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FAST PARTICLE RESONANCES IN TOKAMAKS

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Abstract

Most of the recent studies on fast particles deal with the stabilization-destabilization of magnetohydrodynamics modes. Here we present another aspect of fast particle resonances in tokamaks and review some recent results on the dynamics of energetic electrons and alpha particles. The interaction between the runaway population and the magnetic ripple is analyzed. The occurrence of an induced radiative slowing down is demonstrated and its potential is assessed. Then, the resonant channeling of alpha particle free energy, to either fast electrons or fast ions, by means of lower hybrid waves, is investigated, and the efficiency of this process is analyzed.

1-INTRODUCTION

In Tokamaks, the control of the plasma is achieved by means of quasistatic magnetic field at the fluid level and radio-frequency electromagnetic fields at the kinetic level. Besides quasilinear heating [1], and current generation [2], which have been successfully demonstrated, ash removal, impurity control and enhanced fusion can also be performed, in principle, by means of waves. All these active processes rely on the occurrence of a resonant transfer of energy and momentum between an electromagnetic field and the electron or ion population. The aim of this paper is to present a comprehensive review of the various physical aspects of some new results on the problem of wave induced dynamics of energetic electrons and alpha particles in tokamaks.

Let us consider a wave characterized by its wave vector and pulsation $(k_\perp, k_\parallel, \omega)$, and a charged particle described by its momentum and energy $(p_\perp, p_\parallel, Mc^2\gamma)$. The parallel and perpendicular indexes refer to the direction of the magnetostatic field. Throughout this review we shall use m_e and e, the electron mass and charge, as the units of mass and charge, and c, the velocity of light, as the unit of velocity. Given a wave and a particle, a resonant wave-particle interaction takes place provided that the resonance condition,

$$\omega\gamma - \frac{k_\parallel p_\parallel}{M} = n\omega_c \tag{1}$$

is fulfilled (n is any integer and ω_c the cyclotron frequency of the particle). Besides these Landau and cyclotron resonances, in the short wavelength ($k_\perp p_\perp > \omega_c$), high frequency regime ($\omega > \omega_c$), the large n cyclotron resonances become a transient perpendicular Landau resonance if, [1,3]

$$\frac{k_\perp p_\perp}{M\gamma} \geq \omega - \frac{k_\parallel p_\parallel}{M\gamma} . \tag{2}$$

This condition simply indicates that there exists one (two) point(s) on the cyclotron orbit where the particle velocity matches the wave perpendicular phase velocity. When Eq. (1) or (2) is fulfilled, an efficient transfer of energy and momentum between the wave and the particles takes place. Moreover, if the quasilinear theory applies, this coupling results in phase space diffusion along the paths

$$\frac{dp_\perp}{dp_\parallel} = \frac{M\omega\gamma - k_\parallel p_\parallel}{k_\parallel p_\perp} . \tag{3}$$

To complement this momentum diffusion, and to ensure canonical momentum conservation, a quasilinear guiding center position $(x_\perp)$ diffusion appears in the direction perpendicular to the static magnetic field and the the wavevector. The associated diffusion path is

$$\frac{dx_\perp}{d\gamma} = \frac{k_\perp}{\omega\omega_c} . \tag{4}$$

When one addresses the issue of fast particle dynamics in tokamaks two specific processes are to be considered carefully. (i) As far as the relativistic electron population is concerned, the cyclotron frequency suffers a dramatic decrease, Eq. (1), more precisely the cyclotron frequency of a relativistic electron is ω_c[GHz]/2π =28.B[T]/γ.

The consequences of this relativistic downshift is the recent discovery that relativistic runaway particles interact with the magnetostatic toroidal ripple [4], and the lower hybrid waves [5]. (ii) As far as the energetic ion population is concerned, the relativistic factor γ has no impact on the dynamics, however, owing to the large value of the *Larmor* radius, the wave induced radial diffusion Eq. (4) becomes a dominant process.
When the perpendicular energy of a particle $E_\perp$ is changed by an amount $\Delta E_\perp$, the associated displacement can be as large as $\Delta x_\perp[\mathrm{cm}]/\Delta E_\perp[\mathrm{MeV}] = N_\perp/6.\ B[\mathrm{T}]$ ($N_\perp = k_\perp/\omega$ is the perpendicular index of the wave, and B the magnetic field). For example, a one MeV alpha particle which slows down as a result of its interaction with lower hybrid waves can experience a shift of the order of 10. cm. It has been recently recognized that this process provides an elegant and efficient way to release the free energy content of the inhomogeneous alpha particle population [6].

