Ultrahigh intensity laser–plasma interaction: A Lagrangian approach*

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A Lagrangian analysis simplifies considerably the description of the ultrahigh intensity regime of laser–plasma interaction, and facilitates the identification of several new and important effects. First, the vacuum figure "8" orbit is shown to be unstable, with respect to a stochastic instability leading to collisionless heating. Second, in the generation of plasma wakes using ultrahigh intensity laser pulses, it can be shown that, for long-duration laser pulses, the plasma wake is insignificant, but, through proper phasing of a set of short-duration pulses, a dramatic amplification of the wake amplitude occurs. Third, in the generation of third-harmonic waves using ultrahigh intensity, long-duration laser pulses, it can be seen that a mismatch in the phase velocity limits severely the power conversion, but a conversion efficiency free from saturation might indeed be possible by employing either a density-modulated plasma or an ionized buffer gas.

I. INTRODUCTION

The basic physical processes involved in laser–plasma interaction, up to 10^{17} W/cm^2, are now well understood; on the other hand, a large number of fundamental issues remain open in the study of the ultrahigh intensity (UHI) relativistic interaction regime. Recent advances in pulse compression now make possible the exploration of laser–plasma interactions in this UHI (above 10^{18} W/cm^2) regime.[1]

In this new regime of laser intensity, the quiver velocity of the electrons is relativistic, so that both the single-particle and collective responses become nonlinear functions of the incident field. This nonlinearity has been identified as the source of new nonlinear, collective,[2] and single particle[3] processes.

The nonlinearity parameter of an electromagnetic transverse wave, with vector potential A, is eA/mc, the normalized quiver momentum of the electron, where c is the speed of light, e and m the electron charge and mass. When eA/mc reaches one, the electric field can accelerate an electron up to its rest energy in one wavelength.

When an UHI laser pulse interacts with a plasma, three time scales are involved, the pulse is characterized by its mean frequency, ω, and its frequency width, $\delta\omega$. The plasma is fully characterized by the plasma frequency, ω_p. Apart from relativistic effects, there are then three regimes to consider: (i) $\omega_p<\delta\omega<\omega$, (ii) $\omega_p\sim\delta\omega<\omega$, and (iii) $\delta\omega<\omega_p<\omega$; furthermore, because of the relativistic increase of the electron mass, the effective plasma frequency is in fact decreased, so that we should also consider a fourth regime, (iv) $\delta\omega<\omega<\omega_p$.

In this paper, we describe and analyze certain basic collective processes relevant to the UHI interaction in the so called short pulse (i) and long pulse (iii) regimes in which nonuniformity transverse to the wave-vector direction is irrelevant. The validity of this one-dimensional (1D) model[2] requires the transverse size of the laser pulse to be larger than c/ω_p. The physical interpretation of these two interactions regimes is illustrated in Figs. 1 and 2. These space-time diagrams, introduced in Ref. 4, are particularly convenient in analyzing, for UHI interactions, the processes which take place in the direction of propagation of the pulse.

In Fig. 1, in the short pulse case when the pulse passes by an electron, the relativistic ponderomotive deflection displaces the electron in the direction of the pulse propagation. But, because the pulse is so short, the plasma does not have enough time to set up a collective response, so that the electron behaves nearly as an electron interacting with an UHI pulse in vacuum. Nevertheless, as a result of the ponderomotive displacement, the plasma does set up a collective response in the form of an electrostatic wake behind the pulse.

In the long pulse regime, depicted in Fig. 2, the situation is completely different. In this regime, each electron is displaced in the direction of the pulse as the pulse passes it by, but, after a short transient response (not represented in Fig. 2), a nonlinear oscillation, driven by the wave, and modulated by the plasma collective effects, is set up. In this regime, both the energy and the momentum transferred from the pulse to an electrostatic wake behind the pulse turns out to be very small, except if a nonadiabatic process, such as an ionization or an instability, takes place. Although wake generation is negligible, phase-matched harmonic generation becomes interesting and will be considered in Sec. V.

