Information Content of Transient Synchrotron Radiation in Tokamak Plasmas

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A brief, deliberate perturbation of hot tokamak electrons produces a transient synchrotron radiation signal, in frequency-time space, with impressive informative potential on plasma parameters; for example, the dc toroidal electric field, not available by other means, may be measurable. Very fast algorithms have been developed, making tractable a statistical analysis that compares essentially all parameter sets that might possibly explain the transient signal. By simulating data numerically, we can estimate the informative worth of data prior to obtaining it.

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Suppose that superthermal electrons are heated briefly in a tokamak plasma. The change in the electron distribution function, particularly at high energy, is manifest in a change, or increment, in the synchrotron emission. Since the excitation is brief, the changes incurred in both the electron distribution function and the accompanying synchrotron emission are transient. Thus, the *incremental* synchrotron radiation is a two-dimensional pattern $R(\omega,t)$ in frequency-time space. The details of this pattern are governed by plasma parameters; for example, the higher the plasma density, the faster the decay of the incremental radiation. Our problem is to deduce these parameters by viewing the transient radiation.

The use of synchrotron emission to deduce plasma properties is an established and important technique. Generally, the emission is used for information on the electron temperature; recently there have been attempts to uncover further details of the electron momentum distribution function f.[1-7] An one-dimensional f was deduced elegantly in a relativistic electron ring geometry.[8] In these studies, the deduction of details of the electron distribution function was based on the synchrotron emission from the entire distribution of electrons; consequently, only one-dimensional data (in frequency) could be used to constrain f. Other studies have recognized some utility in transient radiation.[9,10]

Here, we explore the consequences of deliberate, brief heating of the plasma (e.g., by lower-hybrid waves) to produce radiation directly attributable to this probe. Although requiring the burden of the initial perturbation, the transient 2D radiation response is far more informative. An example of this response, shown in Fig. 1, exhibits radiation at several harmonics from electrons initially with about 700 keV parallel energy, or tail electrons in a 20-keV reactor plasma. In Fig. 1(a) the parallel dc electric field corresponds to 0.02 V/m at density $10^{14}/\mathrm{cm}^3$; in Fig. 1(b) it is -0.0067 V/m. The electric fields here are easily distinguishable by their radiation response; the challenge, however, is to make far finer discriminations, in the presence of noise, and when several parameters are simultaneously unknown.

The parameters that might be inferred from the radiation response include the effective ion charge state Z_{eff}, the direction of the magnetic field, the position and

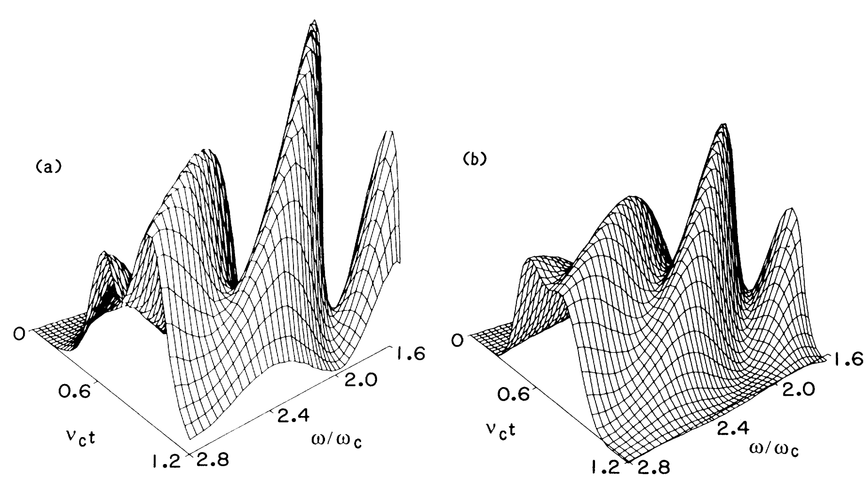

FIG. 1. Radiation response $R(\omega,t)$ (arbitrary units) at extraordinary polarization for different electric fields. (a) $\mathcal{E}=0.3$. (b) $\mathcal{E}=-0.1$.