This paper is organized as follows. In the next section, we show that, owing to the relativistic downshift of the cyclotron frequency, the energetic runaway population is coupled to the magnetic ripple. Then, in section 3, we explain how the combination of the induced quasilinear pitch angle scattering with the spontaneous cyclotron emission gives rise to an energy blocking mechanism which might be useful as high energy runaway electrons are to be controlled for safer tokamak operations. Section 4 reviews the interaction of energetic alpha particles with intense lower hybrid waves. Section 5 explores the potentiality of free energy extraction with these waves. Section 6 concludes and discuss some future directions of research.

2-ENERGETIC RUNAWAY- MAGNETIC RIPPLE RESONANT INTERACTION

Usually tokamaks operate in the regime of weak or zero electric field. However, low density inductive operation at the beginning of the discharge or the occurrence of major disruption at the end of the discharge leads to the appearance of a small population of energetic runaway electrons [7]. The dynamics of this population is fairly simple, the energy gain is driven by the inductive electric field, while synchrotron radiation provides the major energy loss process. The balance between these driving and damping mechanisms gives rise to an ultimate energy upper bound:

$$\gamma_0 = \left(\frac{3RV_l}{4\pi r_e}\right)^{1/4}, \quad \gamma_0 = 167.\ [R[\mathrm{m}].V_l[\mathrm{v}]]^{1/4} \tag{5}$$

where $r_e = 2.8\ 10^{-15}$ m is the classical electron radius, V_l the loop voltage, and R the tokamak major radius. This limit is typically of the order of tens, up to few hundreds of MeV.

Because of the finite number of coils (N) the toroidal magnetic field is slightly modulated. This ripple has usually a negligible effect on the thermal particle orbits, but can induce a dramatic loss of fast ion when a resonance occurs, i.e. when some multiple of the bounce frequency matches some multiple of the precession frequency. Similarly, it has been recently recognized that, when some multiple of the transit frequency equals some multiple of the relativistic cyclotron frequency, the Fourier component of the ripple acts as a resonant perturbation on the unperturbed relativistic trajectories [4]. To investigate this process, we start with the Fourier decomposition of the transverse part (δB) of the ripple,

$$\delta B(r,\theta,\varphi) = \sum_{mn} \delta B_{mn}(r) \cos(m\theta) \cos(nN\varphi) \tag{6}$$

θ and φ are the poloidal and toroidal angles and r the label of the drift surface. The amplitude of these components decreases exponentially with the toroidal mode number (n), and, given the large shift of the runaway drift surfaces, to first order in the inverse aspect ratio, the m=0 and m=1 poloidal components have roughly the same order of magnitude. As a result of the relativistic downshift of the cyclotron frequency, before reaching γ_0, a relativistic runaway will resonate with these various harmonic of the tokamak ripple when the cyclotron frequency matches the periodicity of the ripple as seen in the electron rest frame. This resonance can be written as a condition on the energy:

$$\gamma_n = \frac{R\omega_c}{nN}, \qquad \gamma_n = 586. \frac{R[m] . B[T]}{nN} . \tag{7}$$

In order to study the impact of these resonances we have to perform an Hamiltonian analysis of the particles orbits. To do so we introduce a set of parallel and perpendicular actions: $P_\perp = p_\perp^2/2\gamma\omega_c$ and $P_\parallel = p_\parallel$ conjugate to the drift s along a magnetic field line and the gyroangle α. The relativistic Hamiltonian describing this quasi slab model is the sum of an unperturbed part (first term) plus the ripple perturbation (second term).

$$H(P_\perp,P_\parallel,s,\alpha) = \sqrt{1+P_\parallel^2 + 2\gamma\omega_c P_\perp} + \\ + \sum_{mn} \frac{Rq\delta B_{mn}}{2(nNq+m)} \sqrt{2\omega_c P_\perp/\gamma} \cos[(nNq+m)s/Rq \pm \alpha)] \tag{8}$$

(q is the safety factor). This ripple perturbation becomes resonant when the argument of the cosine becomes stationary with time ($s = ct$, $\alpha = \omega_c t/\gamma$) so that we recover the condition Eq. (7).