The interesting effects in the UHI regime which have been predicted and described in the literature, such as

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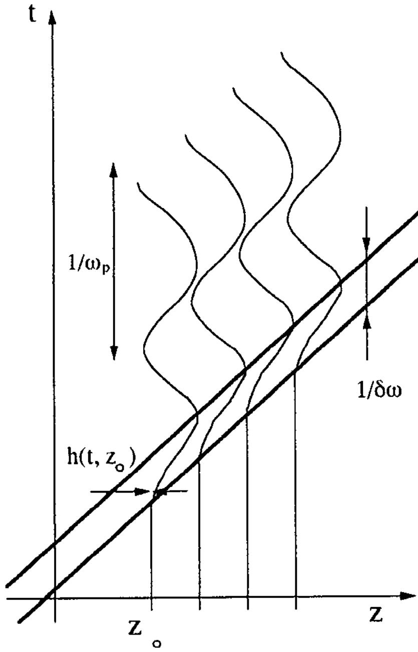

FIG. 1. Space-time orbit in a circularly polarized UHI short pulse.

wake generation and harmonic generation,[2] are collective processes. But, in order to understand laser–plasma interaction at intensities above 10^{18} W/cm^2, the very first issue logically to address is the stability of the nonlinear single particle motion in the field of an UHI laser wave. We will carry out this task in Sec. III and show that, besides the adiabatic regime leading to the ponderomotive force, a 2-D wave can display a large number of new resonances called Compton resonances identified in Ref. 3. Above a stochasticity threshold, which can be easily fulfilled, these resonances can overlap and the electron orbit becomes chaotic.

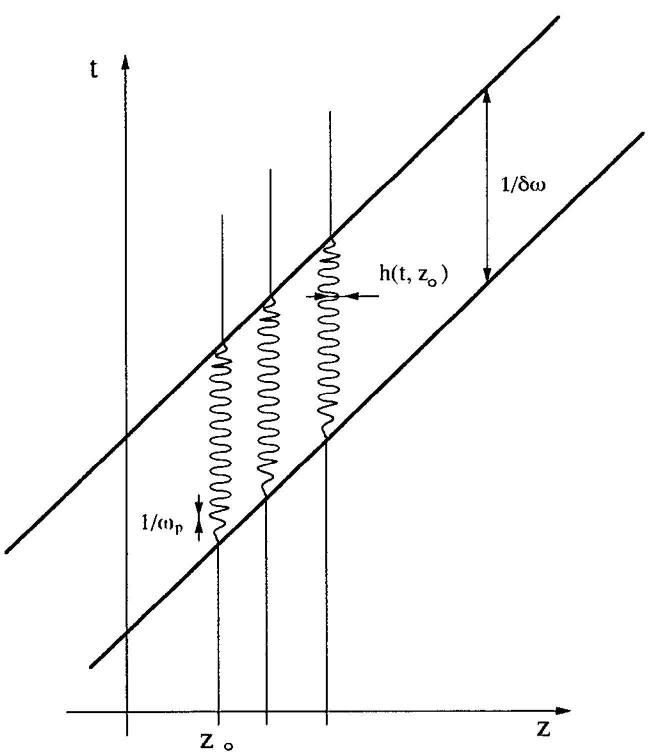

FIG. 2. Space-time orbit in a linearly polarized UHI long pulse.

To study phase-matched harmonic generation, wake generation and amplification, and relativistic stochastic heating, we use a single theoretical framework relevant both to the UHI single particle and collectives processes: A relativistic Lagrangian description of the electron response.

In the next section, we set up this formalism, and we clarify the relationship to the Eulerian quasistatic description introduced in Ref. 2. Throughout this paper, except when needed for clarity, we use $e=m=c=\omega=1$.

II. LAGRANGIAN DESCRIPTION

Consider an UHI laser pulse, linearly polarized along the x axis and propagating along the z axis, in an infinite homogeneous plasma. Each electron is labeled by its unperturbed position, z_0, x_0, and, as a result of the interaction with the pulse, it describes an orbit $x(t,z_0,x_0)$, $h(t,z_0,x_0)=z(t)-z_0$. This orbit is the solution of the Lorentz's equations:

$$\begin{aligned}\frac{d\mathbf{p}}{dt}&=\frac{\partial \mathbf{A}}{\partial t}-\mathbf{v}\times\left(\frac{\partial}{\partial \mathbf{r}}\times\mathbf{A}\right)-\omega_p^2 h,\\ \frac{d\gamma}{dt}&=\mathbf{v}\cdot\frac{\partial \mathbf{A}}{\partial t}-\omega_p^2 h\cdot\frac{dh}{dt},\end{aligned}\tag{1}$$

