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Pycnonuclear reaction and possible chain reactions in an ultra-dense DT plasma

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Abstract

In ultra dense matter, fusion reactions might be observable even at zero temperature. We show that these so-called "pycnonuclear reactions" can be appreciable in the laboratory, and that explosive fusion chain-reactions are possible. Moreover, we predict the possibility of fission-like chain reactions. We assess the enhancement due to electron screening and consider the local field correction as well as relativistic effects.

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1. Introduction

In cold ultra-dense matter, fusion reactions are quite frequent due to the overlapping of the wave functions between neighbor ions. These reactions are called the pycnonuclear fusion reactions, where "pycno" means compact. While the most violent pycnonuclear reactions occurred in the initial phase of the universe, the reactions are still observable in dense stars [1], and there have been propositions that these reactions can be observable even in the laboratory [2,3]. The calculations here support the possibility for observing pycnonuclear reactions in the laboratory. We further predict that even explosive fission-like chain reactions are possible.

The calculation of the fusion rate is usually based on the assumption that the Coulomb interactions with all other nuclei and electrons can be neglected.

This is a good approximation for a sufficiently ideal plasma. The first order correction to this vacuum picture is the Salpeter enhancement theory whose validity has been questioned or defended many times [4,5] even though it is a very small effect in an ideal plasma. For an intermediate regime of density n and temper-

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ature T, while most of nuclei are bound by strong Coulomb interactions, reacting high-energy nuclei are still free. The electrostatic effects still can be, if large, expressed as a multiplicative factor [6,7].

However, in an ultra dense plasma, even the reacting nuclei are bound in a Coulomb lattice. To obtain the fusion reaction rate in this regime, quite different methods must be used. While the so-called cold fusion reactions [8,9] also have employed this pycnonuclear fusion concept, it must be emphasized that the pycnonuclear fusion reaction itself is generally accepted theory [1], even if general acceptance has not been accorded to all the effects to which it has been associated.

A prominent feature of the pycnonuclear reactions is that the fusion rate is extremely sensitive to the density, but almost independent of the temperature. Prodigious work has been done in 1950s and 1960s in this field. However, the main interest has been mainly in stellar evolution, such as white dwarf [1,10,11], and the extra heat in the Jupiter [12]. The reason for this limited interest is apparently that the pycnonuclear reaction is estimated to be rare in laboratory devices compared with the thermonuclear reaction when the temperature is reasonably hot, i.e., $T > 100$ eV, and the density is not ultra dense, i.e., $n < 10^{27}\ \mathrm{cm}^{-3}$ [10].

In a thermonuclear-reaction regime, the fusion reactions are discontinuous events since an energetic particle must meet another energetic ion in a head-on collision, but, in a pycnonuclear regime, the neighbor ions fuse all the time. This time advantage is one of physical aspects, along with the electron screening and the ion–ion correlation, which make the regime attractive [3]. If these features can be tapped in a certain density and temperature, the pycnofusion might be observed or used for a reactor. The interest in the solid fusion has been recently revived in this direction including concepts for a new reactor [2,3,13–15].

In this Letter, we show that, in a very dense plasma, an alpha particle from a D–T fusion is mainly stopped by ions not by electrons. Because little energy goes into the electrons, the pycnonuclear reaction rate might be big enough to be detected experimentally even if it is not economically feasible to capture this energy: in contrast, note that, in the thermonuclear reaction regime, the possibility of circumventing energy flow into electrons is achieved only intrusively [16], although there is great utility in this regime as well [17]. We also propose an explosive chain-reaction scenario. We also compute the enhancement from electron screening, with inclusion of the local field correction [18,19] and the relativistic effect [19,20].

This Letter is organized as follows. In Section 2, the pycnonuclear regime is descirbed; the initial setting of the problem is presented. In Section 3, we estimate the fusion rate by WKB method and show that the pycnonuclear reactions might be observable in an experiment. In Section 4, we show the fusion rate is further enhanced by electron screenings. In Section 5, we show that a very fast alpha particle is not stopped by electrons but by ions; it can catalyze the fusion probability vastly, and thereby induce more than one fusion events before it slows down. In Sections 6 and 7, a summary and a discussion are given.

2. Pycnonuclear reactions

A plasma becomes strongly coupled when $\Gamma = (4\pi/3)^{1/3}(e^2/aT) > 180$, where $a = n^{-1/3}$ is the inter-particle spacing [21–23]. To first approximation, let us treat the ions as in a solid. Each ion then makes a zero point oscillation as if in a harmonic potential, and the wave functions of the neighbor ions overlap (Fig. 1). The probability of a fusion per second between the nearest neighbors, P, is then proportional to

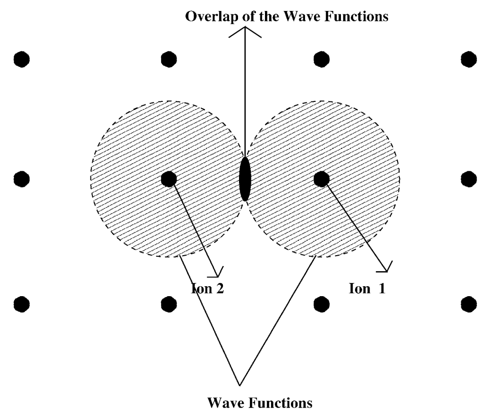

Fig. 1. Pycnonuclear reaction rate is proportional to the overlapping amplitude of the wave function.

the overlapping probability, which is given as

$$P = \left(\frac{2S}{\hbar}\right) r_G \left|\psi(r_n)\right|^2, \tag{1}$$

where $r_G = 2\mu e_1^2 e_2^2/\hbar^2$ is the radius of the Gamow nucleus, r_n is the radius of the nucleus, S, which is almost constant at low energy ($E <$ MeV), is the structure factor dependent upon the strong interaction, and ψ is the Schroedinger wave function of the fusing ion pair. The radius of the Gamow nucleus and the radius of the nucleus is comparable.