This Hamiltonian formulation of the runaway dynamics allows to calculate the width of each resonance and the distance between two neighboring resonances. When the ratio of these two quantities, the Chirikov parameter S,[8] becomes of the order of one, the particle motion becomes stochastic and quasilinear diffusion takes place.

$$S = 4nNq\sqrt{\frac{\delta B_{0n}}{2B}\frac{p_\perp}{p_\parallel}} = O[1] \tag{9}$$

The region of phase space ($P_\perp$, γ) where S becomes larger than one, S_1, S_2, S_3, are depicted on Figure -1. When the runaway orbit enters one of these instability zones the parallel kinetic energy is converted into perpendicular energy at a rate given by quasilinear theory.

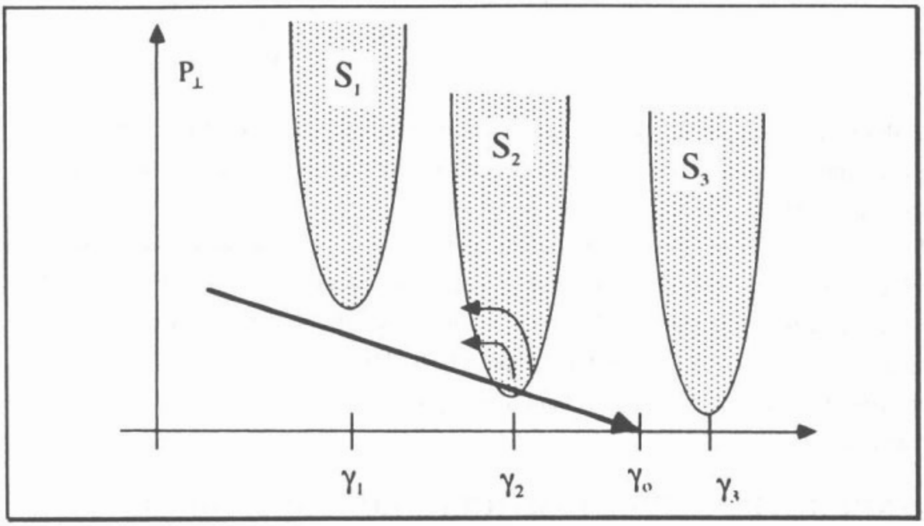

Figure -1- *When a runaway approaches the velocity of light and becomes relativistic it reaches the ultimate synchrotron limit γ_0, however, there exist some instability region* (S_1, S_2, S_3) *where a quasilinear conversion of the parallel energy into perpendicular one occurs, and a further cyclotron dissipation of this energy takes place, resulting in an anomalous energy spectra near γ_n (here γ_2).*

3-RUNAWAYS RESONANT SLOWING DOWN

The Chirikov criterion Eq. (9) ensures that stochastic quasilinear diffusion takes place, but, because of this perpendicular heating, spontaneous cyclotron losses appears. If this loss overcomes the inductive (parallel) energy gain, during the time needed to cross the resonance, the global effect of this resonance is to act as an energy blocking mechanism and the runaway population remains trapped in the vicinity of the ripple resonance Eq. (7).

Let us call τ_d the time needed to diffuse up to a pitch angle where cyclotron losses overcome the loop voltage free fall acceleration, and τ_r the time needed to cross a resonant stochastic region (S>1). Runaway slowing down and subsequent blocking takes place if $\tau_d < \tau_r$. The anomalous runaway energy spectra of Tore Supra during low field operation has been explained as a result of the n=2 resonance with the ripple. The following table reproduces the quantitative results, N=18, B=1.8T, q=2.0, V_l= 1.0v, R=2.4m.

n	E[MeV]	$\delta B_{0n}/B[10^{-2}]$	S	τ_d[ps]	τ_r[s]
1	74.7	1.8	3.0	36.0	0.1
2	37.3	0.03	0.77	$14.\ 10^4$	0.025

Similarly, an unsuspected pitch angle has been observed with an infrared camera on Textor, and seems also to be a consequence to the occurrence of a resonance of the relativistic runaway with the ripple [9].

