/c^3$, where ν_0 is the normalizing frequency characteristic of previous work.[1,2,4] As pointed out in Ref. 3, the relativistic scattering dynamics are equivalent to the nonrelativistic slowing down of electrons with relativistically augmented mass. On account of this effect, the background electrons can absorb far more of the test electron momentum than can the background ions.

Note that Eq. (2a) is an ordinary differential equation for λ and hence may be exploited to parametrize t in terms of λ. Therefore, substituting for t in Eq. (2b) using Eq. (2a), and then fortuitously finding the integration possible in terms of elementary functions, it may be found that

$$\zeta(t)=\zeta\frac{\lambda(t)}{\lambda}\left[\frac{\gamma[\lambda(t)]-1}{\gamma[\lambda(t)]+1}\right]^{(1+Z_i)/2}\times\left[\frac{\gamma(\lambda)+1}{\gamma(\lambda)-1}\right]^{(1+Z_i)/2}, \tag{4}$$

where here ζ and λ without the time argument denote values at $t=0$. Note that $j_{\parallel}(t,\vec{p})=-ep_{\parallel}(t)/m\gamma[\lambda(t)]=ec\zeta(t)/\gamma[\lambda(t)]$, so that again exploiting the parametrization of time in terms of momentum magnitude, Eq. (1) can be cast into the form

$$\frac{J}{P_d}=\frac{\gamma(\lambda)}{\lambda\vec{S}\cdot\hat{\lambda}}[\vec{S}\cdot\hat{\zeta}G(\lambda)+\vec{S}\cdot\hat{\lambda}\zeta dG(\lambda)/d\lambda]\ , \tag{5}$$

where the caretted quantities are unit vectors, where we have normalized J to $-en_0c$ and P_d to νn_0mc^2, and where

$$G(\lambda)=\frac{2}{\lambda}\left[\frac{\gamma(\lambda)+1}{\gamma(\lambda)-1}\right]^{(1+Z_i)/2}\times\int_0^\lambda\left[\frac{x}{\gamma(x)}\right]^3\left[\frac{\gamma(x)-1}{\gamma(x)+1}\right]^{(1+Z_i)/2}dx\ . \tag{6}$$

Suppose that the push that the electrons receive is the result of absorbing a wave with frequency ω and parallel wave number $k_{\parallel}$. Since the ratio of wave energy to momentum is $\omega/k_{\parallel}$, the electron absorbs energy and momentum in this proportion too. We seek now the path in momentum space that is allowable to a particle interacting with the wave, i.e., if the interaction is diffusive we seek what might be called the diffusion path. In the wave frame of reference $\omega=0$ so that the wave can impart momentum but, on the average, no energy to the particle in this reference frame, and the diffusion paths are contours of constant energy. For nonrelativistic motion, these contours are spheres in momentum space concentric with the point $p_{\parallel}=m\omega/k_{\parallel}$, $p_{\perp}=0$. More generally, if by interacting with a wave a particle experiences motion along $\vec{S}'$ in the wave frame, then we must have $\vec{S}\cdot\vec{\nabla}E'$ in that frame, which implies that in the laboratory frame of reference (where the wave frequency is ω) we have

$$\vec{S}\cdot\vec{\nabla}(E-p_{\parallel}\omega/k_{\parallel})=0\ , \tag{7}$$

which implies that the direction of $\vec{S}$ is such that

$$\vec{S}\propto(c^2p_{\parallel}/E-\omega/k_{\parallel})\hat{p}_{\perp}-(c^2p_{\perp}/E)\hat{p}_{\parallel}\ , \tag{8}$$

where $\hat{p}_\perp$ is the unit momentum vector perpendicular to $\hat{p}_{\parallel}$. Note that in the nonrelativistic limit E is nearly constant so that $\vec{S}$ traces the familiar concentric spheres. In the relativistic limit, where $E=(E_0^2+c^2p^2)^{1/2}$, E is no longer constant along $\vec{S}$ so that this simple geometrical picture no longer holds, and $\vec{S}$ follows nonconcentric ellipsoids for $\omega/k_{\parallel}c<1$ and nonconcentric hyperboloids for $\omega/k_{\parallel}c>1$.

