Electron-ion collisions in intensely illuminated plasmas

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In the presence of a high-frequency intense uniform electric field, the collisions of electrons with ions can be made more frequent or less frequent, depending on the polarization of the hf field, the direction and magnitude of particle velocity, and the ratio of the plasma Debye length to the size of the electron oscillation in the hf field. The stimulated bremsstrahlung emission is calculated for both circularly and linearly polarized fields.
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I. INTRODUCTION

The presence of an electromagnetic wave alters electron-ion collisions, thereby also altering the rate of inverse bremsstrahlung.[1–6] The pioneering papers of Dawson and Oberman[1] and Silin[2] analyzed the linear[1] and nonlinear[2] high-frequency collisional resistivity of the plasma. Both models subscribed to the same physical picture—the wave dissipation arises from a Maxwellian distribution of electrons, oscillating in a spatially uniform electric field, and colliding with a random field of stationary ions. These two calculations were carried out using slightly different techniques for describing electron-ion collisions (Landau collisional integral in Ref. 2 versus explicit random-phase averaging over ion positions in Ref. 1). Decker *et al.*[3] recently extended Dawson's technique to large hf fields and confirmed the nonlinear results of Silin.

These calculations[1–3] all begin by assuming a Maxwellian distribution of the electrons, so that the average effect of the full distribution of electrons is calculated. This obscures possibly interesting effects that may arise in the collisions of individual electrons with ions in the presence of hf field. An attempt was made to describe a so-called ''correlated collision'' of an oscillating electron,[3] but a formal treatment of a collision of a test electron with an ion was not carried out. The more formal treatment here shows that these correlated collisions are, in fact, already treated in the calculation of Dawson and Oberman, modified to include finite hf fields. Some other effect would be required to explain the apparently enhanced energy exchange with the wave found in numerical simulations.[3]

We find at high laser intensities the appearance of a number of new effects, in addition to inverse bremsstrahlung, all due to the discrete nature of the plasma, and all of which require a much more detailed understanding of electron-ion binary collisions. The purpose of this paper is to analyze scattering, on an ensemble of Debye-shielded ions, of a single oscillating electron, drifting with arbitrary velocity. This enables us not only to calculate the rate of inverse bremsstrahlung for a Maxwellian plasma, but also the rate of inverse bremsstrahlung for an arbitrary distribution of electrons. This generality is important, because different populations of electrons, in fact, make contributions of different sign to the overall energy exchange with a wave.

The paper is organized as follows: In Sec. II we consider two simple examples of scattering of an oscillating electron on a single ion—when the hf field is linearly polarized in the direction of the slow drift of the oscillation center of the particle and when hf field is circularly polarized in the plane normal to the direction of the oscillation center drift. Correlated collisions, which arise because of the repeated interactions of an oscillating electron with the same ion, are quantified for these two exactly tractable cases. Qualitative insights obtained from these examples will be utilized in Sec. III, where we derive an averaged energy, exchanged between an oscillating electron and hf field, in the presence of a random ensemble of immobile ions. We find that the effect of correlated collisions is naturally included in the nonlinear Dawson-Oberman model, thus making an introduction of the *ad hoc* correlation coefficient (as suggested in Ref. 3) unnecessary. Section IV concludes and outlines the directions for future work.

II. COLLISIONS WITH A SINGLE ION

Consider the scattering on an infinitely heavy ion, in a charge state Z, of a single electron, oscillating in a dipole electric field

$$\vec{E}(\vec{r},t)=\vec{E}\sin\omega t \tag{1}$$

and drifting freely along $\hat{z}$ axis with velocity $\vec{v}=v\vec{e}_z$. The Coulomb field of the ion is considered a first-order perturbation to the zeroth-order motion of the electron, which consists of drift and quiver. This implies that the frequency of electron-ion collisions is much smaller than the frequency of hf field. This assumption is violated for slow electrons, which are subject to a reduced rate of inverse bremsstrahlung, as shown by Langdon.[4] However, here we are concerned mainly with the fast electrons at the tails of the distribution function, where the main deviations from Maxwellian occur. By considering a dipole electric field, the calculation simplifies considerably. In a uniform electric field, a canonical transformation to the oscillating frame exists, which makes the drift momentum of the electron and the position of its oscillating center canonical variables. In nonrelativistic dynamics, used throughout this paper, the infinite

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phase velocity of the wave (corresponding to uniform field) is invariant under Galilean transformations. Parenthetically, in relativistic dynamics, the calculations simplify by assuming that the phase velocity of the wave is precisely equal to the speed of light in a vacuum,[7] which similarly remains invariant under Lorentz transformations.

For an electromagnetic wave, propagating with phase velocity $v_{\rm ph}$, close to the speed of light in vacuum, the wave frequency is in the regime $\omega \gg \omega_p$, where $\omega_p = (4\pi n_e e^2/m)^{1/2}$ is the plasma frequency, and n_e, $-e$, and m are electron density, charge, and mass. The dipole field, Eq. (1), approximates an electromagnetic wave of finite wavelength, $\lambda_0 = 2\pi c/\omega$, when (i) $v_{\rm osc} \ll c$, so that the high-frequency electron motion is unaffected by the field inhomogeneity; and (ii) $v \ll c$, so that the Doppler shift of the frequency of the electromagnetic wave, caused by the drift motion of the electron, is negligible, thus avoiding wave-particle resonances.

In addition, much of our attention will be devoted to the regime where the wavelength of the hf wave λ_0 is much larger than the Debye length $\lambda_D = v_{th}/\omega_p$. In this regime, electron-ion collisions dominate over electron-electron collisions in dissipating the energy of an electromagnetic wave. This dominance arises because, to exchange energy, the net product $q\vec{v}\vec{E}$, summed over the colliding particles, must change in a collision. Because $\lambda_D = v_{th}/\omega_p$, the electric field is constant over a Debye length, i.e., for all collisions. Then, there is no energy dissipated in electron-electron collisions, since these collisions conserve current.