where $\mathbf{p}$, $\mathbf{v}$, and γ are, respectively, the momentum, velocity, and energy of the electron, and $\mathbf{A}(t,z)$ is the vector potential of the laser pulse. The last term on the right-hand side of both equations is a restoring force, arising from the application of Gauss' theorem to the perturbed electron density.[5,6] This electrostatic force describes the collective plasma response to all orders and avoids the use of a scalar potential in this Lagrangian representation. Equation (1) can be solved either numerically[7] or analytically.[4] Then, given the Lagrangian displacement, we calculate the Eulerian electron density,

$$n(t,z)=\omega_p^2\int dz_0\,\delta[z-z_0-h(t,z_0,\mathbf{A})].\tag{2}$$

To make the description self-consistent, we use the conservation of the transverse canonical momentum, $\gamma\, dx/dt=dx/d\tau=\mathbf{A}$, where τ is the proper time, to obtain, with the Lorentz gauge,

$$\frac{\partial^2\mathbf{A}}{\partial z^2}-\frac{\partial^2\mathbf{A}}{\partial h^2}=\omega_p^2\mathbf{A}(z,t)\int dz_0\frac{\delta[z-z_0-h(t,z_0,\mathbf{A})]}{\gamma(t,z_0,\mathbf{A})}.\tag{3}$$

Equations (1) and (3) are a closed, self-consistent description of the 1-D UHI interaction. The integral on the right-hand side of Eq. (3) can be performed, and we are led to the evaluation of the inverse of $\gamma(1+\partial h/\partial z_0)$. All the electrons have the same orbit translated in space and time. This translation accommodates the delay between their excitation, as depicted on Figs. 1 and 2, namely,

$$z=z_0+\int_0^{\tau(t,z_0)}p(u)du,\quad t=\frac{z_0}{V}+\int_0^{\tau(t,z_0)}\gamma(u)du,\tag{4}$$

where V is the slope of this space-time translation, and $p=dh/d\tau$. Near the front of the pulse, $V=c$; in the bulk, V

is to be calculated self-consistently with Eq. (3). Differentiating z and t, with respect to z_0, we finally obtain $\gamma(1+\partial h/\partial z_0)=\gamma-p/V$. This latter identity shows the connection between this Lagrangian method and the Eulerian quasistatic description.[2]

If we neglect the collective term in Eq. (1), we can solve[8] this equation for an arbitrary laser pulse $A(t-z)$, to get the exact motion

$$x(\tau)=\int_0^\tau A(u)du,$$
$$h(\tau)=\frac{1}{2}\int_0^\tau A^2(u)du,\quad t(\tau)=\tau+\frac{1}{2}\int_0^\tau A^2(u)du. \tag{5}$$

It is evident from this exact solution that as the pulse passes by the particle, no net exchange of energy and momentum between the wave and the particle takes place. The remaining effect of the passing wave packet on the electron, after it has passed, is only a spatial displacement of the electron in the direction of the wave propagation.

The situation is completely different in the long pulse regime, where the motion within the pulse, rather than just the net effect of the pulse, now assumes importance. Suppose a vector potential of the form $A=A(t-z)\cos(t-z)$, with $A(t-z)$ is now an envelope varying on a time scale much slower than ω_p^{-1}. Apart from insignificant, short, transient effects when the pulse first encounters, or ceases to encounter the electron, the electron oscillates in the wave with no drift, since the wave envelope is smoothly varying. The oscillatory solution, without drift, is the well-known figure 8 vacuum orbit.[9] To obtain this solution the invariant of the motion $\gamma-p$ which we set equal to 1 in Eq. (5) is now given by $\gamma-p=M=\sqrt{1+A^2/2}$ the effective mass of the electron;

$$x(\tau)=\frac{A}{M}\sin[M\tau(t)],\quad h(\tau)=\frac{A^2}{8M^2}\sin[2M\tau(t)],$$
$$t(\tau)=M\tau+\frac{A^2}{8M^2}\sin[2M\tau(t)]. \tag{6}$$

The key to describing the various plasma responses in the UHI regime is to use the exact motion, Eqs. (5) and (6), as a starting point for an $\omega_p/\delta\omega$ expansion in the short pulse regime[4] or for an $\omega^2_p/A\omega^2$ expansion in the long pulse one.[10]