Substituting in Eq. (5) for the diffusion path as given in Eq. (8) allows us to write

$$J/P_d = G(\lambda)/\beta_w + (\gamma\zeta/\lambda)(dG/d\lambda)\,, \tag{9}$$

where $\beta_w \equiv \omega/k_{\parallel}c$. In spite of its concise form, Eq. (9) is exceedingly general and informative, and it remains only to extract relevant information from it.

Consider the various limits of Eq. (9). In the nonrelativistic limit we have $\gamma \to 1$ and we find

$$\frac{J}{P_d} = \frac{6\lambda\zeta}{5+Z_i} + \frac{2\lambda^3}{(5+Z_i)\beta_w}\,, \tag{10}$$

which is an equivalent representation of Eq. (10) in Ref. 2. In the case of Landau damping, $\beta_w \to \zeta$, as the wave is resonant only with electrons traveling at its own parallel phase velocity. For high-phase-velocity waves, most resonant electrons have larger parallel momentum than perpendicular momentum, so that we can take $\lambda \to \zeta$, and obtain $J/P_d \to 8\zeta^2/(5+Z_i)$ as found previously. In the case of purely perpendicular acceleration, such as with electron cyclotron heating with $\beta_w \to \pm\infty$, and using the above instance of $\lambda \to \zeta$, we obtain $J/P_d = 6\zeta^2/(5+Z_i)$.

Note that the largest J/P_d may be obtained by taking β_w small but λ large. This corresponds to a method of maximizing current-generation efficiency by utilizing waves with high content of momentum ($\omega/k_{\parallel}$ small) to interact with relatively collisionless electrons. Such waves exist in a plasma but their excitation may be impractical, as discussed in Ref. 4. For these waves we have $J/P_d = 2\lambda^3/[(5+Z_i)\beta_w]$.

As can be seen from Eq. (10) and the discussion following it, J/P_d increases nonrelativistically with increasing momentum of the resonant electrons because of their decreasing collisionality with increasing kinetic energy. This, in fact, is the motivation behind the scheme of current drive in tokamaks with fast traveling waves.[1] However, this scaling ceases to hold for $\lambda >> 1$, where from Eq. (10) we calculate the limiting efficiency of $J/P_d \to 2$ for parallel acceleration with $\beta_w \to 1$. On the other hand, the efficiency of current drive by selective electron heating with infinitesimal parallel-momentum input, say by electron cyclotron heating, goes to zero for λ large. This is unfortunate, though not discouraging, in that the efficiency of current drive is theoretically bounded. Note that current drive by relativistic electron beams yields the same efficiency as by parallel acceleration by waves in this limit and is bounded identically. Figure 1 exhibits these cases. The only scheme for current drive by wave diffusion that can exceed the limiting efficiencies shown in Fig. 1 utilizes a high-momentum content driver. Here, if $p \to \infty$, we have $J/P_d \to 2/\beta_w$, which can be large for β_w small.

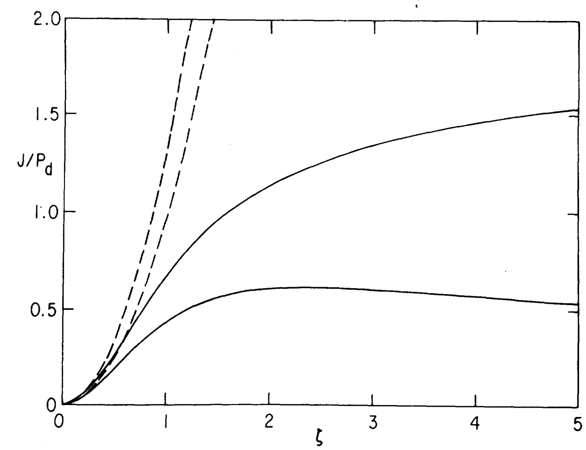

FIG. 1. J/P_d vs ζ. The upper solid and the upper broken curves are for parallel diffusion while the lower solid and lower broken curves are for perpendicular diffusion. The broken curves indicate the nonrelativistic treatment. Here $Z_i=1$ and $\lambda=\zeta$ are assumed.

In summary, what has been uncovered is a bound on the limits of current-generation efficiency by any means which rely on the acceleration of superthermal electrons. The utility of this finding is, among other things, to bound, but not extinguish, the recent enthusiasm over new techniques of current drive for tokamaks. If the maximum efficiencies for parallel or perpendicular acceleration are not sufficient, then means for exciting the low-β_w waves described here may become an even more tantalizing goal.

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