We consider small-angle collisions, introducing an artificial cutoff at impact parameter $\rho = b$, where b is the distance of closest approach, given by

$$b = \frac{2Ze^2}{mv^2}. \tag{2}$$

The validity of assuming that the collisions are small-angle for $\rho > b$ is discussed later in this section. The zeroth order electron trajectory is given by

$$\vec{r}_0(t) = \vec{\rho} + \vec{v}\cdot(t - t_j) - \vec{\epsilon}_0 \sin \omega t, \tag{3}$$

where

$$\vec{\epsilon}_0 = -\frac{e\vec{E}}{m\omega^2}, \tag{4}$$

and where t_j characterizes the electron phase with respect to hf field, and can be chosen such that $0 < \omega t_j < 2\pi$.

Consider now a small-angle scattering of an electron moving along its zeroth-order trajectory, given by Eq. (3), by a stationary Debye-shielded ion at the origin. The force acting on the electron, moving along its trajectory $\vec{r}_e(t)$, is given by

$$\vec{F}_{\rm ion}(t) = i4\pi Ze^2 \int_{-\infty}^{t} dt' \int \frac{d^3k}{(2\pi)^3} U(k) \times \epsilon(k, t - t')\vec{k} e^{i\vec{k}\cdot\vec{r}_e(t')}, \tag{5}$$

where $U(k)$ is a three-dimensional Fourier transform of the unshielded ion potential, and $\epsilon(k, t - t')$ is a time-dependent dielectric function of the plasma. In a reference frame of an oscillating electron, ions are oscillating with frequency ω. Hence, the ion field that the electron experiences, consists of both a dc component (Debye-shielded by other electrons), and ac components at harmonics of $\omega \gg \omega_p$ (unshielded). Hence, we choose $\epsilon(k, t - t')$ to be such that

$$\tilde{\epsilon}(k, \Omega = 0) = \frac{k^2}{k^2 + k_D^2}, \qquad \tilde{\epsilon}(k, \Omega = n\omega) = 1, \tag{6}$$

where

$$\tilde{\epsilon}(k, \Omega) = \int d\tau\, e^{i\Omega\tau} \epsilon(k, \tau),$$

and $k_D^2 = 4\pi e^2 n_e / m v_{th}^2$. To remove large-angle collisions, we soften the Coulomb potential at distances smaller than the distance of closest approach by choosing $U(k)$ as

$$U(k) = \left(\frac{1}{k^2} - \frac{1}{k^2 + k_{\rm max}^2}\right), \tag{7}$$

where $k_{\rm max} = 1/b$. This procedure for ''softening'' the diverging Coulomb potential for small distances was originally suggested, in the context of electron-ion collisions in a magnetized plasma, by Montgomery *et al.*[8] For the ideal plasma considered here, $\lambda_D \gg b$, so, effectively, for small-impact collisions $\tilde{\epsilon} \approx 1$.

Note that the precise procedures for ''softening'' the Coulomb potential at small distances and Debye shielding at large distances do not affect the final result. An equally accurate description of these effects can be achieved by, for example, limiting the integration domain in k-space to $|k| < k_{\rm max}$, using

$$U(k) = \frac{1}{k^2} H(k_{\rm max} - k), \tag{8}$$

where $H(x)$ is a Heaviside step function. The choice of the ion potentials in the forms (7), (8) in this section enables us to obtain the closed form solutions for two illustrative examples, analyzed below. In Sec. III a slightly different form of the ion potential will be used,

$$U(k) = \frac{1}{k}\sqrt{\left(\frac{1}{k^2} - \frac{1}{k^2 + k_{\rm max}^2}\right)}, \tag{9}$$

which, like Eqs. (7), (8), removes the large angle collisions.

To calculate the rate of electron-ion collisions, assume, without loss of generality, that $\vec{\rho} = \vec{e}_x \rho$, thus fixing x-z to be the collision plane, with direction z chosen to be along the instantaneous electron velocity. In the absence of the laser field, the x-component of the ion force could be integrated between $t = -\infty$ and $t = +\infty$, obtaining the deflection angle $\delta\theta = \delta v_x / v$ as a function of impact parameter. The rate at which the colliding electron gains transverse momentum leads, for elastic scattering, to a decrease of v_z by amount $\delta v_z = v\cdot(\delta\theta)^2/2$ to conserve energy. In the presence of hf field, however, electron-ion collisions are not necessarily elastic, so that calculating the deflection angle gives only a pitch-angle contribution to the scattering rate. Calculating $\delta\theta$ in this manner, however, allows us, among other things, to discuss the possibility of correlated collisions, introduced in Ref. 3.

Integrating Eq. (5) over time, and using Eqs. (3) and (7), then gives the deflection angle

$$\delta\theta = i\frac{4\pi Ze^2}{mv}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}U_n(k)k_xJ_n(\vec{k}\cdot\vec{\epsilon}_0)\times e^{i\vec{k}\cdot\vec{\rho}}\int_{-\infty}^{+\infty}dt\ e^{i(\vec{k}\cdot\vec{v}-n\omega)t}e^{-i\vec{k}\cdot\vec{v}t_j}, \tag{10}$$

where

$$U_n(k) = U(k)\tilde{\epsilon}(k,n\omega). \tag{11}$$

Performing first the time integration and introducing a spherical coordinate system, with

$$k_z = k\cos\theta,\quad k_x = k\sin\theta\cos\phi,\quad k_y = k\sin\theta\sin\phi, \tag{12}$$

simplifies Eq. (10) to

$$\delta\theta = i\frac{4\pi Ze^2}{mv^2}\sum_{n=-\infty}^{n=+\infty}e^{-in\omega t_j}\int_0^\infty\frac{k^2dk}{(2\pi)^3}U_n(k)\int_0^{2\pi}d\phi\times\int_{-1}^{1}d(\cos\theta)\sin\theta\cos\phi e^{ik\rho\sin\theta\cos\phi}\times J_n(\vec{k}\cdot\vec{\epsilon}_0)2\pi\delta(\cos\theta-n\omega/kv). \tag{13}$$

Note that only those Fourier components of the spatially inhomogeneous stationary field of the ion, which resonate with some harmonic of the electron quiver, contribute to particle deflection.