width in velocity space of the brief heating, and the dc parallel electric field E. The inference of details of the heating pulse has been considered previously.[11] Knowing the viewing angle is the same as knowing the current profile, since, with the toroidal magnetic field given, the poloidal magnetic field is deducible from the viewing angle. The current profile and the ion-charge-state profile, both of interest in tokamak experiments, are already measured through other means, but further resolution using the synchrotron emission is useful.

If is of particular interest here, however, to deduce the dc parallel electric field, something entirely unavailable otherwise. Typically less than a volt per meter in a tokamak, this field is far too small to be inferred through atomic phenomena, and cannot be measured directly by probes because the plasma is too hot. Its effect is manifest, however, in the dynamics of superthermal electrons, exactly those that synchrotron radiate most profusely. It turns out that both E and the other parameters of interest can be deduced almost *orthogonally*; i.e., ignorance or even misinformation concerning some parameters does not impair significantly the inference of other parameters. The radiation response changes in very different ways when different parameters are varied. It is a conclusion of this work that simultaneous inferences can be made with surprising success.

We exploit here a fortuitous separation of time scales $1/\omega \ll \tau_{\text{det}} \ll \tau_c \ll \tau_{\text{par}}$. From the first inequality we have that the radiation frequency ω ($\sim$300 GHz) is sufficiently characterized on the instrumental detection time scale of τ_{det}, which can be 50 μs. (It is a similar separation of time scales that makes possible the 2D frequency-time speech spectrogram used in the machine recognition of human speech.) The amount of processible information that would be available in a nontransient analysis is multiplied by τ_c/τ_{det} (typically 10^2–10^3), the number of detector observations in a slowing down time, τ_c, of a superthermal electron. The parameters change on the longer time scale τ_{par}, so that their values may be treated as constant during the transient analysis, with the opportunity, by repeating the probe, to average the results of several transient analyses.

Why is there this great multiplication in the amount of processible information? To be sure, any frequent measurement of the radiation records a great deal of data, but unless something is liable to change on that time scale, those data are redundant. It would not be informative to measure temperature (which governs the background radiation) every 50 μs, were the temperature already known to change only on the time scale of a second. Only by producing a transient signal, do we endow the time measurements with informative potential.

Perturbing the high-velocity, superthermal electrons has many advantages: These electrons synchrotron radiate most copiously, but lose energy slowly, so that there can be more independent time points in the radiation pattern $R(\omega,t)$. Dominated by Coulomb collisions and the dc electric field, these electrons mainly flow along the magnetic field, largely immune to temperature fluctuations and other turbulence in the bulk of the ion or electron distributions. This understanding of the dynamics of fast electrons has received considerable experimental verification.[12-16] What makes the inverse problem of determining parameters from the radiation response of fast electrons tractable is that relatively few parameters govern this response, and powerful analytic tools exist for finding the response given the parameters.

The incremental or transient radiation response is defined as $R(\omega,t;\theta) \equiv R_{\text{tot}}(\omega,t;\theta) - R_{\text{back}}(\omega,t;\theta)$, where R_{back} is the background radiation associated with a relatively constant distribution function and R is the incremental radiation specifically due to an externally imposed impulsive momentum-space flux $\mathbf{\Gamma}(\mathbf{p},t)$. We can then write the distribution function f as $f = f_M(1+\phi_B+\phi)$, where f_M is a Maxwellian distribution, ϕ_B describes the relatively constant deviation from Maxwellian of the background distribution, and ϕ describes the time-dependent distribution specifically associated with the source $\mathbf{\Gamma}$. For problems of interest, in terms of contributing to the collision integral, both ϕ_B and ϕ may be treated as small, so that f obeys the linearized Fokker-Planck equation. The evolution of ϕ is then governed, after the brief excitation, by Coulomb collisions and the dc electric field,