III. ADIABATICITY, RESONANCES, AND CHAOS

Consideration of the single particle motion in UHI laser fields logically precedes the more complex motion that involves collective effects. The motion of an electron in a one dimensional, infinite, linearly polarized UHI wave, depicted on Fig. 3, is the combination of a drift and a nonlinear oscillation. For an inhomogeneous two dimensional UHI wave, what has been treated so far in the literature is only the "adiabatic" regime, i.e., the regime in which the particle motion is not stochastic in the wave. So even before addressing collective plasma effects, one must inquire

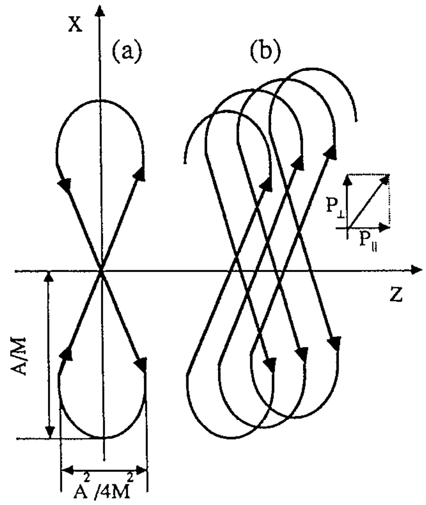

FIG. 3. Orbit in a 1-D linearly polarized UHI wave.

about the stability of this drifting figure 8 motion, which, in fact, may not itself be stable, with respect to electromagnetic and electrostatic perturbations.

The uniform drift translation is described by a parallel momentum, $P_\parallel$, in the wave propagation direction, z, a perpendicular momentum, $P_\perp$, in the polarization direction, x, and a relativistic average energy, E. As shown in Ref. 3, these variables provide a simple set of actions on which to base a Hamiltonian analysis of the instabilities. If we consider time as an additional configuration variable, then the proper time becomes the new time associated with the extended phase space. The Hamiltonian, derived previously,[3] but now generalized to include electrostatic perturbations, becomes

$$H_0(\mathbf{r},t,\mathbf{P},-\gamma)=1+\mathbf{p}^2-(\gamma+\phi)^2$$
$$=1+[\mathbf{P}+(\mathbf{A}+\mathbf{a})]^2-[\gamma+\phi]^2, \tag{7}$$

where $\mathbf{a}$ is an electromagnetic perturbation, and ϕ a potential one, arising, for example, from the collective effects through space charge self-consistency, i.e., Eq. (2).

We introduce the actions $(P_\perp, P_\parallel, E)$, angles (θ,φ,ξ), and, to perform the canonical change of variables, we use a generating function S:$S(P_\perp, P_\parallel, E,x,z,t)=P_\parallel x+P_\perp y-Et-P_\perp A/P_\parallel-E \quad \sin(z-t)-A^2/8P_\parallel-8E\sin(2z-2t)$. With the help of this generating function, the unperturbed Hamiltonian can be expressed in terms of these actions variables describing the drift of the figure 8 orbit:

$$H_0(P_\perp, P_\parallel, E)=M^2+P_\parallel^2+P_\perp^2-E^2. \tag{8}$$

To address the orbital stability problem, let us consider a set of harmonic, transverse, and longitudinal perturbations, $\mathbf{a}(\mathbf{r},t)=\mathbf{a}\sin(k_\parallel z+k_\perp y-\Omega t)$, and $\phi(\mathbf{r},t)=\phi\sin(k_\parallel z+k_\perp y-\Omega t)$. The Hamiltonian then becomes

$$H(\mathbf{r},t,\mathbf{P},-\gamma)=H_0+2\mathbf{P}\cdot\mathbf{a}(\mathbf{r},t)+2\mathbf{A}(\mathbf{r},t)\cdot\mathbf{a}(\mathbf{r},t)$$
$$+2\gamma\phi(\mathbf{r},t). \tag{9}$$

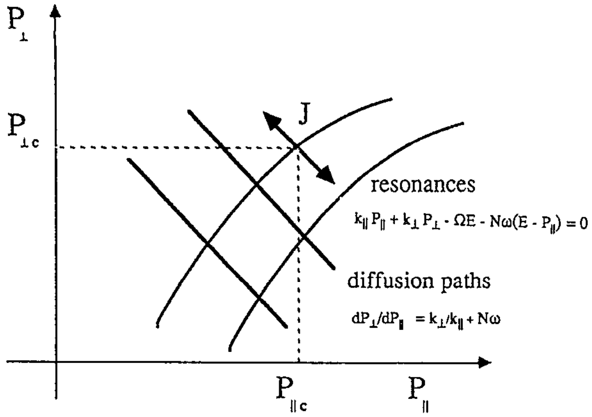

FIG. 4. Resonances and diffusion paths in action space.