A. Oscillations along velocity

For a hf field polarized in the direction of particle motion, Eq. (13) reduces to

$$\delta\theta = -\frac{2Ze^2}{mv^2}\sum_{n=-\infty}^{n=+\infty}e^{-in\omega t_j}J_n(n\omega\epsilon_0/v)\int_{k_{0n}}^{\infty}dk\ kU_n(k)\times\chi_nJ_1(\chi_n\rho), \tag{14}$$

where $k_{0n}=n\omega/v$ and $\chi_n=\sqrt{k^2-k_{0n}^2}$. Using the integral identity

$$\int_0^\infty dx\frac{x^2}{x^2+k^2}J_1(ax) = kK_1(ak), \tag{15}$$

we obtain

$$\delta\theta = -\frac{b}{\rho}[k_D\rho K_1(k_D\rho)-\sqrt{k_D^2+k_{\max}^2}bK_1(\sqrt{k_D^2+k_{\max}^2}\rho)] - \frac{b}{\rho}\sum_{n=1}^{n=+\infty}2\cos(n\omega t_j)[k_{0n}\rho K_1(k_{0n}\rho) - q_n\rho K_1(q_n\rho)]J_n\left(\frac{n\omega\epsilon_0}{v}\right), \tag{16}$$

where

$$q_n = \sqrt{k_{0n}^2+k_{\max}^2},$$

and we made use of Eq. (11) in calculating the $U_n(k)$, resulting in the analytical expression (16).

In Eq. (16), the $n=0$ term is separated from the other terms to make obvious that only the $n=0$ term in the series is independent of t_j. In Sec. III we show that only $n\neq 0$ terms contribute to the energy exchange between the wave and the electron. Hence, we call the $n=0$ contribution to the scattering ''elastic.'' In the absence of the hf field, the $n=0$ term is the only term contributing to the scattering, which is only in pitch-angle. For $b\ll\rho\ll\lambda_D$, the elastic term is approximately equal to b/ρ, which reduces to the Rutherford formula. Note that the $n\geqslant 1$ terms are exponentially small in $n\omega\rho/v$. Thus for $\rho\gg v/n\omega$, there is little interaction, since the collision time is much longer than the wave period. For both elastic and inelastic contributions, the second term in square brackets in Eq. (16) is a consequence of the softening of the ion potential for $\rho\leqslant b$; it makes the infinite series of Eq. (16) converge.

The rate of pitch-angle scattering, ν_{ei}, is defined by

$$\frac{v}{\nu_{ei}}n_i\int 2\pi\rho\ d\rho\left\langle\frac{(\delta\theta)^2(\rho)}{2}\right\rangle = 1, \tag{17}$$

where n_i is the ion density, and where we have used the fact that the deflection angle $\delta\theta$ only depends on the magnitude of the impact parameter (something that is not true when the electric field is polarized at an angle to the particle velocity). Angular brackets in Eq. (17) denote averaging over the random particle phase ωt_j. Substituting Eq. (16) into Eq. (17), obtain

$$\nu_{ei} = \pi vb^2n_i\left\{\ln\Lambda_0+\sum_{n=1}^{n=+\infty}2J_n^2(n\omega\epsilon_0/v)\times\left[\left(\frac{1}{2}+\frac{n^2\omega^2}{k_{\max}^2v^2}\right)\ln\left(1+\frac{k_{\max}^2v^2}{n^2\omega^2}\right)-1\right]\right\}, \tag{18}$$

where $\Lambda_0=\ln(k_{\max}/k_D)$ is the Coulomb logarithm.[9]

In practice, the number of terms necessary to keep in the series (18) is finite, since the expression in square brackets in Eq. (18) becomes small for $n\omega b/v\gg 1$. Keeping only $n_{\max}=v/\omega b$ and assuming that $n\omega b/v<1$, one obtains

$$\nu_{ei}\approx\frac{A}{v^3}\left[\ln(k_{\max}\lambda_d)+\sum_{n=1}^{n=n_{\max}}2J_n^2(n\omega\epsilon_0/v)\ln(k_{\max}v/n\omega)\right], \tag{19}$$

where

$$A = \frac{4\pi Z^2e^4n_i}{m^2}. \tag{20}$$

For comparison, note that the classical expression for the rate of electron-ion collisions in the absence of an external hf electric field is

$$\nu_0(v) = \frac{A}{v^3}\ln\Lambda_0. \tag{21}$$

Comparing Eq. (19) to Eq. (21) shows that the $n=0$ (elastic) term is identical to scattering rate in the zero-field case.

The analytic solution presented here facilitates a consideration of the recent conjecture of correlated collisions.[3] Decker *et al.* conjecture that the nonlinear extension of the Dawson-Oberman model to $v_{\rm osc}>v$, developed in Ref. 3, may not adequately describe the subtle effect of an electron repeatedly returning back to the same ion in the course of its oscillation. They argue that the collision frequency, analytically obtained from the nonlinear Dawson-Oberman model, has to be multiplied by a numerical factor, roughly equal to the number of oscillations an electron makes as it crosses the Debye sphere of an ion. This numerical factor is found to be $C\approx\omega/\omega_p$.[3]

In comparison, Eq. (18) does not exhibit this large increase in collisional frequency by the factor C for the regime $v_{\rm osc}\gg v$. To see this immediately, take Eq. (18) in the limit $\omega\to\infty$, keeping the oscillation velocity $\epsilon_0\omega$ fixed, and find the zero field collision frequency, given by Eq. (21). Now this result is clearly expected; as the oscillation amplitude decreases, the particle spends exactly the same amount of time in the vicinity of any point along its trajectory as it would in the absence of the hf field. The qualitative explanation of Ref. 3 must fail here. While an oscillating electron may repeatedly pass by the same ion (as noted in Ref. 3), the amount of time it spends in the vicinity of this ion is similarly reduced. Clearly, there is no need to introduce additional numerical factors to understand the role of correlated collisions in the case presented here. Note that, in the case of very high oscillation velocity $v_{\rm osc}\gg v$, simply replacing the particle velocity v by $v_{\rm osc}$ in Eq. (21) leads to a much smaller collision frequency than predicted by Eq. (19). Here, in comparison to this replacement, the collision frequency does appear to be enhanced because collisions are correlated. Indeed, while the electron speed is equal to $v_{\rm osc}$, it does return back to the same ion during the next cycle of oscillation.