The above equation has been derived by Salpeter [10] and Ichimaru [6], but Eq. (1) is smaller by 4-times than Salpeter's result and greater by π-times than Ichimaru's. The cause of the discrepancy has not been identified yet; the difference might stem from a normalization convention. We use Eq. (1) in our computation, but the qualitative conclusion in this Letter is independent of the precise numerical constant.

To obtain a more exact rate than by WKB or in time-dependent case, we need to obtain the overlapping amplitude,

$$\left|\psi(r_n)\right|^2 = \int \left|\psi\left(\mathbf{R}, |\mathbf{r}| = r_n\right)\right|^2 d^3\mathbf{R}, \tag{2}$$

where R is the center of mass, $r = r_1 - r_2$ is the relative coordinate, and $r_n \cong 0$ is the radius of the nucleus that is very small compared with any length involved. Let us define the penetration probability as

$$p_f = r_n^3 \left|\psi(r_n)\right|^2. \tag{3}$$

The probability for a fusion per second between 1 and 2 is then $P(\mathrm{s}^{-1}) = (2S/\hbar r_n^2) p_f$. The penetration probability p_f is obtained from Eq. (2) after solving the Schroedinger equation of two neighbor particles, i.e.,

$$\left[-\frac{\hbar^2\nabla_1^2}{2m_1} - \frac{\hbar^2\nabla_2^2}{2m_2} + V(r_1, r_2)\right]\psi_0 = E_0\psi_0, \tag{4}$$

where ψ_0 (E_0) is the ground state (energy), and $V(r_1, r_2) = v_{1,2} + \sum_i (v_{1,i} + v_{2,i})$ for all ions i except 1 and 2 with $v_{j,i}$ being the inter-particle potential between the particle j and the particle i. As an example, consider a D–D plasma with the body-centered cubic lattice. We obtain $V(r_1, r_2)$ in Eq. (4) by summing up and solve Eq. (4). Then, we obtain the penetration probability p_f from Eqs. (2) and (3). Finally, we obtain P (the probability for fusion per second between 1 and 2) from Eq. (1). Each deuterium has 8 neighbors to have the fusion rate $8 \times P$, and the total fusion rate per second per volume is then $4 \times nP$ ($\mathrm{cm}^{-3}\,\mathrm{s}^{-1}$).

Let us introduce a few approximations necessary in obtaining p_f, the probability amplitude at the nuclear radius. Firstly, we fix $R = C$ at the equilibrium position so that $V(r) = V(R = C, r)$ is just function of r. The Schroedinger equation, Eq. (4), is then a 1-body problem rather than 2-body. Secondly, we only solve the 1-dimensional Schroedinger equation in the radial direction.

While V depends on the direction of r, the spherically-symmetric $v_{1,2}$ dominates V as $r \to 0$. This approximation has been, in fact, adopted many times [6,10,12]. The radial equation is then

$$\left[-\frac{1}{2}\frac{\hbar^2}{\mu}\frac{d^2}{dr^2} + V(r)\right]\phi_0 = E_0\phi_0, \tag{5}$$

where $\phi_0 = r\psi_0$ from Eq. (4). Thirdly, we use

$$V(r) = U(r) = Z_1 Z_2 e^2\left(\frac{1}{r} + \frac{1}{|r - 2a|}\right), \tag{6}$$

where $a = \sqrt{3}/2(2/n)^{1/3}$ is the distance between the nearest neighbor in a bcc lattice. We now impose periodic boundary condition such that $\phi(0) = \phi(2a)$, and the position at $r = 0$ is the same position at $r = 2a$. A particle with the mass μ is at the center of cylinder with the equilibrium position at $r = a$. The penetration probability is given as $p_f = r_n|\phi_0(r_n)|^2$. We employ a normalization in length and time:

$$r = asx, \quad s = \left(\frac{\hbar^2}{4a\mu Z_1 Z_2 e^2}\right)^{1/4},$$
$$t = \frac{1}{\omega_i}\tau, \quad \omega_i = \sqrt{\frac{4Z_1 Z_2 e^2}{a^3\mu}}, \tag{7}$$

where $x(\tau)$ is a new dimensionless length (time), Eq. (5) then becomes

$$\left[-\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{4s^2}\left(\frac{1}{x} + \frac{s}{2 - sx}\right)\right]\phi_0 = \frac{E_0}{\hbar\omega_i}\phi_0 \tag{8}$$

with the boundary condition $\phi_0(0) = \phi_0(2/s)$. For an example, if we consider a D–T plasma with $n_D = n_T$ and $\rho = 1.4 \times 10^6$ ($\mathrm{g\,cm}^{-3}$), then $s = 0.249$, $\omega_i = 2.0 \times 10^{17}$ (s), and the nuclear radius $r_n = 4s^3$ is 0.075, and the potential U has the minima at $x_0 = 4$. The potential U is drawn in the unit of renormalized length (x) and energy ($E/\hbar\omega_i$) for $0 < x < 8$.