$$f_M\,\partial\phi/\partial t + q\mathbf{E}\cdot\nabla_{\mathbf{p}} f_M\phi - C(\phi) = 0\,, \qquad (1)$$

with initial condition $f_M\phi(\mathbf{p},t=0) = Q(\mathbf{p})$, which is the result of the impulse $\mathbf{\Gamma}$. The incremental or transient radiation response, viewed at angle θ with respect to the magnetic field, is then

$$R(\omega,t;\theta) = \int d^3p\, f_M\phi(\mathbf{p},t) I(\omega,\mathbf{p};\theta)\,, \qquad (2)$$

where the radiation intensity I can be of ordinary or extraordinary polarization; for the latter, $I = I^X$, we have

$$I^X(\omega,\theta,\mathbf{u}) = \sum_n \frac{e^2\omega^2}{2\pi c\lambda^2}\left(\frac{u}{\gamma}\right)^2 (1-\mu^2) J_n'^2\left(n\frac{u}{\gamma}(1-\mu^2)^{1/2}\frac{\cos\theta}{\lambda}\right)\delta(\omega - n\omega_c/\gamma\lambda)\,, \qquad (3)$$

where n is the cyclotron harmonic, J_n' is the derivative of the nth Bessel function of the first kind, $\omega_c = eB/mc$ is the cyclotron frequency of nonrelativistic electrons, $\mathbf{u} = \mathbf{p}/mc$, $\gamma^2(u) \equiv 1+u^2$, $\mu \equiv p_\parallel/p$, and $\lambda = 1 - u\mu\sin\theta/\gamma$ is the extent of the Doppler shift through viewing the radiation at angle θ.

Very fast algorithms have been developed for solving for the radiation response $R(\omega,t)$. The fast algorithms, which make feasible a statistical analysis that would otherwise be unthinkable, exploit several properties of Eqs. (1)–(3).

First, note that Eqs. (1)-(3) admit several scale-invariant transformations of the radiation response $R(\omega,t)$. Having solved for $R(\omega,t;\Theta)$, where Θ is a set of parametric dependences which includes the magnetic field amplitude B, the electric field E, the density n, and the perturbation amplitude Q, we also have for any constants α_1, α_2, and α_3,

$$R(\omega,t;\alpha_1 B,\alpha_2 Q,\alpha_3 n,E) = \alpha_1\alpha_2 R(\omega/\alpha_1,t/\alpha_3;B,Q,n,E/\alpha_3)\,. \quad (4)$$

The impulsive heating can be arranged to affect only nonrunaway electrons, so that Eq. (4) simplifies further through the linearization $R=R_0+ER_1$.

Second, note that since Eq. (1) is linear in ϕ, a Green's function, ψ, for the radiation response can be defined. We write the radiation response as an integral over initial condition $Q(\mathbf{p})$,

$$R(\omega,t;\theta)=\int d^3u\, \psi(\omega,\mathbf{p},t;\theta)Q(\mathbf{p})\,. \quad (5)$$

The Green's function makes efficient the simultaneous consideration of many perturbations $Q(\mathbf{p})$.

Third, choosing to perturb electrons on the tail of the distribution function, superthermal but not runaways, makes possible an analytic solution for ψ. For these electrons, energy diffusion by collisions is ignorable compared to energy loss. The Green's function for the radiation response, ψ, solves the relativistic Fokker-Planck adjoint equation,[17] which we write as

$$\frac{\partial\psi}{\partial\tau}-\mathscr{E}\frac{\partial\psi}{\partial u_\parallel}+\frac{1}{u_3}\left[\gamma^2 u\frac{\partial\psi}{\partial u}-\gamma\frac{1+Z_{\rm eff}}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu}\psi\right]=0\,, \quad (6)$$

written for superthermal excitation in the high-velocity limit, and in terms of the normalized variables $\tau=\nu_c t$, $\nu_c=nq^4\ln\Lambda/4\pi m^2\epsilon_0^2c^3$, and $\mathscr{E}=qE/mc\nu_c$, and to be solved with the following initial condition $\psi(\omega,\mathbf{u};\theta,\tau=0)=I(\omega,\mathbf{u};\theta)$.