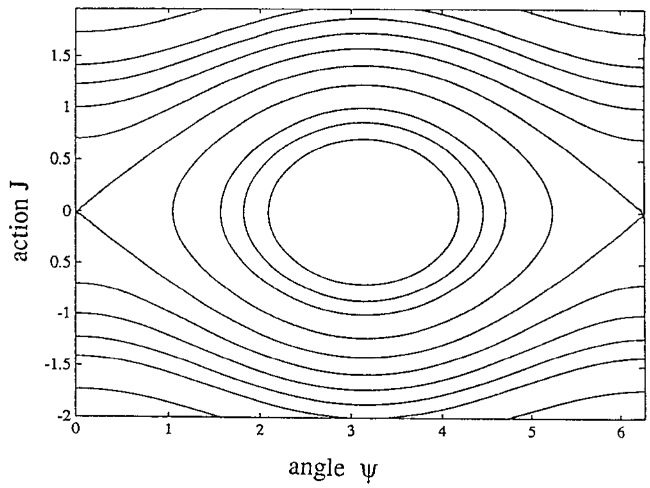

FIG. 5. Angle-action orbit near a nonlinear Compton resonance.

We can express this Hamiltonian in terms of the actions, angles variables, and after some algebra, the final result can be in terms of a sum of harmonic interactions weighted by a combination of Bessel functions, i.e., $H(P_\perp, P_\parallel, E, \theta, \varphi, \phi) = H_0(P_\perp, P_\parallel, E) + \Sigma_N V_N \cos[k_\parallel \varphi + k_\perp \theta + \Omega\xi + N\omega(\varphi+\xi)]$, where the sum is to be taken over all integers. It may be shown that $V_N \alpha a \alpha \phi$, and that the coupling coefficient can be put in terms of generalized or ordinary Bessel functions. If we plug the unperturbed motion into the argument of the perturbating cosines, some small resonant denominators appear as a result of the occurrence of a stationary phase. Therefore, it is necessary to consider more carefully the dynamics of the electron in the vicinity of the Compton resonances,

$$k_\parallel P_\parallel + k_\perp P_\perp - \Omega E - N\omega(E - P_\parallel) = 0. \tag{10}$$

Consider a point $(P_{\parallel c}, P_{\perp c}, E_c)$, which fulfills this resonance condition [if $\omega \approx \Omega$, $k_\parallel \approx k_\perp$, and $N \approx O(1)$, this gives resonant energies of the order of few MeV] and assume that the Nth resonant interaction can be considered without the influence of other resonances. Near $(P_{\parallel c}, P_{\perp c}, E_c)$ in the action space depicted on Fig. 4, the motion takes place along the direction of a reduced action $J = (P_\parallel - P_{\parallel c})/k_\parallel + N\Omega$. It turns out that this action is conjugate to the angle $\psi = k_\parallel \varphi + k_\perp \theta + \Omega\xi + N\omega(\varphi+\xi)$, so that this resonant motion is described by the equations

$$\begin{aligned} \frac{dJ}{ds} &= aV_N(P_{\perp c}, P_{\parallel c}, E_c)\sin[\psi], \\ \frac{d\psi}{ds} &= [2(k_\parallel + N\Omega)^2 + 2k_\perp^2 - 2(\omega + N\Omega)^2]J. \end{aligned} \tag{11}$$

The phase portrait of this nonlinear oscillator is depicted in Fig. 5, where the coupling coefficient V_N is rescaled to one. This universal structure of a nonlinear resonance is completely characterized by the island half-width in action space, i.e., the distance between the separatrix and the center in Fig. 5, which may be calculated to be

$$\Delta J = \sqrt{2a|V_N|/|k_\perp^2 + (k_\parallel - \omega)(k_\parallel + \omega + 2N)|}.$$

On the basis of this formula, the Chirikov criterion[11] tells us that the electron response becomes chaotic when the sum of the half-width of two neighboring islands becomes larger than the distance between the resonances. Above this stochasticity threshold, the random phase approximation (RPA) can be applied to Hamilton's equations. This RPA results in a quasilinear kinetic equation describing the stochastic heating of the electron population which takes place along the diffusion paths

$$\frac{dP_\perp}{dP_\parallel} = \frac{k_\perp}{k_\parallel} + N\omega. \tag{12}$$

Thus, as a result of this heating, both transverse (**a**) and longitudinal (ϕ) perturbations are damped.