For this U, the ground state is computed by evolving the Schroedinger equation in imaginary time using a pseudo-spectral method.

3. Pycnoreactions in laboratory

We now use WKB for an order-of-magnitude-estimation of p_f. From Eqs. (7) and (8), we note that the wave-packet-width of the center (e.g., $x = 4$ in Fig. 2) is of order as. The wave amplitude at the center is then $|\phi_0(r = a)|^2 \cong 1/as$. From WKB, the ratio of the wave amplitude at the nuclear radius to that at the center is

$$R = \frac{|\phi_0(r_n)|^2}{|\phi_0(a)|^2} = \sqrt{\frac{E_a}{E_G}} \exp\left(-\pi\sqrt{\frac{E_G}{E_a}}\right), \quad (9)$$

where $E_G = 2\mu e^4/\hbar^2$, and $E_a = e^2/a$. Since $p_f \cong r_n|\phi_0(r_n)|^2 = r_n|\phi_0(a)|^2 R$, we have

$$p_f \cong \frac{1}{s}\frac{E_a}{E_G} \exp\left(-\pi\sqrt{\frac{E_G}{E_a}}\right). \quad (10)$$

Let us now use Eqs. (1) and (10) to determine whether these fusion reactions can be observable in laboratory regime for various plasmas and various densities. We consider two densities ($\rho = 10^3\ \mathrm{g\,cm^{-3}}$, $\rho = 2 \times 10^6\ \mathrm{g\,cm^{-3}}$), and three kinds of plasma (D–D, P–D, D–T).

First, we consider a plasma with $\rho = 10^3\ \mathrm{g\,cm^{-3}}$. The form factor S, the Gamow energy E_G, p_f, and P are given in Table 1. We note that $P = 2.83 \times 10^{-24}$ ($\mathrm{s^{-1}}$) for P–D plasma, $P = 1.2 \times 10^{-28}$ ($\mathrm{s^{-1}}$) for D–D, and $P = 8.02 \times 10^{-33}$ ($\mathrm{s^{-1}}$) for D–T. We easily see that the P–D reaction is the most active one; P–D has smallest form factor S, but has the smallest Gamow energy to have the largest exponential term in Eq. (10). In an ICF plasma, with this density, the confinement time τ_c is an order of 10 picosecond [40], and the fraction of fusion reactions per particle in the P–D plasma during the confinement is $O(P\tau) \cong 10^{-32}$, which is too small to be detected.

Second, we consider a plasma with $\rho = 1.4 \times 10^6$ ($\mathrm{g\,cm^{-3}}$) (Table 2). We note that $P = 8.42 \times 10^2$ ($\mathrm{s^{-1}}$) for P–D, $P = 7.3 \times 10^5$ ($\mathrm{s^{-1}}$) for D–D, and $P = 1.57 \times 10^6$ ($\mathrm{s^{-1}}$) for D–T. The most reactive plasma is D–T since D–T has the most largest form factor S; in such a ultra dense plasma, the differences in the exponential terms associated with various plasmas are not of critical importance.

We now estimate the fraction that will be burned during the compression of the fuel. If the pellet is compressed to $\rho = 10^6$ ($\mathrm{g\,cm^{-3}}$), then compared with $\rho = 10^3$ ($\mathrm{g\,cm^{-3}}$), the dimension D of the pellet is 10 times smaller, and the sound wave velocity C_s is 10 times faster. The confinement time D/C_s is then less than $0.01 \times 10^{-11} = 10^{-13}$ (s) with which $\tau P \cong 1.0 \times 10^{-7}$ for a D–T pellet. This is appreciable fraction, and the pycnonuclear reaction might be observed in the laboratory for the future even if it is not presently possible to compress D–T to such an ultra-dense condition.

Table 1
For various plasma, $S(E)$ in the unit of (MeV ∗ Barn), E_G, E_a, p_f and P for $\rho = 10^3\ \mathrm{g\,cm^{-3}}$

	$S(E)^1$	E_G (keV)	E_a (keV)	p_f	P (1/s)
$d(p,\gamma)^3$He	2.50×10^{-7}	60	0.10	1.74×10^{-38}	2.83×10^{-24}
$d(d,p)t$	5.29×10^{-2}	100	0.09	8.05×10^{-49}	6.23×10^{-29}
$d(d,n)^3$He	4.97×10^{-2}	100	0.09	8.05×10^{-49}	5.85×10^{-29}
$t(d,n)^4$He	11.0	120	0.08	3.46×10^{-55}	8.02×10^{-33}

Table 2
For various plasma, $S(E)$ in the unit of (MeV ∗ Barn), E_G, E_a, p_f and P for $\rho = 1.4 \times 10^6\ \mathrm{g\,cm^{-3}}$

	$S(E)^1$	E_G (keV)	E_a (keV)	p_f	P (1/s)
$d(p,\gamma)^3$He	2.50×10^{-7}	60	1.17	6.62×10^{-12}	8.42×10^2
$d(d,p)t$	5.29×10^{-2}	100	1.11	4.83×10^{-15}	3.78×10^5
$d(d,n)^3$He	4.97×10^{-2}	100	1.11	4.83×10^{-15}	3.5×10^5
$t(d,n)^4$He	11.0	120	1.03	6.77×10^{-17}	1.57×10^6