This new process can have an interesting application if runaway electrons remain a problem for the next tokamak generation. We can envision to build a wiggler (permanent or energized by eddy currents during disruptions) creating an artificial ripple at the plasma edge, typical value in the range, $\delta B/B \approx 10^{-4}$, N≈100, are safe for bulk confinement, but give rise to resonant slowing down and blocking of the energetic runaways at a safer level.

4-ENERGETIC ALPHA PARTICLES-LOWER HYBRID WAVES RESONANT INTERACTION

Lower hybrid waves are a demonstrated mean of driving the toroidal currents in tokamaks. The theory of noninductive current drive is now well understood, and the experimental data base well established [2].

However, it has been pointed out, some time ago, that the lower hybrid wave might be absorbed by energetic alpha particles [10]. The condition for quasilinear absorption have been investigated in detail by Karney [3], and, above the stochasticity threshold, the interaction can be viewed as a perpendicular Landau resonance.

Besides this perpendicular Landau resonances, the energetic alpha particles collide primarily with electrons, so that the Fokker Planck description of the relaxation involves mainly a viscous damping. Finally, owing to the large value of the *Larmor* radius for MeV alpha particles, the drift term Eq.(4) is to be included in the kinetic equation describing the dynamics of this population. Consider a quasi-slab model, and assuming that the perpendicular index is purely poloidal, this kinetic equation for the energetic alpha particle distribution function F $(E_\perp,x_\perp)$ reads [11],

$$\frac{\partial F(E_\perp,x_\perp,t)}{\partial t} = \frac{\partial E_\perp F}{\partial E_\perp} + \left[\frac{\partial}{\partial E_\perp} \pm \frac{\partial}{\partial x_\perp}\right] D(E_\perp,x_\perp)\left[\frac{\partial}{\partial E_\perp} \pm \frac{\partial}{\partial x_\perp}\right] F + \frac{N_\alpha(x_\perp)}{\sqrt{E_\alpha - E_\perp}} \quad (10)$$

the time t is normalized to $0.09\, T_e^{3/2} M_\alpha/\sqrt{2\pi} n_e \ln[\Lambda]$, the radial guiding center position $x_\perp$ to $v_\alpha^2 N_\perp/2\omega_{c\alpha}$, and the perpendicular energy $E_\perp$ to $M_\alpha v_\alpha^2/2$ = 3.5MeV the birth energy of the alpha population, T_e and n_e are the electron temperature and density.

The first term on the right hand side describes collisional slowing down, the second the quasilinear interaction with lower hybrid waves, $D(E_\perp)=D\theta(E_\perp-E_{lh})(E_\perp-E_{lh})^{-1/2}$, θ is the Heaviside function and $D = 3T_e^{3/2}M\omega/E_{lh}^3 16\sqrt{2\pi}\, n_e \mathrm{Ln}\Lambda\, (V_o/V_\alpha)^2$ where V_o the quiver velocity of the alpha particles and $E_{lh}= E_\alpha/N_\perp^2$. The third term is the alpha particle source: $N_\alpha(x_\perp)\, dx_\perp$ is the total number of fusion events taking place at the point $x_\perp$ in the slice $dx_\perp$, this production rate is proportional to the (averaged) fusion cross section and the square of the density.