IV. WAKE GENERATION AND AMPLIFICATION

Nonlinear electrostatic plasma waves have recently attracted interest because of their ability to accelerate electron and photon.[12] In this section we will address two new issues related to wake generation, and in doing so, we will answer two important questions: First, why does a long smooth pulse not generate a wake? Second, how can the proper phasing of a set of short pulses be put at work to amplify wake generation?

For the long pulse case (Fig. 2) and the Eq. (1), the relevant equations are

$$\begin{aligned} \frac{d(\gamma - p)}{d\tau} &= \omega_p^2(\gamma - p)\cdot h, \\ \frac{d(t-z)}{d\tau} &= (\gamma - p), \end{aligned} \tag{13}$$

where, for simplicity we take $V=c$. Recasting this Lorentz equation in an Hamiltonian form reveals the underlying physics and considerably simplifies the analysis. We introduce the following variables:

$$Q=(\gamma-p), \quad \varphi=(t-z). \tag{14}$$

Consider a circularly polarized UHI long pulse, $\mathbf{A}(\varphi)$, and introduce the variable P, canonically conjugate to Q with respect to the time φ, then Eqs. (13) are simply Hamilton's equation associated with the Hamiltonian

$$H_0(P,Q,\varphi)=P^2+\frac{\omega_p^2}{4}\left(\frac{1+A^2(\varphi)}{Q}+Q\right). \tag{15}$$

This Hamiltonian can describe a one dimensional nonlinear oscillator driven by a time-dependent force. If the driving force vary on a time scale longer than ω_p, then the action, $I=\int PdQ$, of the free nonlinear oscillator is an adiabatic invariant. No exchange of energy and momentum takes place between the UHI pulse and the electron; moreover, as a consequence of this adiabatic invariance, no ponderomotive displacement takes place as the pulse passes by the electron. Thus, given the fact that the adiabatic invariance is guaranteed to exponential accuracy, with respect to the parameter $\partial A/\omega_p\partial\varphi$, in the long pulse regime the wake amplitude is insignificant except if a nonadiabatic process, such as an ionization, takes place.

When $\omega_p/\partial A/\partial\varphi$ becomes small, the adiabatic theory applied to the Hamiltonian, Eq. (15), no longer applies, and the ponderomotive potential gives rise to a displacement. The physical interpretation of this short pulse regime is straightforward: If $\omega_p<\delta\omega$, the pulse duration is shorter than the time needed for plasma electrons to set up a collective response, i.e., the motion inside the pulse is dominated by the response to the transverse wave. In this regime, studied in Ref. 4, the collective longitudinal response can be treated as a perturbation. The total ponderomotive displacement and the velocity change due to the interplay between the collective effect and the ponderomotive force are then

$$h=\frac{1}{2}\int_0^T A^2(u)du-\frac{1}{2}\omega_p^2\int_0^T ds\int_0^s du\int_0^u dv\,A^2(v)$$
$$\times[1+A^2(s)], \quad v=-\frac{1}{2}\omega_p^2\int_0^T ds\int_0^s A^2(v)dv. \tag{16}$$

These position and velocity coordinates now become the initial condition of the nonlinear oscillation behind the pulse, i.e., of the wake. Thus the wake can be completely characterized in terms of these two quantities, h and v. In fact, a short UHI pulse in a plasma can be viewed as a quasiparticle characterized by these two scalar quantities. Two pulses with different frequency, amplitude, and polarization will have the same effect on the plasma, provided the integrated quantities h and v are equal.

Based on this Lagrangian picture of wake generation, it becomes clear how one might devise a way to amplify the generation process. Consider Fig. 6, which depicts an electron interacting with a set of short pulses so that the cumulative effects of the pulse add up to induce a larger wake. This resonant effect takes place if the delay between two pulses is such that the pulses interact with the electron just at the maximum amplitude of the nonlinear oscillation. The delay, T_{N+1}, between the N and $N+1$ pulses is