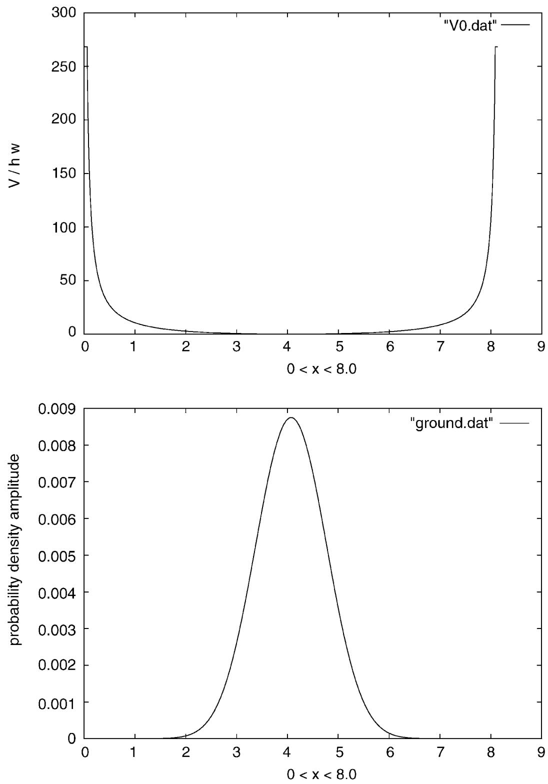

Fig. 2. For D–T plasma with $\rho = 1.4 \times 10^6\ \mathrm{g\,cm^{-3}}$, the potential normalized by $\hbar\omega_i$ in y-axis and as in x-axis (above figure) and the ground state with y-axis normalized by the condition: $\sum_i |\psi(i)|^2 = 1$ with i being the mesh-point, and x-axis by as (below figure).

As shown in Fig. 2, a numerical computation is carried out to obtain p_f. A WKB treatment is no longer valid and the numerical treatment is necessary because the ground state wave function has a width of $\delta x \cong 1$ and the inter-particle spacing a is just $a \cong 4$. Thus, we numerically calculate $p_f = r_n|\phi|^2 = 1.339 \times 10^{-15}$ for $\rho = 1.4 \times 10^6$, which is much larger than that from WKB by a factor 100. We obtain, from p_f, $P = 0.3 \times 10^8$ (s^{-1}). Assuming the confinement time 10^{-13} s, more than 0.001% of the fuel will fuse.

4. Enhancement by electron screening

In the last section, the bare inter-particle potential $v(1,2) = e^2/r$, which does not take into account the electron screening, is used. The electron screening reduces the repulsion between the fusing ions and so enhances the fusion probability. How much the screening enhances the fusion rate is calculated here.

We now use $v_s(q) = 47\pi e^2/q^2\epsilon(q,0)$ with the dielectric function ϵ, instead of $v_0(q) = 4\pi e^2/q^2$ where $v_0(q)$ is the Fourier-transform of the bare Coulomb potential. The calculation of ϵ has been studied in strongly coupled plasma [6,20,24,25]. The expression for ϵ with the local field correction [26,27] is

$$\epsilon(q,0) = 1 - \frac{v(q)\chi(q)}{1 + v(q)G(q)\chi(q)}, \tag{11}$$

where $\chi(q)$ is the free electron polarizability, and $G(q)$ is the local correction factor [26]. For $G(q)$, we use the fitting formula from [27]:

$$\begin{aligned} G(q) = {} & AP^4 + bP^2 + C \\ & + \left[AP^4 + \left(B + \frac{8}{3A} \right) P^2 \right. \\ & \left. - C\frac{4 - P^2}{4P} \log\left(\left| \frac{2+P}{2-P} \right| \right) \right], \end{aligned} \tag{12}$$

where $P = q/q_{\mathrm{F}}$ with $q_{\mathrm{F}} = (3\pi^2 n_e)^{1/3}$ being the Fermi wave number. We obtain $A = 0.029$, $B = 0.086$, $C = 0.005$ when $\rho = 1.4 \times 10^6$ ($\mathrm{g\,cm^{-3}}$) (for further reference, see [27]). For the electron polarizability χ, we use the conventional static Lindhard RPA polarizability χ_0 [25] and the relativistic polarizability χ_1 [19,20,28]. χ_0 is defined to be

$$\begin{aligned} \chi_0(q,0) = {} & -3\frac{n_e m_e}{\hbar^2 q_{\mathrm{F}}^2} \\ & \times \left[\frac{1}{2} + \frac{1}{4Q}(1 - Q^2)\log\left(\left| \frac{Q+1}{Q-1} \right| \right) \right], \end{aligned} \tag{13}$$

where $Q = 1/2q/q_{\mathrm{F}}$, and χ_1 is given as

$$\begin{aligned} \chi_1(q,0) = {} & -\frac{q_{\mathrm{TF}}^2}{4\pi e^2}\left[\frac{2}{3}(1+b^2)^{1/2} - \frac{2bQ^2}{3}\sinh^-(b) \right] \\ & - \frac{q_{\mathrm{TF}}^2}{4\pi e^2}\left[+(1+b^2)^{1/2}\frac{1 + b^2 - 3b^2Q^2}{6b^2Q} \right. \\ & \left. \times \ln\left(\left| \frac{1+Q}{1-Q} \right| \right) \right] \\ & + \frac{q_{\mathrm{TF}}^2}{4\pi e^2}\left[\frac{1 - 2b^2Q^2}{6b^2Q}(1 + b^2Q^2)^{1/2} \right. \\ & \left. \times \log\left(\left| \frac{(1+b^2Q^2)^{1/2} + Q(1+b^2)^{1/2}}{(1+b^2Q^2)^{1/2} - Q(1+b^2)^{1/2}} \right| \right) \right], \end{aligned} \tag{14}$$