An analytic solution is available as follows: Separate ψ and the initial conditions into Legendre harmonics $[\psi(u,\mu,\tau)=\sum_k P_k(\mu)\psi_k(u,\tau)]$, expand in the electric field $[\psi_k(u,\tau)=\psi_k^{(0)}+\mathscr{E}\psi_k^{(1)}+\cdots]$, and then integrate the equation for $\psi_k^{(0)}$ along characteristics to obtain[11]

$$\psi_k^{(0)}=I_k(x)\left[\frac{1+\gamma(u)}{u}\right]^{\alpha_k}\Bigg/\left[\frac{1+\gamma(x)}{x}\right]^{\alpha_k}, \quad (7)$$

where $\alpha_k\equiv k(k+1)(Z_{\rm eff}+1)/2$, and the characteristic function $x(\tau,u)$ can be written as $x=g^{-1}[g(u)-\tau]$, with $g(u)\equiv u-\tan^{-1}u$; g^{-1} is defined such that $g^{-1}[g(u)]=1$. The equation governing $\psi_k^{(1)}$, to be solved with homogeneous initial conditions, is driven by the kth Legendre harmonic of $\partial\psi^{(0)}/\partial u_\parallel$; fortunately, this inhomogeneous term can be simplified enormously so that $\psi_k^{(1)}$ can be put into an efficient closed form.

These fast algorithms enable us to consider essentially all competing parameter sets that might possibly explain our obtained data. More than that, we can estimate the worth of data prior to obtaining it. Suppose that experimental measurements are of the following form $R_x(\omega,t)=R(\omega,t)+\tilde{R}(\omega,t)$, where the extraneous signal $\tilde{R}(\omega,\tau)$ is Gaussian noise, uncorrelated in both frequency and time, with $\langle\tilde{R}\rangle=0$ and $\langle\tilde{R}^2\rangle=\sigma^2$. Given this model for data generation, and given a set of plasma parameters $\{\Theta\}$, we can express the probability $P(R_x\,|\,\Theta;\sigma)$ of generating a specific data set R_x in the presence of noise characterized by σ. Given an *a priori* distribution $P(\Theta)$ for the parameter set $\{\Theta\}$, by Bayes's theorem we can write $P(\Theta\,|\,R_x;\sigma)=P(R_x\,|\,\Theta;\sigma)P(\Theta)/P(R_x)$. The probability distribution of the plasma parameter set $\{\Theta\}$, given that the data were obtained in the presence of noise σ and generated with the specific plasma parameter set $\{\Theta_p\}$, can now be written as

$$P(\Theta\,|\,\Theta_p;\sigma)=\sum_{\{R_x\}}P(\Theta\,|\,R_x;\sigma)P(R_x\,|\,\Theta_p;\sigma) =\lim_{N_R\to\infty}\frac{1}{N_R}\sum_{j=1}^{N_R}P(\Theta\,|\,R_x^{(j)};\sigma)\,, \quad (8)$$

where, in the first equality, the summation over all possible data sets $\{R_x\}$ is both unfeasible and, in practice, unnecessary; the second equality obtains, since, by construction, $P(\Theta\,|\,R_x;\sigma)$ is sampled with probability $P(R_x\,|\,\Theta_p;\sigma)$. Generally $N_R\sim 80$ suffices to approximate $P(\Theta\,|\,\Theta_p;\sigma)$. Of course, the fast algorithms for generating $R(\omega,t)$ are indispensable, since R must be obtained for each competitive data set.