Starting from Eq. (10) various models can be considered to calculate the power transfer from the lower hybrid wave to the energetic alpha population $P_{LH\text{->}\alpha}$ [12]. First, for small amplitude lower hybrid waves (D->0), one can assume that collisional slowing down overcomes quasilinear diffusion, within this approximation we obtain the following scaling,

$$P_{LH\rightarrow\alpha} \approx D.\theta(N_\perp-23.). \quad (11)$$

However this weak field limit neglects completely the ability of lower hybrid waves to create a fast ion tail above the birth energy at 3.5 MeV. When this process is accounted for, the ($D \to \infty$) scaling of the transfer becomes [11]:

$$P_{LH\to\alpha} \approx D^{4/5}.\theta(N_{\perp}-23.)\,. \tag{12}$$

This later scaling is to be contrasted with the electronic one $P_{LH\to \text{electrons}} \approx D/K+D$ which displays a saturation in the intense field regime [13]. Nevertheless, both previous models neglect the drift term in Eq(10). The sign of this drift term can be either, positive or negative, depending of the poloidal orientation of the perpendicular wavevector. This gives rise to an additional absorption or to a destabilization because of the free energy contained in the spatial gradient of the alpha particles distribution. A simple linear calculation ($D \to 0$) with a slab model and an inhomogeneity characterized by a gradient length gives a standard formula of the type $P_{LH\to\alpha} \approx D.\,(1 \pm \omega^*/\omega).\,\theta(N_{\perp}-23.)$, but the drift frequency ω^* of the non thermal alpha particle distribution becomes of the order of the wave frequency when the gradient length becomes of the order of 10. cm. Thus, in this weak field linear regime, there is little hope to induce transparency or to extract free energy from the alpha particle population.

However, in the strong field regime ($D \to \infty$), when quasilinear diffusion is larger than collisional slowing down, it has been recently shown that lower hybrid waves can be used to release the alpha particle free energy [6].

5-ALPHA PARTICLES FREE ENERGY RELEASE

To study this power extraction process we consider a slab model and a finite beam of intense lower hybrid waves. The interaction of the alpha particles with the wave is restricted in space to $0 < x_{\perp} < A.$ and in energy to $E_{lh} < E_{\perp} < \infty$.

The dynamic behaviour of an alpha particle in phase space ($E_{\perp}$, $x_{\perp}$) is depicted on Figure -2. In order to quantify this strong field dynamic behaviour we introduce $G(E_{\perp},x_{\perp},t)$, the amount of normalized energy absorbed from the wave, within time t, by an alpha particle with coordinates ($E_{\perp},x_{\perp}$) at t=0. The total energy delivered to the wave, $E_{\alpha\to LH}(t)$, within time t is given as a sum over the source.

$$E_{\alpha\to LH}(t)=\frac{1}{2}\int_0^A dx_{\perp}\int_{E_{lh}}^{\infty} dE_{\perp}\int_0^t dt'\;\; G(E_{\perp},x_{\perp},t-t')\;\frac{N_{\alpha}(x_{\perp})}{\sqrt{E_{\alpha}-E_{\perp}}}\,. \tag{13}$$

The asymptotic form (D->∞) of the Green function G has been calculated in Ref. [6] and the final result writes,

$$G(E_\perp,x_\perp,t) = (x_\perp - A/2)\theta(P-A-E_{lh}) + (x_\perp - E_\perp + E_{lh})\theta(A+E_{lh}-P)/2 + \\ + \frac{1}{4}\frac{E_{lh}}{1-s}[-3t - \frac{4}{s}(e^{-t/2} - 1) + 2s(e^{t/2} - 1)], \quad (14)$$

where $P = x_\perp + E_\perp$, and $s = 2E_{lh}/(P+E_{lh})$. This energy transfer Green function is the sum of two terms, an instantaneous energy exchange: the first two terms on the right hand side of Eq. (14), and a subsequent collisional relaxation: the last term on the right hand side of Eq. (14). This analytical result has been recently checked numerically, and it turns out that the energy transfer Green function is not sensitive to the value of D over a wide range of magnitude, so that Eq. (14) provides an accurate basis to evaluate the potentiality of this method of releasing alpha particle free energy.