For an electron drifting in the direction of the electric field with velocity approximately equal to the velocity of its oscillation, the largest increase in the collisional frequency

$$\nu_{ei}>\nu_0(v) \tag{22}$$

is expected, because of the stagnation points along the electron trajectory, where its total speed is close to zero. Near those points an electron spends more time near the ion, thereby increasing the mean squared scattering angle $\delta\theta$. This effect is accounted for in the present formalism. The effect of the stagnation points along the electron trajectory becomes most pronounced when $v_{\rm osc}\approx v$. Consequently, Eq. (19) indicates that the net increase in the collision frequency is maximized for $v_{\rm osc}\approx v$ [since Bessel functions $J_n^2(nv_{\rm osc}/v)$ achieve their maxima at $v_{\rm osc}/v\approx 1$].

Note that the conclusion that the collision frequency increases in the presence of external field (22) is valid only when an electron travels in the direction of the hf field. In fact, as we show in the next example, a significant reduction in the collision frequency can be expected for an electron traveling perpendicularly to the hf field

The validity of the assumption of small-angle scattering for electron-ion distances larger than b can be checked. In the absence of external field, the deflection angle $\delta\theta=b/\rho$. Requiring $\delta\theta<1$ implies $\rho>b$, justifying the smoothing of the Coulomb potential for distances smaller than b. In the presence of external field, an estimate of the $\delta\theta$ is obtained by using Eq. (16). The contribution of the inelastic terms to angular deflection depends on the relative phase between the electron and the hf field. The largest angular deflection, for fixed impact parameter ρ, is experienced by electrons with $t_j=0$ and $v_{\rm osc}/v\approx 1$. The latter condition is derived from the fact the first zero of the Bessel function of order n is always larger than n, so all the terms in the series add up with positive signs. The contribution of the inelastic terms then dominates over the elastic term for small distances, resulting in

$$\delta\theta\propto-\frac{b}{k_0\rho^2}. \tag{23}$$

Hence, the cutoff for small-angle scattering is actually at the larger distance $\rho_{\rm cut}\approx\sqrt{b/k_0}$.

This increase in scattering can be traced to correlated collisions, since the electron stays longer in the vicinity of an ion. In this paper, for simplicity, this refinement of the cutoff is neglected, because: (i) for most electrons $t_j\neq 0$, hence different orders of inelastic contribution may interfere destructively; (ii) we are mainly concerned here with strongly illuminated plasmas, with $v_{\rm osc}\gg v$, where the contribution of the inelastic terms is smaller than that of the elastic term.

Note that here only the rates of pitch-angle scatterings are calculated, which are sufficient here for describing momentum transfer. In Sec. III the rates of energy-exchanging collisions are studied. These collisions, which produce bremsstrahlung or inverse bremsstrahlung, are described entirely by inelastic terms.

B. Circularly polarized wave

Consider now the scattering on a single ion of an electron, which oscillates in the field of a circularly polarized (CP) EM wave. For simplicity, the plane of polarization is chosen perpendicular to the direction of the electron motion. The zeroth order electron trajectory is then given by

$$\vec{r}_0(t)=\vec{\rho}+\vec{v}\cdot(t-t_j)-\epsilon_0(\vec{e}_x\sin\omega t+\vec{e}_y\cos\omega t). \tag{24}$$

A calculation similar to that leading to Eq. (14) yields

$$\delta\theta=\frac{2Ze^2}{mv^2}\sum_{n=-\infty}^{n=+\infty}e^{-in(\omega t_j-\pi/2)}\frac{\partial}{\partial\rho}\int_{k_{0n}}^{\infty}dk\,kU_n(k)\times J_n(\chi_n\epsilon_0)J_n(\chi_n\rho), \tag{25}$$

which can be simplified to give

$$\delta\theta=-bG_0(\rho,\epsilon_0)+b\sum_{n=1}^{n=+\infty}2\cos(n\omega t_j-n\pi/2)G_n(\rho,\epsilon_0), \tag{26}$$

where

$$\begin{aligned}G_0&=k_DI_0(k_D\epsilon_0)K_1(k_D\rho)\quad\text{if }\rho>\epsilon_0\\&=k_DK_0(k_D\epsilon_0)I_1(k_D\rho)\quad\text{if }\rho<\epsilon_0,\end{aligned} \tag{27}$$

and

$$G_n = k_{0n} I_n(k_{0n}\epsilon_0) K_n'(k_{0n}\rho) - q_n I_n(q_n\epsilon_0) K_n'(q_n\rho) \quad \text{if} \ \rho > \epsilon_0$$
$$= k_{0n} K_n(k_{0n}\epsilon_0) I_n'(k_{0n}\rho) - q_n K_n(q_n\epsilon_0) I_n'(q_n\rho) \quad \text{if} \ \rho < \epsilon_0. \tag{28}$$

In deriving Eq. (27), for $\epsilon_0 \gg b$, it was not necessary to soften the Debye-shielded Coulomb potential at small distances. Note that for the amplitude of the electron oscillation much smaller than the Debye length, Eq. (27) simplifies to

$$G_0 = k_D K_1(k_D\rho) \quad \text{if} \ \rho > \epsilon_0$$
$$= k_D^2 \rho \ \ln(k_D\epsilon_0) \quad \text{if} \ \rho < \epsilon_0. \tag{29}$$

As Eq. (29) indicates, for small oscillation amplitudes $\epsilon_0 < \rho$, the elastic contribution to the electron deflection for the CP field is the same as in the previous example of the linearly polarized (LP) field. However, when the impact parameter is smaller than the orbit size, Eq. (29) indicates that the electron deflection may drop significantly if $\epsilon_0 \ll \lambda_D$. Physically, this is because, as soon as the electron's orbit is large enough to cross the $y-z$ plane, ion kicks in positive and negative x-directions destructively interfere with each other. In fact, for the *unshielded* Coulomb potential, the cancellation is *exact*. The amount of uncanceled scattering is proportional to the degree of Debye shielding, characterized by k_D. Hence, if the orbit size is much smaller than the Debye length, the electron-ion interaction can be neglected for impact parameters smaller than the orbit size. For an electron drifting perpendicularly to the polarization plane of a circularly polarized field, the dc component of an ion field (as experienced by an electron) would not lead to large-angle scattering for any impact parameter. There is then no need to smooth the Coulomb potential at small distances; a natural cutoff, arising at $\rho = \epsilon_0$, replaces the usual small-angle scattering cutoff at $\rho = b$. As a result, $\ln(\lambda_D/\epsilon_0)$ replaces the Coulomb logarithm.