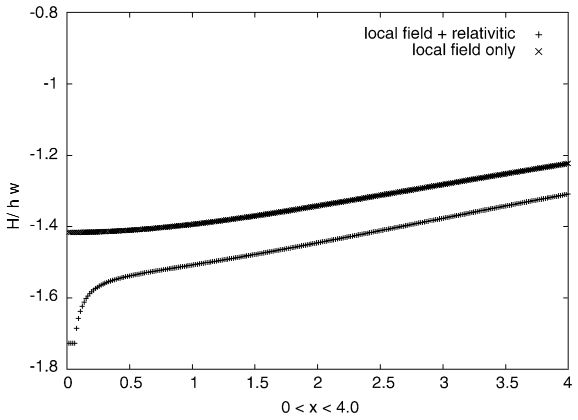

Fig. 3. The screening potential of $H(r)$ defined in Eq. (15).

where $q_{\mathrm{TF}} = (12\pi m_e e^2 n)^{1/2}/\hbar q_{\mathrm{F}}$ is the Thomas–Fermi number, and $b = \hbar q_{\mathrm{F}}/m_e c$. At this density, we have $r_s = 0.023$ and the relativistic effect is important (the relativistic effect is important when $r_s = (3/4\pi n_e)^{1/3}(m_e e^2/\hbar^2) < 0.1$).

The difference between the bare Coulomb potential and the screened one is

$$H(r) = \frac{e^2}{8\pi^3}\int d^3q\left[v(q)\left[1 - \frac{1}{\epsilon(q,0)}\right]\exp(iq\cdot r)\right]. \tag{15}$$

$H(r)/h\omega_i$ is plotted with the normalization in Section 2 with the local field correction, and with local field and relativistic correction in Fig. 3. Our result is in agreement with Ichimaru [19] except that we have a more strong screening at $r \cong 0$ in $H(r)$ in the case of the relativistic correction.

A numerical computation is carried out to obtain p_f as in Section 3. We obtain $p_f = 1.835 \times 10^{-15}$ only with the local field correction, and $p_f = 1.856 \times 10^{-15}$ with the local field and the relativistic correction. Compared to the bare Coulomb potential, we see 37% and the 39% increase of the rate, respectively.

5. Chain reactions

In this section, it is shown that an energetic alpha particle from a fusion reaction is slowed down primarily by ions not by electrons, and that the alpha particle might catalyze more than one fusion event before it slows down. The situation is similar to the case of the fission chain-reactions in which a neutron produces more than one neutron. The consideration here is limited to D–T with $\rho = 1.4 \times 10^6$ (g cm^{-3}), but the same considerations apply to a D–D pellet with the same density.

5.1. Slowing down of an alpha particle

In a fully degenerate plasma, when the velocity of an ion is smaller than the electron Fermi-velocity, the electronic stopping power becomes almost independent of the density and proportional to the ion velocity [25]. The formula is written here as

$$\frac{dE}{dt} = C(\chi)\frac{8}{3\pi}\frac{m_e^2 Z^2 e^4}{\mu\hbar^3}E, \tag{16}$$

where μ is the ion mass, E is the ion energy, m_e is the electron mass, $\chi^2 = e^2/\pi\hbar v_{\mathrm{F}}$, v_{F} is the Fermi velocity, and $C(\chi) \cong 1/2[\log(1 + 1/\chi^2) - 1/(1 + \chi^2)]$ [29]. The above formula is valid if $v \ll v_{\mathrm{F}}$ and $r_s \ll 1$, where v is the ion velocity and $r_s = (me^2/\hbar^2)(3/4\pi n_e)^{1/3}$ [25,29–36]. For a D–T plasma with $\rho = 1.4 \times 10^6$ g cm^{-3}, we can estimate $C(\chi) \cong 2.3$ and the ion–electron collision frequency as

$$\nu_{i,e} = 4.0 \times 10^{13}\left(\frac{Z^2}{\mu}\right)\frac{1}{\mathrm{s}}, \tag{17}$$

where μ is the nucleus mass in the unit of the proton mass. When an ion with energy E slows down due to collisions with electrons, deuteriums and tritium, the fraction of the ion energy into electrons is given as

$$r_e = \int_0^E \frac{\sum_j \nu_{i,e}(E)}{\nu_{i,e}(E) + \sum_j \nu_{i,j}(E)}\,dE, \tag{18}$$

where $\nu_{i,j}(E)$ is the ion–ion collisions frequency. For $\nu_{i,j}(E)$, we use the classical formula:

$$\nu_{i,j} \cong 1.8 \times 10^{-7}\left(\frac{n_j Z_j Z_i^2}{m_j}\Lambda_{i,j}\right)\sqrt{m_i}\,\frac{1}{E_i^{3/2}}, \tag{19}$$

where E is in eV, m in esu, and n in cgs unit. Assuming equal concentrations of D and T with $\rho = 1.4 \times 10^6$ g cm^{-3}, we obtain $r_e = 0.96$ for a 4.0 MeV alpha particle. Most of the alpha particle energy then goes to the ions, and its mean-free-path is roughly estimated to be as $l \cong v/(\sum_j \nu_{i,j}) \cong 10^{-5}$ cm.