Carrying out a program of examining $P(\Theta\,|\,\Theta_p;\sigma)$ with various sets of plasma and heating parameters unknown, we find that the *a priori* probabilities $P(\Theta)$ can be improved upon meaningfully. To taken an example of particular interest, consider the simultaneous viewing of radiation from the core periphery of a tokamak, where in a coarse model, the two regimes have, respectively, densities n_c and n_p, and electric fields $\mathscr{E}_c$ and $\mathscr{E}_p$. In other relevant respects, such as viewing angle or ion charge state, the two regimes are presumed identical. One detector then sums

$$R(\omega,t)=Q_cR(\omega,t;n_c,\mathscr{E}_c)+Q_pR(\omega,t;n_p,\mathscr{E}_p)\,,$$

where Q_c, Q_p, $\mathscr{E}_c$, and $\mathscr{E}_p$ are assumed unknown, but n_c and n_p are known from other measurements. Of course, were $n_c=n_p$, there would be no distinguishing the radiation source. However, even a 10% variation in density is exploitable. As shown in Fig. 2, the marginal probability distribution $P(\mathscr{E}_c,\mathscr{E}_p)$ (the joint probability summed over all $\{Q_c,Q_p\}$) reveals the true parameters $\mathscr{E}_c=0.08$, $\mathscr{E}_p=0$, i.e., a loop voltage on axis not yet relaxed via

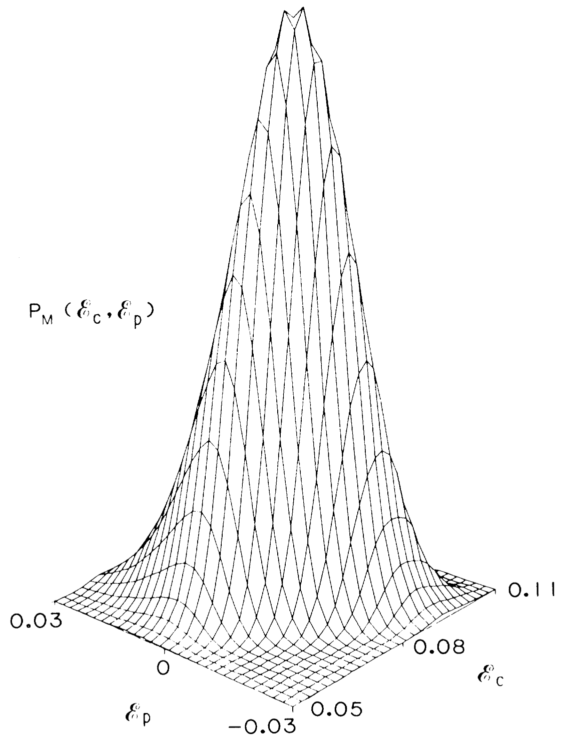

FIG. 2. Marginal joint probability distribution of $\mathcal{E}_c$ and $\mathcal{E}_p$, given the radiation response. The *a priori* distribution was flat over the 5466 data sets considered.

magnetic diffusion. Here, data were simulated on a 40×40 grid in frequency-time space, with noise σ of 10% of the maximum signal $R(\omega,t)$. In practice, purely experimental noise can be kept much lower and a larger differential in density makes this discrimination much easier.

The model that we employ can be improved upon in several ways, particularly in accounting for cross-field transport due to imperfect magnetic surfaces.[18,19] Accounting for losses of the fast electrons[20] can probably be done analytically by introducing only a few new parameters; the fast algorithms should remain useful and the inference problem should remain tractable. Of course, in many instances the model as presented may suffice.

In summary, the relatively modest diagnostic system that we propose includes both the brief, probing rf signal that leads to the incremental synchrotron signal, and an array of frequency detectors with submillisecond time resolution. In this purposefully *constrained* problem, a great deal of data yield information on but a few choice parameters, and powerful analytic tools make feasible a numerical analysis of data that would otherwise be unthinkable. The information obtainable is novel, reliable, and likely quite useful.

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