However, the ac component of an ion field (as experienced by an electron) can lead to large-angle scattering, so the validity of the small-angle approximation for distances larger than b has to be examined. Using Eqs. (27) and (28) and assuming $t_j = \pi/2$, and $v_{\text{osc}} \ll v$, yields

$$\delta\theta \approx -b/(\rho - \epsilon_0),$$

for $\rho \approx \epsilon_0$. Physically, this clearly means that the deflection is largest when an electron orbit almost touches the ion, since $(\rho - \epsilon_0)$ is the distance of an electron orbit from the ion. Hence, smoothing the ion potential at distances smaller than b indeed removes large-angle collisions, as could be expected at weak illuminations. Using Eqs. (27) and (28), one can similarly show that at intense illuminations $v_{\text{osc}} \gg v$ small-angle scattering is also ensured, since

$$\delta\theta \approx -\frac{v}{v_{\text{osc}}} \frac{b}{\rho - \epsilon_0}.$$

The electron-ion collision frequency can then be written as

$$\nu_{ei} = \frac{A}{v^3} \left[\Lambda_{\text{el}} + \sum_{n=1}^{n=\infty} \Lambda_{\text{inel}}^{(n)} \right], \tag{30}$$

where

$$\Lambda_{\text{el}} = \frac{(k_D\epsilon_0)^2}{2} I_0^2(k_D\epsilon_0)[-K_1^2(k_D\epsilon_0) + K_0(k_D\epsilon_0)K_2(k_D\epsilon_0)] - \frac{(k_D\epsilon_0)^2}{2} K_0^2(k_D\epsilon_0)[-I_1^2(k_D\epsilon_0) + I_0(k_D\epsilon_0)I_2(k_D\epsilon_0)], \tag{31}$$

and

$$\Lambda_{\text{inel}}^{(n)} = \frac{v}{\omega\epsilon_0} \left(\frac{1}{n} - \frac{1}{\sqrt{n^2 + k_{\max}^2 v^2/\omega^2}} \right). \tag{32}$$

In evaluating $\Lambda_{\text{inel}}^{(n)}$, we assumed that $v_{\text{osc}} \gg v$. For $\epsilon_0 \ll \lambda_D$, $\Lambda_{\text{el}} \approx \ln(\lambda_D/\epsilon_0)$; in the opposite limit, $\epsilon_0 \gg \lambda_D$, Λ_{el} can be shown to scale as $\Lambda_{\text{el}} \approx (2k_D\epsilon_0)^{-1}$. This indicates that the collision frequency drops by a significant factor as $k_D\epsilon_0$ approaches unity. Apparently, as the size of the orbit exceeds the Debye distance, the individual electron-ion collisions become less efficient, because electrons spend most of their time away from the ion, where the ion field is Debye-shielded by other electrons.

Assuming $k_0\epsilon_0 > k_D\epsilon_0 \gg 1$ and using the large-argument expansion for the Bessel functions, one finds

$$\nu_{\text{pitch}}(\epsilon_0) = \frac{A}{v^3} \left(\frac{1}{k_D\epsilon_0} + \frac{1}{k_0\epsilon_0} \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{\sqrt{k_{\max}^2/k_0^2 + n^2}} \right). \tag{33}$$

This series is clearly convergent, since, for $n \gg n_{\max}$, each term of the series scales as $1/n^3$. In practice, the infinite series (33) can be terminated at $n = n_{\max}$. Using

$$\sum_{n=1}^{n_{\max}} \frac{1}{n} \approx \ln n_{\max}$$

for $n_{\max} \gg 1$, we obtain

$$\nu_{\text{pitch}}(\epsilon_0) \approx \frac{A}{v^3} \left[\frac{1}{k_D\epsilon_0} + \frac{1}{k_0\epsilon_0} \ln \Lambda_1 \right], \tag{34}$$

where $\Lambda_1 = k_{\max} v/\omega$. For k_0/k_D of order a few, and $\ln(k_{\max} v/\omega) \gg 1$, the last term will dominate.

The nonlinear reduction in the collision rate is roughly given by

$$Q = \frac{\nu_{\text{pitch}}(\epsilon_0 = 0)}{\nu_{\text{pitch}}(\epsilon_0)} \approx k_0\epsilon_0 \frac{\ln(k_{\max}/k_D)}{\ln(k_{\max}/k_0)}. \tag{35}$$

Note that the reduction in the collision frequency is not due simply to an increase in the velocity v to the oscillation velocity v_{osc}.

III. NONLINEAR INVERSE BREMSSTRAHLUNG

This section presents a calculation of the average energy exchanged between an oscillating electron and the hf field, in the presence of a random ensemble of immobile ions. The collisional damping of a hf wave, also known as inverse bremsstrahlung, originally studied in Ref. 1 for weak fields, was later revisited by a number of authors who have included nonlinear[2,3,10] and quantum-mechanical[11] effects. Using a different approach, the nonlinear results of Ref. 3 for a linearly polarized hf field, are confirmed. In addition, the nonlinear inverse bremsstrahlung of a circularly polarized wave is obtained.