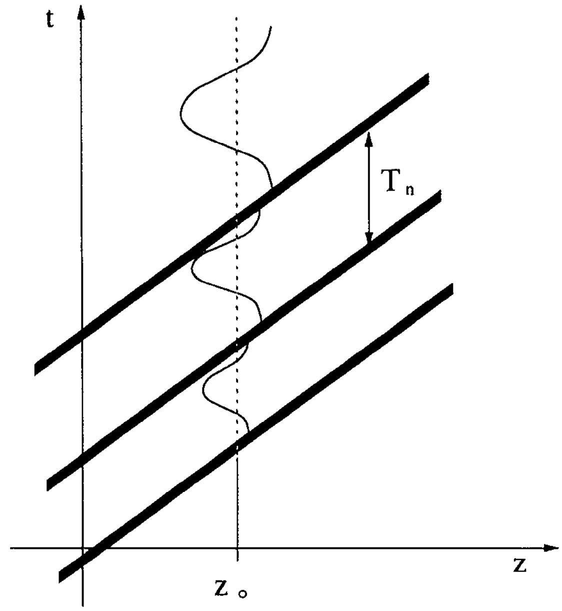

FIG. 6. Space-time orbit with a phased set of short UHI pulses.

$$\Omega_N T_{N+1}=4K(k_N)+\frac{\omega_p^2 h_N^2}{2}\int_0^{4K(k_N)} cn^2(u)du,$$
$$\Omega_N=\omega_p\sqrt{1+\frac{\omega_{p'}^2 h_N^2}{4}}, \quad k_N=\frac{\omega_p^2 h_N}{2\Omega_N}, \tag{17}$$

where K and cn are elliptic integral and function. This resonance condition depends on the Nth amplitude because the free oscillations are nonlinear. The displacement after the Nth pulse is given approximately by Nh, and the energy transfer to the wake scales as N^2. Note that if the same amount of energy were delivered to the plasma in the form of a long pulse, rather than a set of phased short pulses, the wake would be exponentially small. The proposed resonant scheme which can result in significant energy transfer, operates if the nonlinear plasma oscillations in the wake of the pulse remain coherent for at least several plasma periods.

V. HARMONIC GENERATION AND PHASE MATCHING

While long pulses do not generate significant wakes, long pulses are useful for generating harmonics. To study third harmonic generation, we use the slowly-varying envelope and phase approximation and consider a pump wave $\mathbf{A}(z,t)=A(z,t)\cos[t-z+\phi(t)]\mathbf{e}_x$, and a third harmonic wave $\mathbf{a}(z,t)=a(t)\cos[3(t-z)+\varphi(t)]\mathbf{e}_x$. The phase mismatch between the fundamental and the harmonic is given by $\theta(t)=\varphi(t)-3\phi(t)$. In an expansion that essentially orders the density small, to the lowest order in the plasma density, the figure 8 motion does not gives rise to

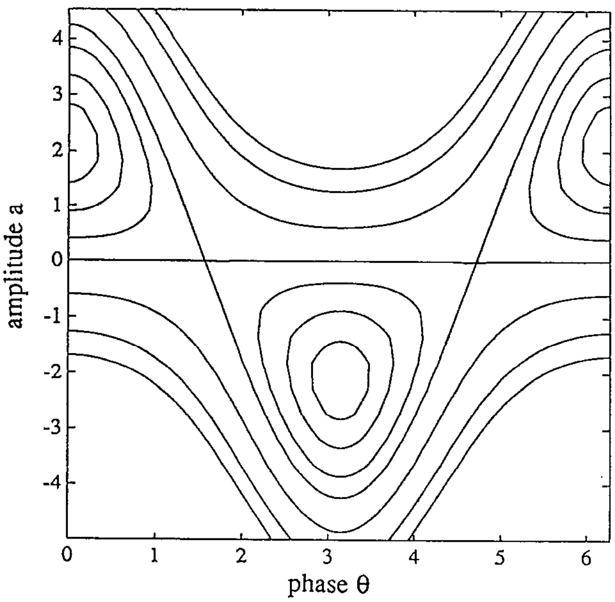

FIG. 7. Amplitude-phase orbit for harmonic generation.

harmonic generation, but, as shown in Ref. 9, to order density squared, a and θ obey the set of equations

$$\frac{da}{dt}=-\omega_p^4\frac{A^3}{64M^4}\sin(\theta),$$
$$\frac{d\theta}{dt}=-\frac{4\omega_p^2}{3M}-\omega_p^4\frac{A^3}{64M^4}\frac{\cos(\theta)}{a}. \tag{18}$$