5.2. Chain reactions

We now estimate how many fusion reactions an alpha particle catalyzes. The width of a ground state wave packet of D or T is $O(as)$ as shown in Section 2 and is not small compared to a. An alpha particle penetrates and distorts many times the wave packet of D or T before it slows down. Such events enhance the rate. The number of such penetrations for each alpha particle can be estimated $N = n\pi(as)^2 l \cong 10^4$. A simulation showing the enhancement in the fusion probability for each penetration is given in Fig. 4.

The wave packet is initially in the ground state as in Fig. 2, with the local-field and relativistic correction taken into account. At $\tau_0 = 0.23$, an alpha particle penetrates the wave packet at position $x_p = 3$, which is at the edge of the center of the wave packet in Fig. 2. We record the penetration probability p_f during $0 < \tau < 1$, and the wave amplitude $|\phi_0(y)|^2$ right after the alpha particle has passed (see Fig. 4). For the effect of the alpha particle penetration, in addition to $U(x)$ in Eq. (5), we add the whizzing-by potential δU:

$$\delta U(x,t) = 2e^2\left[\frac{1}{\sqrt{(x-x_0)^2 + v_a^2(t-t_0)^2 + b_c^2}}\right] - 2e^2\left[\frac{1}{\sqrt{(x)^2 + v_a^2(t-t_0)^2 + b_c^2}}\right], \quad (20)$$

where $t_0 = (1/\omega_i)\tau_0$, $x_0 = asr_0$, and v_a is the velocity of a 3.5 MeV alpha particle. We introduce b_c in ad-hoc fashion to avoid large angle scattering which cannot be included in 1D simulation. We choose $b_c = 8 \times 10^{-12}$ cm which is big enough to avoid the small angle scattering by a MeV particle. We evolve the equation in time by the real time pseudo-spectral method.

In Fig. 4, the wave probability distribution in x was shown right after the alpha particle whizzed by ($\tau = 0.23$), and the time histories of p_f in Section 2 is also shown around $0 < \tau < 1$. We see the enhancement of p_f by $O(10^9)$.

We have done the same simulation except the penetration point $x_p = x_0 = 4.0$, which is the center of the packet, and we have observed the same enhancement as in the case of $x_p = 3.0$. We have done the same simulation for the penetration point $x_p = 2.0$ which is far off from the center of the wave packet. The wave distortion is negligible, but still see the enhancement of p_f by $O(10^5)$.

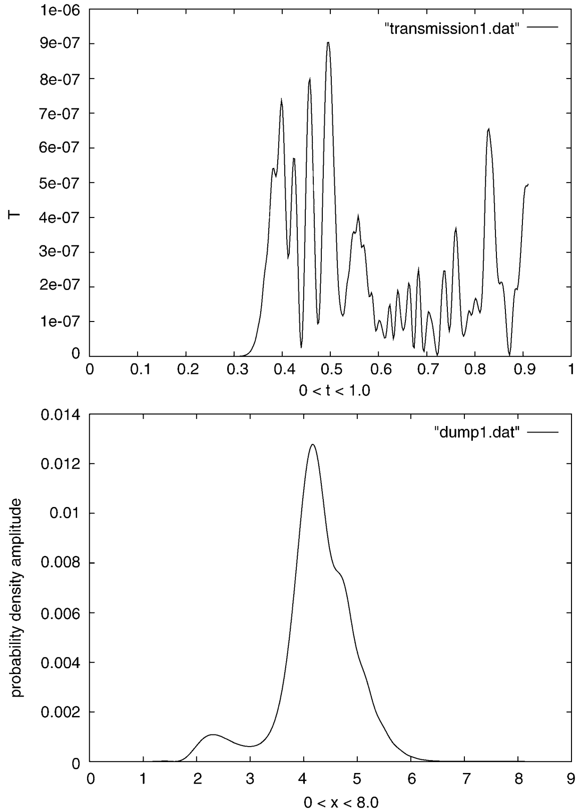

Fig. 4. The above figure: the tunneling probability p_f for $0 < \tau < 1$ (y-axis: p_f, x-axis: normalized by 0.5×10^{-17} s). The below figure: the wave function $|\phi|^2$ right after the penetration of the alpha particle at $y = 3.0$.

Given the fact that $p_f = 1.3 \times 10^{-15}$ without an alpha particle, the time needed for a D–T pair to fuse is $O(1/P) = 10^{-8}$ s, which is far longer than the confinement time. As the alpha particle passes-by, the time needed becomes 10^{-17} due to the enhancement of p_f by $O(10^9)$, and this is far shorter than the confinement time $\tau_c \cong 10^{-11}$ s. The alpha particle, since it catalyzes 10^4 such events before slowed down, produces more than one alpha particle if the enhanced penetration probability persists more than $10^{-17}/10^4 = 10^{-21}$ s for each event.