The energy exchange between the electron and the EM wave is estimated to second order in (Ze^2) by integrating the rate of energy exchange $-e\vec{E}\cdot\vec{v}$ along the perturbed electron trajectory. The energy exchange obtained is then averaged over a random distribution of ions. Since the presented calculation considers the interaction of an electron with a single ion exactly, the effects of the correlated collisions are naturally included. Having obtained the rate of energy exchange for a particle with a given momentum, the overall rate of inverse bremsstrahlung can then be obtained by integrating, over the electron momentum distribution, the rate of energy exchange for particles with a given momentum. For a Maxwellian distribution, the results of Ref. 3 are recovered. On the other hand, for an anisotropic distribution, the inverse bremsstrahlung is quite different: A hf wave can be amplified by the particles which primarily move parallel to the wave polarization.

Defining a vector-potential of the hf wave as

$$\vec{E}(t)=\frac{d\vec{A}_0}{dt}, \tag{36}$$

the total energy exchange between the wave and the electron can be calculated as

$$\delta E=-e\int_{-\infty}^{+\infty}dt\ \vec{v}\cdot\vec{E}$$

$$=\frac{e}{m}\int_{-\infty}^{+\infty}dt\ \vec{F}_{\mathrm{ion}}(\vec{r}_e)\cdot\vec{A}_0, \tag{37}$$

where the last step involved integration by parts. Assuming that the EM field is linearly polarized, as given by Eq. (1), the vector potential $\vec{A}_0$ can be expressed as

$$\vec{A}_0=\frac{m\omega}{e}\vec{\epsilon}_0\cos\omega t. \tag{38}$$

In Eq. (37), $\vec{r}_e(t)$ is the location of the electron at time t. The lowest order nonvanishing contribution to inverse bremsstrahlung can be obtained by evaluating the integral in Eq. (37) along the perturbed electron trajectory: $\vec{r}_e(t)=\vec{r}_0(t)+\vec{r}_1(t)$. Hence, Eq. (37) can be recast in the form

$$\delta E=\frac{e}{m}\int_{-\infty}^{+\infty}dt(\vec{r}_1\cdot\nabla)\vec{F}_{\mathrm{ion}}\cdot\vec{A}_0, \tag{39}$$

where the first-order perturbation to the electron trajectory is obtained by integrating

$$\ddot{\vec{r}}_1=\frac{i4\pi Ze^2}{m}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}U(k)\vec{k}J_n(\vec{k}\cdot\vec{\epsilon}_0)\times e^{i\vec{k}\cdot\vec{\rho}}e^{-i\vec{k}\cdot\vec{v}t_j}e^{i(\vec{k}\cdot\vec{v}-n\omega)t}. \tag{40}$$

Note that, to obtain Eqs. (38) and (39), we used the spatial homogeneity of the hf wave, thereby neglecting any possible wave-particle resonances. Hence, Eq. (39) contains no direct wave-particle energy exchange, apart from the collisional exchange.

Integrating Eq. (40) over time, yields

$$\vec{r}_1=\Re\frac{-4\pi iZe^2}{m}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}U_n(k)\vec{k}J_n(\vec{k}\cdot\vec{\epsilon}_0)\times e^{i\vec{k}\cdot\vec{\rho}}e^{-i\vec{k}\cdot\vec{v}t_j}\frac{e^{i(\vec{k}\cdot\vec{v}-n\omega)t}}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}, \tag{41}$$

where ε is an infinitesimal positive number introduced to ensure causality. Rewriting the ion force as

$$\vec{F}_{\mathrm{ion}}(t)=-i4\pi Ze^2\int_{-\infty}^{t}dt'\int_{-\infty}^{+\infty}\frac{d^3k}{(2\pi)^3}U(k)\epsilon(k,t-t')\times\vec{k}e^{-i\vec{k}\cdot\vec{r}_0(t')}, \tag{42}$$

and substituting Eq. (42) into Eq. (39), results in

$$\delta E=-\Im\frac{(4\pi Ze^2)^2\omega}{m}\sum_{n=-\infty}^{n=+\infty}\sum_{m=-\infty}^{m=+\infty}\int\frac{d^3k}{(2\pi)^3}\times U_n(k)\int\frac{d^3k_1}{(2\pi)^3}U_n(k_1)\frac{J_n(\vec{k}\cdot\vec{\epsilon}_0)J_m(\vec{k}_1\cdot\vec{\epsilon}_0)}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}\vec{k}_1\cdot\vec{\epsilon}_0e^{i(\vec{k}_1-\vec{k})\cdot\vec{v}t_j}e^{-i(\vec{k}_1-\vec{k})\cdot\vec{\rho}}\int_{-\infty}^{+\infty}dt\ e^{-i(\vec{k}_1-\vec{k})\cdot\vec{v}t}\times e^{-i(n-m)\omega t}\cos\omega t. \tag{43}$$

Only the terms with $m=n\pm1$ survive the averaging over random t_j, which reduces Eq. (43) to

$$\delta E=-\Im\frac{(4\pi Ze^2)^2\omega}{mv}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}\times U_n(k)\int\frac{d^3k_1}{(2\pi)^3}U_n(k_1)\frac{nJ_n(\vec{k}\cdot\vec{\epsilon}_0)J_n(\vec{k}_1\cdot\vec{\epsilon}_0)}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}\times e^{-i(\vec{k}_1-\vec{k})\cdot\vec{\rho}}2\pi\delta(k_{1z}-k_z). \tag{44}$$

The rate of energy exchange, averaged over collisions with many ions, is given by

$$\frac{dE}{dt}=vn_i\int d^2\rho\,\delta E(\vec{\rho}). \tag{45}$$

Substituting δE from Eq. (44) into Eq. (45) yields

$$\frac{dE}{dt}=-\Im\frac{(4\pi Ze^2)^2n_i\omega}{m}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}\times U_n^2(k)k^2\frac{nJ_n^2(\vec{k}\cdot\vec{\epsilon}_0)}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}. \tag{46}$$

As Eq. (46) indicates, the $n=0$ term vanishes since in the absence of external field a collision of an electron with an immobile ion is elastic. Following a very similar procedure, the rate of collisional energy exchange between an electron and a circularly polarized wave can be derived. If $\vec{e}_\perp$ is the normal to the plane of polarization, the rate of bremsstrahlung is given by

$$\frac{dE^{\text{circ}}}{dt}=-\Im\frac{(4\pi Ze^2)^2 n_i\omega}{m}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}\times U_n^2(k)k^2\frac{nJ_n^2(|\vec{k}\times\vec{e}_\perp|\epsilon_0)}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}. \tag{47}$$