The phase portrait of this dynamical system is depicted on Fig. 7 (where a/A is normalized to $3\omega^{2,p}a^2/538M^3$). The third harmonic wave describes the separatrix orbit and no phase locking occurs, so that, instead of growing linearly with time, the amplitude oscillates.[9,13] In the $eA/mc\approx 1$ regime, the detuning length is of the order of $(\omega/\omega_p)^2$ times the laser wavelength, and the power conversion saturates at the level $P_3/P_1\approx O[10^{-3}(\omega_p/\omega)^4]$. The reason for the absence of secular growth in the third harmonic can be understood as follows. From conservation of momentum in harmonic generation, we expect the wave vectors to fulfill

$$3k(\omega,A)=k(3\omega,A), \tag{19}$$

but, the nonlinear dispersion relation $k(\omega,A)$ is

$$k(\omega,A)=\omega-\omega_p^2/2\omega M. \tag{20}$$

Because in a plasma k is not linear in the frequency as it is in vacuum, Eq. (19) can not be fulfilled.

To generate the third harmonic one might, for example, use a third wave so that momentum is conserved. Such a phase-matching scheme works, provided that the frequency of the wave density modulation, Ω, is a multiple of $4\omega^{2,p}/3\omega M$. A more practical way to generate the third harmonic is to use a buffer gas which can slow down the fast pump wave more than it slows down the slow harmonic wave. It turns out that the circumstances are very favorable for such a scheme: On the one hand, free electrons in plasma give rise to a refraction index smaller than one, and, on the other hand, bound electrons in ions lead to a refraction index larger than one. The right mixture of free electrons and residual bound electrons can then be used as a phase matched media.

VI. CONCLUSIONS

We have analyzed UHI laser–plasma interaction within a Lagrangian framework. Equations (1) and (3) provide a self-consistent description of the UHI interaction below the Lagrangian overtaking threshold (which corresponds to the Eulerian wave-breaking threshold). However, even above this threshold the corresponding Lorentz equation can be derived, but with an additional coupling between neighboring electrons. This coupling gives rise to a degenerate eigenvalue in the linearized system, which signifies a secular solution linearly growing with time, corresponding to the production of fast electrons. Thus, even when the relativistic fluid description breaks down, the relativistic Lagrangian methods can still describe the nonlinear dynamics of the plasma.

In conclusion, the Lagrangian framework developed here turned out to be a very powerful tool for discovering and calculating both various longitudinal collective effects and the single-particle stochastic motion that occur in the new ultrahigh intensity regimes now reached in laser plasma interactions.

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[1] G. Mourou and D. Umstadter, Phys. Fluids. B **4**, 2315 (1992).
[2] P. Sprangle, E. Esarey, and A. Ting, Phys. Rev. Lett. **64**, 2011 (1990); Phys. Rev. A **41**, 4463 (1990); A. Ting, E. Esarey, and P. Sprangle, Phys. Fluids. B **2**, 1390 (1990); E. Esarey, P. Sprangle, J. Krall, A. Ting, and G. Joyce, Bull. Am. Phys. Soc. **37**, 1470 (1992).
[3] J. M. Rax, Phys. Fluids B **4**, 3962 (1992).
[4] J. M. Rax and N. J. Fisch, Phys. Fluids B **4**, 1323 (1992).
[5] J. M. Dawson, Phys. Rev. **113**, 383 (1958).
[6] R. C. Davidson and P. P. Schram, Nucl. Fusion **8**, 183 (1968).
[7] J. N. Bardsley, B. M. Penetrante, and M. H. Mittleman, Phys. Rev. A **40**, 3823 (1989).
[8] J. H. Eberly and A. Sleeper, Phys. Rev. **176**, 1570 (1968).
[9] T. W. Kibble, Phys. Rev. **138**, 740 (1965); **138**, 740 (1965); J. H. Eberly, *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1969), Vol. 7, p. 361.
[10] J. M. Rax and N. J. Fisch, Phys. Rev. Lett. **69**, 772 (1992).
[11] G. M. Zaslavski and B. V. Chirikov, Sov. Phys. Usp. **14**, 549 (1972); B. V. Chirikov, Phys. Rep. **52**, 263 (1979).
[12] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979); S. C. Wilks, J. M. Dawson, W. B. Mori, T. Katsouleas, and M. E. Jones, *ibid.* **62**, 2602 (1989).
[13] P. Sprangle and E. Esarey, Phys. Fluids. B **4**, 2241 (1992).