6. Discussion

The proposed idea raises a few points worthy of consideration. Firstly, it is indeed a surprising result that the alpha particle is stopped by ions rather than electrons. It is also a very useful property since this opens the possibility of a chain reaction as in aneutronic fusion. Nonetheless, our crude estimation and example cases show that it is very hard to make the reaction self-sustaining, and the requirement on the density is more than $10^7\ \mathrm{g\,cm^{-3}}$. However, the possibility is still an open issue, since, compared with the classical calculation, the mean free path of the fusion by-product in reality may be much longer.

Secondly, as more alpha particles are produced, the validity of our treatment no longer holds. The critical number of alpha particles at which this happens is an interesting question. If the alpha particles slowly heat up the plasma, the plasma begins to melt down, and thermonuclear reactions, instead of the pycnonuclear reactions, dominate at some point. Since the method used in the pycnonuclear fusion reactions is valid even at the liquid phase of hydrogen, the breakdown occurs when the coupling parameter Γ becomes smaller than a few tens. For a D–T plasma with $\rho = 10^6\ \mathrm{g\,cm^{-3}}$, the inter-particle potential is around 1 keV, and the temperature must be larger than a few tens of eV for the breakdown. From this consideration, we conclude that our treatment is not valid if the density of the alpha particle is more than $10^{25}\ \mathrm{cm^{-3}}$. However, it is also possible that the chain reactions are so explosive as to exhaust all the fuel even before the alpha particles begin to heat the plasma. Whether the alpha particles slowly heat up plasma or exhaust fuel instantaneously, is an interesting question which is beyond of the scope of this Letter.

Thirdly, as an alpha particle penetrates the wave packet, it deposits, on average, 200 eV of its kinetic energy into the wave packet. If we assume that this energy is equilibrated with the temperature of the wave packet, the enhancement of the fusion rate, compared with the cold plasma, is of order unity only, as shown in [10] (note that the pycnonuclear reaction rate is very insensitive to the temperature). This suggests that the enhancement of the penetration probability is a non-equilibrium phenomenon which exists for a short period time. As shown in the numerical computation in Section 5, the enhanced rate decays in roughly 10^{-18} s. For a excited particle with kinetic energy 200 eV to shed its energy into collective phonons, it needs time at least for it to travel the interparticle spacing, which is roughly 10^{-16}–10^{-17} s. This shows that, while we compute one body problem in the cylindrical boundary condition, the enhancement of the fusion rate is valid still in the time domain of our numerical computation since it has a much faster time scale than the phonon decay.

Fourthly, we assumed in Section 5 that the crossing of the alpha particle is exactly perpendicular. The optimum energy and spatial coordinate of the alpha particle for crossing or the crossing angle, however, remain open questions. A more realistic three dimensional wave function must be computed to address these questions.

Fifthly, there are other neighboring pairs for which the fusion rate might also be enhanced. This fact is ignored in our treatment of the boundary condition. The alpha particle crossing is also treated in only a few simplistic situations. While the fusion is catalyzed by an alpha particle, the enhanced rate decays with time. The major decay mechanism is believed to be a phonon–phonon interaction [38], since the electron stopping can be ignored as shown. The phonon decay rate in 3D is beyond the scope of this work: if the decay rate is faster than 10^{21} ($\mathrm{s^{-1}}$), the chain reaction might be impossible.

The pycnonuclear reactions are only detectable when $\rho \geqslant 1000\ \mathrm{g\,cm^{-3}}$. This dense condition might be achieved in the laboratory and maintained for a short period of time, at least in principle. The proposed scenario here is a volume ignition concept and does not rely on the creation of a hot spot [37]. We showed that the pycnonuclear reaction might be observable in a D–T pellet, the alpha particle produced from a fusion is mainly stopped by ions not by electrons so as to have longer mean-free-path than that calculated from the classical formula, and that explosive chain-reactions might be possible.

But the scenario is not economically feasible. The Fermi energy E_{F} is 140 keV, and the energy gain is only 100 even if we extract all energy from neutrons. This is quite small compared with theoretically possible gain of 1000 [37]. We note that $\rho = 10^6$ ($\mathrm{g\,cm^{-3}}$) is prohibitive in present compression technology due to the limitation of the laser (ion) beam power and the uniformity requirement [39,40]. Furthermore, it is not even clear whether it is theoretically possible to achieve such a dense and cold condition. Let us assume that we start from $T_0 = 300$ K and $\rho_0 = 1\ \mathrm{g\,cm^{-3}}$. For an isentropic compression, we can compute theoretical lower-bound of the final temperature T_1 from $S(\rho_0, T_0) = S(\rho_1, T_1)$, where $\rho_1 \cong 10^6\ \mathrm{g\,cm^{-3}}$. Using free-energy formula from [41] and from $S = (E - F)/T$, we can estimate that $T_1 \cong 250$ eV. Therefore, the theoretical lower-bound of temperature which can be achieved may well be greater than a few eV. To overcome this, we might start from a very cold pellet or hope that some dissipation such as bremsstrahlung will occur during compression so that the temperature will be lower. It may be possible that by mixing the fuel with high Z impurity, the bremsstrahlung losses can be considerable with the assumption that the confinement time is on the order of 0.1 ps.