To compare the rate of bremsstrahlung obtained here with the findings of Refs. 1 and 3, the expression (46) is integrated over a Maxwellian distribution in $\vec{v}$, giving

$$\frac{d\langle E\rangle}{dt}=-\Im\frac{(4\pi Ze^2)^2 n_i\omega}{2m}\sum_{n=-\infty}^{n=+\infty}\int d^3v\; f_0(\vec{v})\int\frac{d^3k}{(2\pi)^3}\times U_n^2(k)k^2\frac{nJ_n^2(\vec{k}\cdot\vec{\epsilon}_0)}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}, \tag{48}$$

where

$$f_0(\vec{v})=\frac{1}{(2\pi v_{th}^2)^{3/2}}\;\exp(-v^2/2v_{th}^2). \tag{49}$$

Using

$$\frac{1}{(\vec{k}\cdot\vec{v}-n\omega-i\varepsilon)^2}=-\frac{\vec{k}}{k^2}\cdot\frac{\partial}{\partial\vec{v}}\frac{1}{\vec{k}\cdot\vec{v}-n\omega-i\varepsilon}=-\frac{\vec{v}}{v^2}\cdot\frac{\partial}{\partial\vec{k}}\frac{1}{\vec{k}\cdot\vec{v}-n\omega-i\varepsilon} \tag{50}$$

and integrating Eq. (48) by parts, yields

$$\frac{d\langle E\rangle}{dt}=-\Im\frac{(4\pi Ze^2)^2 n_i\omega}{m}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}U_n^2(k)\times nJ_n^2(\vec{k}\cdot\vec{\epsilon}_0)\int d^3v\frac{\vec{k}\cdot\partial/\partial\vec{v}f_0(v)}{\vec{k}\cdot\vec{v}-n\omega-i\varepsilon}. \tag{51}$$

Since $\omega\gg\omega_p$, we used $\tilde{\epsilon}(k,n\omega)=1$. Recognizing that the imaginary part of the integral over d^3v is proportional to the imaginary part of the plasma dielectric function,

$$D(\vec{k},\omega)=1-\frac{\omega_p^2}{k^2}\int d^3v\frac{\vec{k}\cdot\partial/\partial\vec{v}f_0(v)}{\vec{k}\cdot\vec{v}-n\omega-i\varepsilon},$$

and noting that in the high-frequency limit $|D|\approx 1$, so that $\Im(1/D)\approx-\Im D$, we find that Eq. (51) is identical to Eq. (20) of Ref. 3. To complete the comparison with Ref. 3, we note that the rate of inverse bremsstrahlung, given by Eq. (51), is derived as a sum of individual energy exchanges with distinct ions. This shows that the assertion (cf. Ref. 3) that the effect of correlated collisions is not contained in Eq. (20) of Ref. 3 is not correct.

Equations (46) and (47) are simplified by noting that

$$\Im\frac{1}{z-i\varepsilon}=\pi\delta(z),$$

yielding

$$\frac{dE}{dt}=\frac{Am\omega}{v^2}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{2\pi}nJ_n^2(\vec{k}\cdot\vec{\epsilon}_0)k^2U_n^2(k)\vec{v}\cdot\frac{\partial}{\partial\vec{k}}\delta(\vec{k}\cdot\vec{v}-n\omega) \tag{52}$$

for a linearly polarized field, and

$$\frac{dE}{dt}=\frac{Am\omega}{v^2}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{2\pi}nJ_n^2(|\vec{k}\times\vec{e}_\perp|\epsilon_0)k^2U_n^2(k)\vec{v}\cdot\frac{\partial}{\partial\vec{k}}\delta(\vec{k}\cdot\vec{v}-n\omega) \tag{53}$$

for a circularly polarized field.

The significance of Eqs. (52) and (53) is that they predict the rate of inverse bremsstrahlung for an *individual* electron, not the averaged quantity for the entire Maxwellian distribution. The rate of energy exchange, given by Eqs. (52) and (53), can be averaged over any distribution function of interest, isotropic or anisotropic . In addition, wave-particle resonance can be used to target electron sub-populations of choice.

To illustrate the usage of Eq. (52), consider the linear $(n=1)$ inverse bremsstrahlung. For the purpose of this calculation, choose $U(k)$ in the form given by Eq. (8). Expanding $J_1(\vec{k}\cdot\vec{\epsilon}_0)\approx(\vec{k}\cdot\vec{\epsilon}_0)/2$ and using Gauss' theorem, Eq. (52) can be integrated by parts. The boundaries of integration are chosen to be surfaces $|\vec{k}|=0$ and $|\vec{k}|=k_{\max}$. In order for a particle to exchange energy with the wave, its velocity should satisfy

$$v>\frac{\omega}{k_{\max}}. \tag{54}$$

Without loss of generality, assume that hf wave is polarized in $x-z$ plane, making an angle θ_1 with the particle velocity. The angle θ_1 satisfies

$$\cos^2\,\theta_1=\frac{(\vec{\epsilon}_0\cdot\vec{v})^2}{\epsilon_0^2 v^2}. \tag{55}$$

Thus separating the total energy exchange, given by Eq. (52), into the boundary contribution dE^b/dt at the surface $|\vec{k}|=k_{\max}$ and the volume contribution dE^v/dt, yields

$$\frac{dE^v}{dt}=\frac{A}{v^3}m\omega^2\left[\epsilon_0^2-\frac{3(\vec{\epsilon}_0\cdot\vec{v})^2}{v^2}\right]\int_{\omega/v}^{k_{\max}}\frac{dk}{k}\left(1-\frac{\omega^2}{k^2v^2}\right), \tag{56}$$

and

$$\frac{dE^b}{dt}=\frac{A}{v^3}m\omega^2\left[\frac{(\vec{\epsilon}_0\cdot\vec{v})^2}{v^2}+\frac{1}{2}\left(\epsilon_0^2-\frac{3(\vec{\epsilon}_0\cdot\vec{v})^2}{v^2}\right)\times\left(1-\frac{\omega^2}{k_{\max}^2v^2}\right)\right]. \tag{57}$$