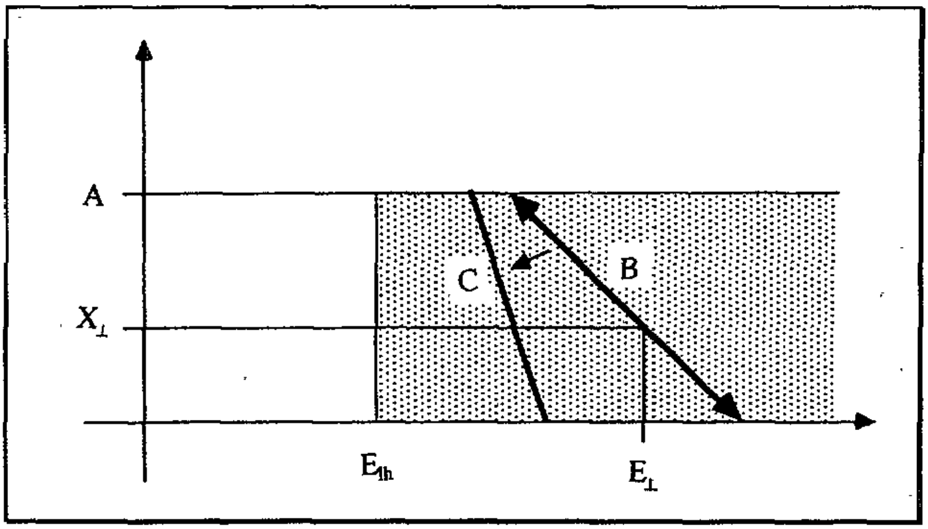

Figure -2- *The orbit of an energetic alpha is the combination of two processes. First the intense lower hybrid wave induces a diffusive spreading along the diffusion path B (Eq. (4)). Then collisions give rise to a rotation of this line and each element of the rotated line (C) is again spread along a diffusion path and so on... Ultimately the particle exits the resonant region (shaded area) and the difference between the exit energy and the initial energy provides the wave's energy gain or loss.*

Consider a reactor with typically 3. GW of fusion power, a fraction of 20% goes to the alpha population, if 10% of this alpha energy can be diverted to the wave, given the efficiency of lower hybrid current drive, this would provide just the amount needed to drive the noninductive current. This 10% figure turns out to be easily achievable provided one uses absorbing boundary conditions rather than reflecting ones on the boundary of the shaded area on Fig(2). The practical implementation of these boundary conditions remains an open question as the understanding of alpha particle transport remains far from complete [14]. Besides stimulated channeling of the alpha particle free energy to suprathermal electron population, it has also been proposed to use this free energy to boost the fusion reactivity as a result of ion tail heating [15]. If the previous energy, Eq. (13), is diverted to the fast D-T tail, the fusion reactions can amplify this energy release $E_{\alpha\rightarrow LH}$ by a factor of the order of 2.

Despite this small gain factor this new internal conversion and amplification scheme appears far more attractive than external power amplification proposed some times ago in Ref. [16].

6-CONCLUSION

Fast particle resonances in tokamaks is an active field of research, besides the stabilization-destabilization of magnetohydrodynamic modes, the control and extraction of the huge energy content of these suprathermal populations, by means of selected resonances, is the main challenge toward an advanced tokamak concept.

Our first study on relativistic runaway particles, and the identification of the new ripple resonance, was triggered by the observation of an anomalous energy spectra during low field operation in Tore Supra [17]. Direct observation of infrared synchrotron radiation on Textor [9] have also confirmed the existence of an unsuspected pitch angle. The synchrotron energy limit can be of the order of a few hundreds of MeV during disruption in the next tokamak generation, thus, for safer operation, a more efficient process than gas puffing is to be devised. A resonance with a weak (10^{-4}), short wavelength (N=100) magnetostatic modulation provides an interesting mechanism to control the energy content of energetic runaway particles.

Our second study on energetic alpha particles, and the identification of a mechanism to release the free energy of this population demonstrates that, *in situ* channeling of fusion power to fast electrons, to accomplish current drive, or to fast ions, to enhance fusion reactivity, is possible. It is possible, also, to envision both the enhanced reactivity and the current drive operating simultaneously, and to use an appropriate set of waves to optimize this process [18].

Such advanced tokamak concepts are now to be tested experimentally. However the present generation of large tokamaks cannot provide a sufficiently dense alpha particle population. Nevertheless, if the complete demonstration of free energy control and release can not be achieved in a straightforward manner, the proof of principle of the mechanism can be demonstrated with the help of fast ion beams.

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