However, the chain reaction scenario presented here is nonetheless interesting in a several aspects. Firstly, this study might be relevant to the burning of D–He-3. D–He-3 has the Gamow-peak energy four times larger than D–T plasma, and by increasing the density by eight times ($\rho = 10^7\ \mathrm{g\,cm^{-3}}$), the fuel will have a similar reaction rate with D–T treat here, and chain-reactions are then possible according to our study. The bremsstrahlung losses in D–He-3 fuels are detrimental to the burning possibility [42–44], but, in pycnonuclear regime, the radiation is greatly reduced because the plasma is very cold. The aneutronic fuel is advantageous because it is cleaner than D–T.

Secondly, the aneutronic fusion scenario in [46] has a few technological problems. One of them is the creation of a hot spot, for which a fusion–fission hybrid concept has been proposed [45]. According to our scenario, D–D or D–T fuel instead of uranium can be used at the center of the pellet without worrying about the initial hot spot.

Thirdly, a new reactor concept has been proposed recently [2,3,13–15]. With $\rho = 200\ \mathrm{g\,cm^{-3}}$ and $T \cong 0.3$ eV, P–D reactions are claimed to be greatly enhanced due to the electron screening and the ion–ion correlation. The cost of the scheme is much smaller than that of the conventional fusion scheme. While we note that there is a controversy about this scheme [2,13], their conclusions are not obvious to us since the ion–ion correlation was treated by us rather conservatively. However, the ion–ion correlation can only change on the time scale $\omega = \sqrt{4\pi n_i e^2/m_i}$, while the alpha particle is within the ion vicinity for a much shorter time, and thereby, the catalytic mechanism proposed here apparently exists in their regime. Furthermore, in such a regime, the wake plasmon by an alpha particle [27,47] might enhance the rate furthermore by a photon absorption process [48,49]. Therefore, if their concept [3] is feasible, the mechanism treated here and others mentioned will ease their severe physical condition.

Lastly, we would like to comment many body effect. We neglect 3-body interactions or higher order cluster effects. It is hard to give a rigorous estimate for how this consideration change the picture we describe. In this strongly coupled plasma, three body and four body interactions are not smaller than two body interactions. However, the fusion reaction occurs mainly between nearest neighbors, so that the enhancement we calculated should be the largest contribution.

To estimate the next order correction, there are three contributions: the first is the effect of neglecting the neighboring nuclei in considering how much energy is imparted by the alpha particle to the fusing nuclei. The second is the effect of the neighboring nuclei on the fusion process. The third is the effect of energy imparted to the neighboring nuclei by the alpha particle, and the subsequent transfer of that energy in the form of an impulse to the fusing nuclei.

For the first, the time scale of the energy transfer from an alpha particle to the fusion nuclei is less than 10^{-18} s. This time is faster than any other time scale in our domain, and the energy deposition from an alpha particle to the fusion nuclei is instantaneous. We can assume then that other ions hold their equilibrium position during this time. We do include the influence of the nearest neighbor ions for this process. More refined theory in the case of equilibrium, including all surrounding particle except electron screening, has been used in [10]. Since the fusing nuclei need move much less than an inter-particle spacing in enhancing the fusion rate, the question is how much are they influenced by the plasma surroundings. Here, it is irrelevant what the plasma parameter is—since the same question can be asked at zero temperature—namely, what is the change in potential for nearby particles in a strongly coupled plasma to move closer or further from each other. Since the plasma is neutral, the potential contribution of the further neighbors is balanced by electron contributions and can be ignored.

For the second effect, the effect on the fusion process itself is already assumed by taking the pycnonuclear rate. Though a more refined theory than ours, with inclusion of all the ions, has been done as in [10], the exact treatment with inclusion of electron screening has not been done to the authors' knowledge. Our treatment is good for qualitative description.

For the third effect, the energy that an alpha particle deposits to the neighboring ions is small. Compared with the energy E_p deposited to the closest ion, the energy from the alpha particle to one neighbor E_n can be estimate as

$$\frac{E_n}{E_p} \cong \frac{s^2}{2}\frac{1}{\log(b_c/as)}\left(\frac{a}{d}\right)^2 \cong 0.003\left(\frac{a}{d}\right)^2, \qquad (21)$$

where a is the inter-particle spacing, d is the closest distance between the neighbor and the alpha particle, $b_c = e^2/E_{\text{alpha}}$ is the closest approach of the alpha particle, and $s \cong 0.25$ is a dimensionless parameter defined in Section 2. The fact that E_n is only 0.1% of E_p suggests that there is very little free energy arising from the nearest neighbors to move the closest fusing particles even closer. The neighbor particle, after it obtains the energy E_n from the alpha particle, sheds its energy through phonons. Assuming that the energy will spread as a spherical wave, the fusion pair can get, at best, $E = (as/d)^2 E_n$ since the area of their wave packet is $(as)^2$ of the sphere with the radius d. This suggests that the energy which will be deposited into the fusing pair via a neighboring ion from the alpha particle will be at most $E = 2.0 \times 10^{-4}(a/d)^4 E_p$. Compared with E_p, this is miniscule, and the further enhancement of the fusion reaction by this perturbation is likely small.

7. Conclusion

We show that, in an ultra dense D–T plasma with $\rho = 10^6$ $(\mathrm{g\,cm^{-3}})$, the pycnonuclear reaction might be observable in the laboratory although it is not yet clear whether such a dense and cold condition can be achieved. We also show that the local field correction and relativistic correction increase the rate by 40%. We also predict a chain reaction regime.

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