Note that the combination

$$\left(\epsilon_0^2-\frac{3(\vec{\epsilon}_0\cdot\vec{v})^2}{v^2}\right)$$

averages out to zero for any isotropic velocity distribution. Hence, because of what remains of the sum of the boundary and volume contributions, the wave is damped for any isotropic distribution function. On the other hand, an anisotropic electron momentum distribution can lead to wave growth. For instance, it follows from Eq. (56) that a stream of particles, moving with constant velocity v along the direction of an EM wave, gives up energy to the wave,

$$\frac{dE}{dt} \approx -\frac{A \ln \Lambda_1}{4v^3} \frac{mv_{\mathrm{osc}}^2}{2}, \tag{58}$$

while a stream of particles moving perpendicularly to the wave absorbs the energy of the wave

$$\frac{dE}{dt} \approx \frac{A \ln \Lambda_1}{2v^3} \frac{mv_{\mathrm{osc}}^2}{2}. \tag{59}$$

In the absence of electron-ion collisions, the hf electric field is $\pi/2$ out of phase with the electron velocity, so no energy is exchanged between the electrons and the wave. When electron-ion collisions are included, the average phase lag can be larger or smaller than $\pi/2$, depending on the orientation of the instantaneous direction of the electron drift with respect to the direction of the wave polarization. Note that many electron-ion collisions are assumed to take place before the direction of the electron drift changes due to pitch-angle scattering.

To calculate the bremsstrahlung friction coefficient for an electron drifting perpendicularly to the plane of a circularly polarized hf wave, assume a large-amplitude hf field such that $v_{\mathrm{osc}} \gg v$, and use $U(k)$ in the form given by Eq. (9). Then Eq. (53) simplifies to

$$\frac{dE}{dt} \approx \frac{A}{v v_{\mathrm{osc}}^2} \frac{mv_{\mathrm{osc}}^2}{2} \frac{v}{v_{\mathrm{osc}}} \sum_{n=1}^{n=\infty} \left(\frac{1}{n} - \frac{1}{n(1+k_{\max}^2 v^2/n^2\omega^2)^{3/2}} \right). \tag{60}$$

Again, the convergent series (60) can be estimated by truncating it at $n_{\max}$. Introducing the rate of nonlinear bremsstrahlung $\nu_{\mathrm{brem}}(\epsilon_0)$ through the identity

$$\frac{dE}{dt} = -\nu_{\mathrm{brem}} v_{\mathrm{osc}}^2/4, \tag{61}$$

obtain

$$\nu_{\mathrm{brem}}(\epsilon_0) \approx -\frac{2A}{v_{\mathrm{osc}}^3} \ln \Lambda_1. \tag{62}$$

Note that, for this particular example of an electron drifting normally to the polarization plane of a CP wave, the rate of nonlinear bremsstrahlung is *negative* (inverse bremsstrahlung). It would be positive, for example, for an electron drifting along the polarization direction of a LP wave.

We now compare the two collision rates for strongly illuminated plasmas. In Sec. II B we derived Eq. (34) for the rate of nonlinear pitch-angle scattering of an electron, drifting perpendicularly to the polarization plane of a CP field. Comparing Eq. (62) to Eq. (34), we find that in a strongly illuminated plasmas, where $v_{\mathrm{osc}} \gg v$, the rate of pitch-angle scattering may exceed that of the bremsstrahlung, namely,

$$\frac{\nu_{\mathrm{pitch}}(\epsilon_0)}{\nu_{\mathrm{brem}}(\epsilon_0)} \approx \frac{v_{\mathrm{osc}}^2}{2v^2}. \tag{63}$$

This means that electron-ion collisions may have a small effect on the high-frequency motion of electrons in a high-intensity wave, but may significantly affect the slow electron drifts.

IV. CONCLUSIONS

In summary, damping of a high-frequency, spatially homogeneous electric field by electron-ion collisions in a plasma has been calculated. First, the energy exchange of an individual electron with the wave is computed in the presence of a single ion. Second, the energy exchange is averaged over random ion positions. Third, the energy exchange averaged over an electron distribution is calculated. This procedure unambiguously accounts for correlated collisions, i.e., the effect of an oscillating electron repeatedly returning to the same ion during a binary collision. The correlated collisions are naturally included in the nonlinear extension of Dawson-Oberman model.[3]

The average force, exerted on an oscillating electron by a single ion, has also been calculated. The nonlinear rate of electron-ion pitch-angle scattering has been evaluated for two simple cases (i) an electron in a linearly polarized hf wave drifting parallel to the polarization direction and (ii) an electron in a circularly polarized hf wave drifting perpendicularly to the polarization plane. In the first case electrons become *nonlinearly opaque*, i.e., they slow down faster. This effect become large when the oscillation velocity becomes comparable with the thermal velocity. In contrast, in the second case, electrons become *nonlinearly transparent*.

It is conjectured that if only selected groups of particles are strongly perturbed by an EM wave, the nonlinearly-induced transparency (or opacity) effect might be used to modify the electron distribution function. Selecting, for example, electrons traveling in a particular direction might be achieved through Landau resonance, or through a cyclotron resonance in magnetized plasmas. To calculate accurately this effect requires extending our analysis to EM waves with nonvanishing wave number. To estimate the magnitude of the effect, assume a circularly polarized EM wave, propagating in the positive z-direction, with phase velocity $v_{\mathrm{ph}} \approx c \gg v_{th}$. In a plasma of density $n_e = 10^{19}$ cm^{-3} and temperature $T_e = 100$ eV, an electromagnetic wave with wavelength $\lambda = 1\mu$ and intensity $I > 5 \cdot 10^{14}$ W/cm^2 might then drive current densities of order $J \approx 40$ MA/cm^2.

Finally, note that, while electron-electron collisions do not contribute to the dissipation of long-wavelength electromagnetic waves, they do contribute to pitch-angle scattering of the electrons. The inclusion of electron-electron collisions, however, appears to be relatively straightforward using the methods presented here, since they are not affected by the presence of a uniform hf field.

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